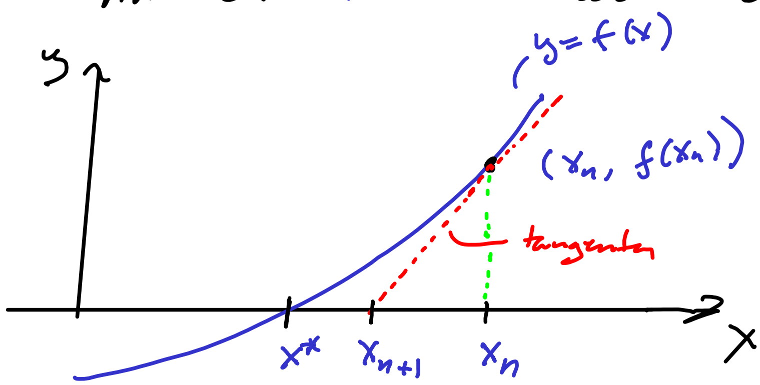
Info om Matematik II

Adress till Adobe Connect: connect.sunet.se/m0043m_ht15

Första träff: Må 2/11 kl. 13.00

5a

Newtons metod finner approximationer till ett x^* som löser ekvationen $f(x) = 0$.



Tangentens ekv.

$$y - f(x_n) = f'(x_n)(x - x_n)$$

Tangenten skär x-axeln i y=0

$$0 - f(x_n) = f'(x_n)(x - x_n)$$

$$-\frac{f(x_n)}{f'(x_n)} = x - x_n$$

$$x = x_n - \frac{f(x_n)}{f'(x_n)}$$

dvs

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$$

5b

$$\cos 3x = x$$

$$\cos 3x - x = 0$$

Låt

$$f(x) = \cos 3x - x$$

$$f'(x) = -3\sin 3x - 1$$

$x_0 = 1$

$x_1 = -0{,}2981$

$x_2 = -0{,}8259$

$x_3 = -0{,}8710$

$x_4 = -0{,}8856$

$x_5 = -0{,}8877$

$x_0 = -1$

$x_1 =$

$\vdots$

$x_0 = 0.5$

$x_1 =$

$\vdots$

Newtonsteget

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{\cos 3x_n - x_n}{-3\sin 3x_n - 1}$$

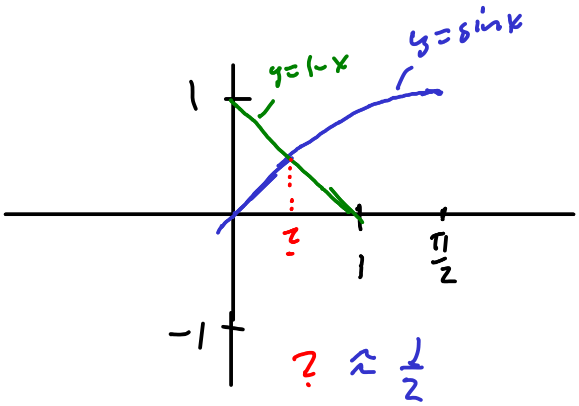
6a

$$f(x) = \sin x + x - 1$$

Metod 1:

$$\sin x + x - 1 = 0$$

$$\sin x = 1 - x$$



$? \approx \frac{1}{2}$

$x_0 = \frac{1}{2}$

Metod 2:

$\sin x \approx x$ då x är nära 0.

ger

$$x + x - 1 = 0$$

$$2x = 1$$

$$x = \frac{1}{2}$$

dvs låt $x_0 = \frac{1}{2}$.

6b

$$f(x) = \sin x + x - 1$$

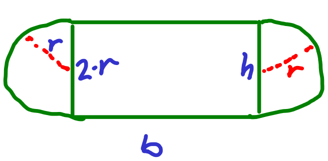
$$f'(x) = \cos x + 1$$

ger

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{\sin x_n + x_n - 1}{\cos x_n + 1}$$

$x_0 = 0.5$
$x_1 = 0.51096$
$x_2 = 0.51097$
$x_3 = 0.51097$

6.3



Radie - halvcirkel	r	[feet]
Bredd	b	⋮
Höjd	h	⋮

$$h = 2 \cdot r$$

Banlängd	L	[feet]
Rektangelarea	A	[feet^2]

$$L = 2 \cdot b + \pi \cdot h$$

$$A = b \cdot h$$

Vi vet $L = 1320$

ger $1320 = 2b + \pi \cdot h$

$$2b = 1320 - \pi \cdot h$$

$$b = 660 - \frac{\pi \cdot h}{2}$$

ger Arean

$$A(h) = b \cdot h = \left(660 - \frac{\pi}{2} \cdot h\right) h$$

$$= 660h - \frac{\pi}{2} h^2$$

Stationära punkter

$$A'(h) = 660 - \pi \cdot h$$

$A'(h) = 0$ $\quad$ $660 - \pi \cdot h = 0$

$$660 = \pi \cdot h$$

$$\boxed{h = \frac{660}{\pi}}$$

$0 \le h$, $\quad h \le b$

$$h \le 660 - \frac{\pi}{2} \cdot h$$

$$h + \frac{\pi}{2} \cdot h \le 660$$

$$\left(1 + \frac{\pi}{2}\right) h \le 660$$

$$h \le \frac{660}{1 + \frac{\pi}{2}} = \frac{1320}{2 + \pi}$$

$$\boxed{0 \le h \le \frac{1320}{2+\pi}}$$

$A'(h)$ existerar för alla h

Ändpunkter $\quad h = 0$ och $h = \frac{1320}{2+\pi}$

h	$A(h)$
0	0
$\frac{660}{\pi}$	69328
$\frac{1320}{2+\pi}$	65910

max

Svar:

Maximal area då $\quad h = \frac{660}{\pi} \approx 210$

$$b = 330 = 660 - \frac{\pi}{2} \cdot \frac{660}{\pi}$$

$\boxed{4}$

$$f(x) = x(x-1)^2 = x(x^2 - 2x + 1) = x^3 - 2x^2 + x$$

Inga asymptoter

Växande/avtagande

$$f'(x) = 1(x-1)^2 + x \, 2(x-1) = (x-1)(x - 1 + 2x)$$

$$= (x-1)(3x-1)$$

		$1/3$		1	
$f'(x)$	$+$	0	$-$	0	$+$
$f(x)$	$\nearrow$	lok. max	$\searrow$	lok. min	$\nearrow$

$$f\left(\tfrac{1}{3}\right) = \tfrac{1}{3}\left(-\tfrac{2}{3}\right)^2 = \tfrac{4}{27} \qquad f(1) = 0$$

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$$f'(x) = (x-1)(3x-1)$$

$$f''(x) = 1\cdot(3x-1) + (x-1)\cdot 3 = 3x-1+3x-3$$

$$= 6x-4 = 2(3x-2)$$

$$f''(x) = 0 \qquad x = \tfrac{2}{3}$$

		$2/3$	
$f''(x)$	$-$	0	$+$
$f(x)$	$\frown$	infl.	$\smile$

$$f\left(\tfrac{2}{3}\right) = \tfrac{2}{3}\left(-\tfrac{1}{3}\right)^2 = \tfrac{2}{27}$$