Tentamen 141029

$$\lim_{x\to 0}\frac{\sin x}{x}=1$$

$\boxed{1a}$

$$\lim_{x\to 0}\frac{\sin 3x}{\tan 7x}=\left[\frac{0}{0}\right]=\lim_{x\to 0}\frac{\sin 3x}{\frac{\sin 7x}{\cos 7x}}=\lim_{x\to 0}\cos 7x\cdot\frac{\sin 3x}{\sin 7x}$$

$$=\lim_{x\to 0}\cos 7x\,\frac{3\cancel{x}\cdot\frac{\sin 3x}{3x}}{7\cancel{x}\cdot\frac{\sin 7x}{7x}}=\lim_{x\to 0}\underbrace{\cos 7x}_{\to 1}\,\frac{3\overbrace{\frac{\sin 3x}{3x}}^{\to 1}}{7\cdot\underbrace{\frac{\sin 7x}{7x}}_{\to 1}}=1\cdot\frac{3\cdot 1}{7\cdot 1}=\frac{3}{7}$$

$\boxed{1b}$

$$\sqrt{x^2}=|x|$$

$$\lim_{x\to -\infty}\left(\sqrt{x^2-x+2}+(x+1)\right)=[\infty-\infty]=\lim_{x\to-\infty}\frac{\left(\sqrt{x^2-x+2}+(x+1)\right)\left(\sqrt{x^2-x+2}-(x+1)\right)}{\sqrt{x^2-x+2}-(x+1)}$$

$$=\lim_{x\to-\infty}\frac{x^2-x+2-(x+1)^2}{\sqrt{x^2\left(1-\frac{1}{x}+\frac{2}{x^2}\right)}-x\left(1+\frac{1}{x}\right)}=\lim_{x\to-\infty}\frac{\cancel{x^2}-x+2-\cancel{x^2}-2x-1}{\underbrace{|x|}_{=-x \text{ då } x<0}\sqrt{1-\frac{1}{x}+\frac{2}{x^2}}-x\left(1+\frac{1}{x}\right)}$$

$$=\lim_{x\to-\infty}\frac{-3x+1}{-x\sqrt{1-\frac{1}{x}+\frac{2}{x^2}}-x\left(1+\frac{1}{x}\right)}=\lim_{x\to-\infty}\frac{\cancel{x}\left(-3+\overbrace{\tfrac{1}{x}}^{\to 0}\right)}{\cancel{x}\left(-\sqrt{1-\underbrace{\tfrac{1}{x}}_{\to 0}+\underbrace{\tfrac{2}{x^2}}_{\to 0}}-\left(1+\underbrace{\tfrac{1}{x}}_{\to 0}\right)\right)}$$

$$=\frac{-3+0}{-\sqrt{1-0+0}-(1+0)}=\frac{-3}{-2}=\frac{3}{2}$$

(c)

$$\lim_{x\to 3}\frac{-x^2+x+6}{2x^2-5x-3}=\left[\frac{0}{0}\right]=\lim_{x\to 3}\frac{\cancel{(x-3)}(-x-2)}{\cancel{(x-3)}(2x+1)}=\frac{-3-2}{2\cdot 3+1}=\frac{-5}{7}=-\frac{5}{7}$$

$\boxed{2}$

(a) $\sqrt{x-2} = x-4 \quad (*)$

Kvadrera

$x-2 = (x-4)^2$

$x-2 = x^2-8x+16$

$x^2-9x+18=0$

$x = \frac{9}{2} \pm \sqrt{\frac{81}{4}-18} = \frac{9}{2} \pm \sqrt{\frac{81}{4}-\frac{72}{4}}$

$= \frac{9}{2} \pm \sqrt{\frac{9}{4}} = \frac{9}{2} \pm \frac{3}{2}$

$x = \frac{12}{2} = 6$ eller $x = \frac{6}{2} = 3$

i $(*)$ ger

$\sqrt{6-2} = 6-4$

$2 = 2$

ok!

$\sqrt{3-2} = 3-4$

$1 = -1$

Falskt

$x=3$ ej lösning

Svar: $x = 6$

(b) $\frac{x+1}{x} < 2$

$\frac{x+1}{x} - 2 < 0$

$\frac{x+1-2x}{x} < 0$

$\frac{1-x}{x} < 0$

		0		1	
$1-x$	$+$		$+$	0	$-$
x	$-$	0	$+$		$+$
$\frac{1-x}{x}$	$-$	ej. def.	$+$	0	$-$

dvs

$x < 0$ eller $x > 1$

(c) $9^x - 6\cdot 3^x = 7$

$(3^2)^x - 6\cdot 3^x = 7$

$3^{2\cdot x} - 6\cdot 3^x = 7$

$(3^x)^2 - 6\cdot 3^x = 7$

Variabelbyte $t = 3^x$

$t^2 - 6\cdot t = 7$

$t^2 - 6t - 7 = 0$

$t = 3 \pm \sqrt{9+7} = 3 \pm \sqrt{16} = 3 \pm 4$

$t = -1$ eller $t = 7$

$3^x = -1$

lösning saknas

$3^x = 7$

$\ln 3^x = \ln 7$

$x \cdot \ln 3 = \ln 7$

Svar: $x = \frac{\ln 7}{\ln 3}$

$\boxed{3}$ $x^3 - y^3 - x\cdot y - x - 2 = 0$

derivera m.a.p x givet att $y = y(x)$

$$3x^2 - 3y^2\cdot y' - 1\cdot y - xy' - 1 - 0 = 0$$

sätt in $(x,y) = (1,-1)$

$$3\cdot 1^2 - 3(-1)^2\cdot y' - 1\cdot(-1) - 1\cdot y' - 1 = 0$$

$$3 - 3\cdot y' + \cancel{1} - y' - \cancel{1} = 0$$

$$3 - 4y' = 0$$

$$3 = 4y'$$

$y' = \frac{3}{4}$ i $(x,y) = (1,-1)$

Tangenten

$$y - (-1) = \frac{3}{4}(x-1)$$

$$y = \frac{3}{4}x - \frac{3}{4} - 1 = \frac{3}{4}x - \frac{7}{4}$$

Normalen $n = -\frac{1}{3/4} = -\frac{4}{3}$ vinkelrätt

$$y - (-1) = -\frac{4}{3}(x-1)$$

$$y = -\frac{4}{3}x + \frac{4}{3} - 1 = -\frac{4}{3}x + \frac{1}{3}$$

$\boxed{4}$ $y = f^{-1}(x)$

$$f(y) = x$$

$y^5 + 2y^3 + y = x$ $\quad x = 4, \quad y = 1$

Derivera m.a.p. x givet $y = y(x)$

$$5y^4\cdot y' + 2\cdot 3y^2\cdot y' + y' = 1$$

sätt in $(x,y) = (4,1)$

$$5\cdot 1^4\cdot y' + 6\cdot 1^2\cdot y' + y' = 1$$

$$12y' = 1$$

$y' = \frac{1}{12}$ då $(x,y) = (4,1)$ dvs. $(f^{-1})'(4) = \frac{1}{12}$

$\boxed{5}$ (a) $\cos 2x = 3\sin x + 2$

$\cos 2x = \cos^2 x - \sin^2 x$

$\cos 2x = 2\cos^2 x - 1$

$\rightarrow \cos 2x = 1 - 2\sin^2 x$

$1 - 2\sin^2 x = 3\sin x + 2$

Variabelbyte: $t = \sin x$

$1 - 2\cdot t^2 = 3\cdot t + 2$

$2t^2 + 3t + 1 = 0$

$t^2 + \frac{3}{2}t + \frac{1}{2} = 0$

$t = -\frac{3}{4} \pm \sqrt{\frac{9}{16} - \underbrace{\frac{1}{2}}_{8/16}} = -\frac{3}{4} \pm \sqrt{\frac{1}{16}}$

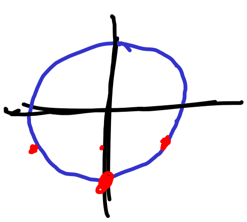
$= -\frac{3}{4} \pm \frac{1}{4}$

$t = -1$ eller $t = -\frac{1}{2}$

$\sin x = -1$ $\sin x = -\frac{1}{2}$

$x = -\frac{\pi}{2} + n2\pi$ eller $x = -\frac{\pi}{6} + n2\pi$

eller



$x = -\frac{5\pi}{6} + n2\pi$

(b)

Ökningen av $f(t)$ är förändringshastigheten av $f(t)$, som ges av $f'(t)$

Om $f'(t)$ är strängt avtagande är dess derivata

$f''(t)$ negativ.

$\boxed{6.1}$ (a) $\quad f(x) = \dfrac{x^2}{(x+1)^2}$

Asymptoter

$$\lim_{x\to\infty} \frac{x^2}{(x+1)^2} = \lim_{x\to\infty} \frac{x^2}{(x(1+\frac{1}{x}))^2} = \lim_{x\to\infty} \frac{\cancel{x^2}\,1}{\cancel{x^2}(1+\underbrace{\tfrac{1}{x}}_{\to 0})^2} = \frac{1}{(1+0)^2} = 1$$

$$\lim_{x\to-\infty} \frac{x^2}{(x+1)^2} = \text{P.S.S} = 1$$

$\boxed{\text{Vågrät asymptot i } y=1}$

Nämnaren blir 0 i $x=-1$ så

$\boxed{\text{Lodrät asymptot i } x=-1}$ med egenskapen

$$\lim_{x\to-1+} \frac{x^2}{(x+1)^2} = \left[\frac{(-1)^2}{(0+)^2}\right] = \infty$$

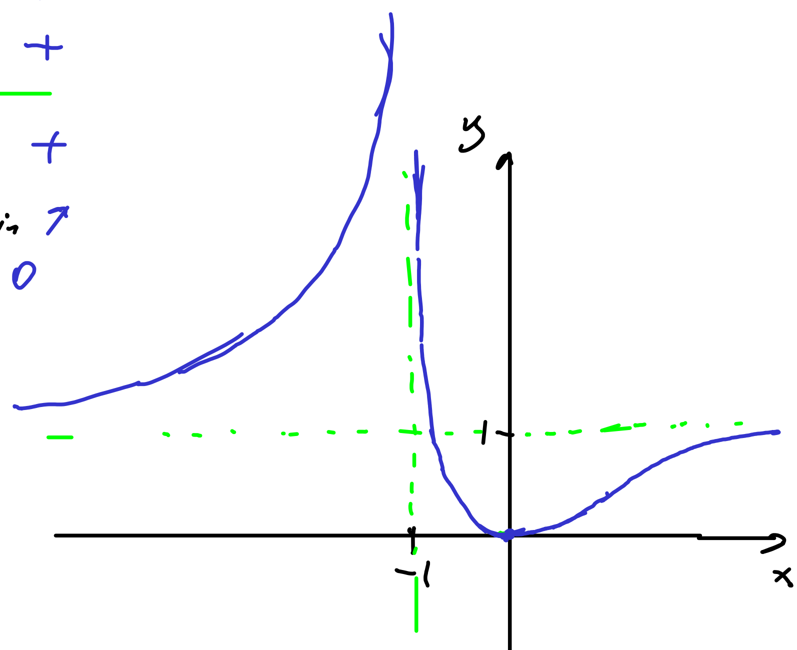
$$\lim_{x\to-1-} \frac{x^2}{(x+1)^2} = \left[\frac{(-1)^2}{(0-)^2}\right] = \infty$$

Lokala extrempunkter

$f(x) = \dfrac{x^2}{(x+1)^2}$

$$f'(x) = \frac{2x(x+1)^{\cancel{2}\,1} - x^2\,2\cancel{(x+1)}\cdot 1}{(x+1)^{\cancel{4}\,3}} = \frac{\cancel{2x^2}+2x-\cancel{2x^2}}{(x+1)^3} = \frac{2x}{(x+1)^3}$$

		-1		0	
$2x$	$-$		$-$	0	$+$
$(x+1)^3$	$-$	0	$+$		$+$
$f'(x)$	$+$	ej def.	$-$	0	$+$
$f(x)$	$\nearrow$	|	$\searrow$	lok min $f(0)=0$	$\nearrow$



$\boxed{6.2}$

v – hastighet [km/h] $\quad 60 \leq v \leq 90$

$K(v)$ – Kostnaden [kr] att köra sträckan med konstant hastighet v.

$$K(v) = \Big(\underbrace{201}_{\text{Chaufför/h}} + \underbrace{12\cdot\left(2+\frac{v^2}{300}\right)}_{\text{Bränslekostnad/h}}\Big)\underbrace{\frac{300}{v}}_{\text{restid}}$$

$$= \left(225 + 12\,\frac{v^2}{300}\right)\frac{300}{v}$$

$$= \frac{67500}{v} + 12v$$

$$K'(v) = -\frac{67500}{v^2} + 12 = \frac{-67500 + 12v^2}{v^2}$$

$K'(v) = 0 \quad v = (\pm)\sqrt{\frac{67500}{12}} = \sqrt{5625} = 75$ km/h

v	$K(v)$
60	1845
75	1800
90	1830

← Min