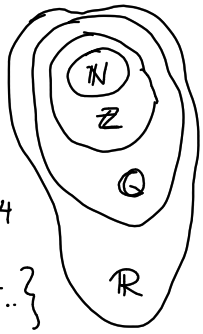
Föreläsning 1

- $\mathbb{N}$ Naturliga tal : $\{0,1,2,\dots\}$
- $\mathbb{Z}$ Heltal $\{\dots,-2,-1,0,1,2\dots\}$

$\mathbb{Q}$ Rationella tal - kvot av heltal $\{\dots \frac{1}{3}, -\frac{187}{14}, \dots 4\}$

$\mathbb{R}-\mathbb{Q}$ Irrationella tal $\{\dots \sqrt{2}, e, \dots \pi \dots\}$



$\mathbb{R}$ Reella tal

$\mathbb{Z}_2$: Binära tal $\{0,1\}$

$\mathbb{C}$ Komplexa tal

. . . . . .

$x \in \mathbb{R}$ (tillhör)

$y = \sqrt{5}$ $y \notin \mathbb{Q}$

. . . . .

Olikhetstecken :

$x < y$ x är mindre än y

$x \leq y$ x är mindre än eller lika med y.

$y \geqslant 0$ y är positivt

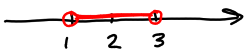
$z < 0$ z är negativt

. . . . . .

$x \in \mathbb{R}$: $1 < x < 3$

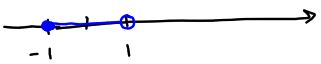
$\Leftrightarrow$ $x \in]1,3[$.

$x \in (1,3)$



$y \in \mathbb{R}$: $-1 \leq y < 1$

$\Leftrightarrow$ $y \in [-1,1[$ $\Leftrightarrow$ $y \in [-1,1)$



$z \in \mathbb{R}$: $z \geqslant 4$ $\Leftrightarrow$ $z \in [4,\infty[$

$w \in \mathbb{R} \Leftrightarrow w \in]-\infty,\infty[$

$\{y \in \mathbb{Z} : -1 \leq y < 2\} = \{-1,0,1\}$

Kvadreringsregler mm.

$x \cdot (y+z) = x \cdot y + x \cdot z$

$(x+y) \cdot (z+w) = x \cdot z + x \cdot w + y \cdot z + y \cdot w$

$(x+y)^2 = (x+y) \cdot (x+y) = x \cdot x + x \cdot y + y \cdot x + y \cdot y$

$= x^2 + 2xy + y^2$

ex) $x^2 + 4x + 4 = x^2 + 2 \cdot 2 \cdot x + 2^2 = (x+2)^2$

$(x-y)^2 = (x-y)(x-y) = \ldots$

$= x^2 - 2 \cdot x \cdot y + y^2$

$(x+y)^3 = (x+y)(x+y)(x+y) =$

$(x^2 + 2xy + y^2) \cdot (x+y) = \ldots$

$= x^3 + 3x^2y + 3xy^2 + y^3$

$1 \cdot x^3 + 3x^2y + 3xy^2 + 1 \cdot y^3$

Pascals triangel

$(a+b)^0$ 1

$(a+b)^1$ 1 1

$(a+b)^2$ 1 2 1 $\quad 1 \cdot a^2 + 2 \cdot ab + 1 \cdot b^2$

1 3 3 1

$(a+b)^4$ 1 4 6 4 1 $\quad 1 \cdot a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + 1 \cdot b^4$

1 5 10 10 5 1

$(x-y)^5 = 1 \cdot x^5 + 5 \cdot x^4 \cdot (-y) + 10x^3(-y)^2 + 10x^2(-y)^3 + 5x(-y)^4 + (-y)^5$

$= x^5 - 5x^4y + 10x^3y^2 - 10x^2y^3 + 5xy^4 - y^5$

osv.

* Konjugatregeln: $(x-y)(x+y) = x^2 + xy - yx - y^2$

$= x^2 - y^2$

- $4 - x^2 = 2^2 - x^2 = \{$Faktorisera Konjugatregeln$\} = (2-x)(2+x)$
- $y^2 - 3 = y^2 - (\sqrt{3})^2 = (y-\sqrt{3}) \cdot (y+\sqrt{3})$

Prioritetsordning.

P : parenteser
E : exponenter
M, D : multiplikation, division
A, S : addition, subtraktion

PEMDAS

- $3 + 4 \cdot 5^2 \cdot (1-2)^{-1}$

  $3 + 4 \cdot 25 \cdot (-1)^{-1}$

  $3 + 100 \cdot \frac{1}{(-1)^1}$

  $3 - 100 = \underline{\underline{-97}}$

- $(-y)^2 = (-y)(-y) = y^2$

  $-y^2 = (-1) \cdot y^2 = -y^2$

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ex) $\dfrac{\left(\frac{x}{y} + \frac{y}{x} + 2\right)}{\left(\frac{1}{x} + \frac{1}{y}\right)} = \dfrac{\frac{x \cdot x}{y \cdot x} + \frac{y \cdot y}{x \cdot y} + \frac{2x \cdot y}{x \cdot y}}{\frac{1}{x} \cdot \frac{y}{y} + \frac{1}{y} \cdot \frac{x}{x}}$ = (gemensam nämnare i täljarna)

$= \dfrac{\frac{x^2 + y^2 + 2xy}{xy}}{\frac{y + x}{xy}} = \dfrac{(x^2 + y^2 + 2xy) \cdot xy}{x \cdot y \cdot (y + x)}$

alt. invertera nämnaren.

$= \dfrac{(x^2 + y^2 + 2xy)}{\cancel{xy}} \cdot \dfrac{\cancel{xy}}{(x+y)} = \dfrac{(x+y)^2}{x+y} = x + y$

Bråktal:

- Vid addition och subtraktion av bråktal krävs gemensam nämnare.

$\frac{2}{3} + \frac{4}{5} = \frac{2 \cdot 5}{3 \cdot 5} + \frac{4 \cdot 3}{5 \cdot 3} = \frac{10 + 12}{15} = \underline{\underline{\frac{22}{15}}}$

- Multiplikation av bråktal: Multiplicera täljare m. täljare och nämnare m. nämnare

$\frac{2}{3} \cdot \frac{4}{5} = \frac{2 \cdot 4}{3 \cdot 5} = \frac{8}{15}$

$2 \cdot \frac{4}{5} = \frac{2}{1} \cdot \frac{4}{5} = \frac{2 \cdot 4}{1 \cdot 5} = \frac{8}{5}$

- Division av bråktal: $\dfrac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} / \frac{c}{d} = \frac{a \cdot d}{b \cdot c}$

Kvadratkomplettering.

- $x^2+2x+1 =$

$= x^2+2\cdot x\cdot 1+1^2 = (x+1)^2$

- $x^2+2x+3 = (x+1)^2 - 1 + 3 = (x+1)^2+2$

$= x^2+2\cdot x\cdot 1+1^2 - 1^2 + 3$

- $x^2+4x-2 = (x+2)^2 - 2^2 - 2$

$= (x+2)^2 - 6$

Koll: $x^2+4x+4-6 = x^2+4x-2$ ok!

- $x^2-3x+1 = \left(x-\frac{3}{2}\right)^2 - \left(-\frac{3}{2}\right)^2 + 1$

$= \left(x-\frac{3}{2}\right)^2 - \frac{9}{4} + \frac{4}{4}$

$= \left(x-\frac{3}{2}\right)^2 - \frac{5}{4}$

- $x^2+px+q = \left(x+\frac{p}{2}\right)^2 - \left(\frac{p}{2}\right)^2 + q$

$x^2+px+q = 0$

$\left(x+\frac{p}{2}\right)^2 - \left(\frac{p}{2}\right)^2 + q = 0$

$\left(x+\frac{p}{2}\right)^2 = \left(\frac{p}{2}\right)^2 - q$

$x+\frac{p}{2} = \pm\sqrt{\left(\frac{p}{2}\right)^2 - q}$ OBS!

$x^2+px+q=0 \iff x = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q}$