F~~8~~: Exponentialfunktioner. Potensfunktioner

F6-7

$\log_2 3 \cdot \log_3 2$ $\qquad$ $\lg A + \lg B = \lg(A \cdot B)$

Basbyte: $\log_a x = \frac{\log_b x}{\log_b a} = \frac{\ln x}{\ln a}$

$\cdot \log_2 3 = \frac{\log_6 3}{\log_6 2}$ $\qquad$ $\cdot \log_3 2 = \frac{\log_6 2}{\log_6 3}$

$\log_2 3 \cdot \log_3 2 = \frac{\log_6 3}{\log_6 2} \cdot \frac{\log_6 2}{\log_6 3} = 1$

Ex) $2 \cdot \log_3(x) + \log_9(x) = 10$ $\qquad$ (>0, >0)

Definitionsmängd: $x > 0$

$\log_9(x) = \frac{\log_3(x)}{\log_3 9} = \frac{\log_3(x)}{\log_3(3^2)} = \frac{\log_3(x)}{2}$

$2 \cdot \log_3 3 = 1$

$\lg 0{,}1 = \lg \frac{1}{10} = \lg 10^{-1} = -1 \cdot \lg 10 = -1 \cdot 1$

$2 \cdot \log_3(x) + \frac{\log_3(x)}{2} = 10$

$\frac{5}{2} \cdot \log_3(x) = 10$

$\log_3(x) = \frac{10 \cdot 2}{5}$

$\log_3(x) = 4$

$3^{\log_3(x)} = 3^4$

$3^{VL} = 3^{HL}$

$x = 3^4 = 81 > 0$ ok!

ex) $\ln(x-1) + \ln(x+5) = \ln 2 + \ln x$ (>0, >0, >0)

$D_f: \begin{cases} x-1>0 \iff x>1 \\ x+5>0 \iff x>-5 \\ x>0 \iff x>0 \end{cases} \therefore D_f: \underline{x>1}$

$\ln(x-1)\cdot(x+5) = \ln(2\cdot x)$

$e^{VL} = e^{HL}$

$e^{\ln(x-1)(x+5)} = e^{\ln(2x)}$

$x^2+5x-x-5 = 2x$

$x^2+2x-5 = 0$

$x = -1 \pm \sqrt{(-1)^2+5}$

$x = -1 \pm \sqrt{6}$

$\begin{cases} x_1 = -1+\sqrt{6} > 1 \\ x_2 = -1-\sqrt{6} < 0 \end{cases}$

Svar: $x = -1+\sqrt{6}$ enda lösningen

Kontrollera alltid lösningarna till logekvationer.

Övn. FN 2.14b) (s.79)

$\ln(x^2-2) \le \ln x$ ($x^2-2>0$, $x>0$)

Df: $x>0$ och $x^2-2>0$

$x^2>2$

$x>\sqrt{2}$ eller $x<-\sqrt{2}$

$\therefore$ Df. $x>\sqrt{2}$

$\ln(x^2-2)-\ln x \le 0$

$\ln\left(\frac{x^2-2}{x}\right) \le 0$

$e^{\ln\left(\frac{x^2-2}{x}\right)} \le e^0$

$\frac{x^2-2}{x} \le 1$ $\quad x$ är positivt.

$\frac{x^2-2}{x} - \frac{1\cdot x}{x} \le 0$

$\frac{x^2-2-x}{x} \le 0$

$Q(x)$: $\frac{(x-2)(x+1)}{x} \le 0$

$x^2-x-2=0$

$x=\frac{1}{2}\pm\sqrt{\frac{1}{4}+\frac{2\cdot 4}{4}}$

$x=\frac{1}{2}\pm\frac{3}{2}$

$x_1=2$, $x_2=-1$

Teckentabell:

	-1		0		2	
$x-2$	$-$	$-$	$-$	$-$	0	$+$
$x+1$	$-$	0	$+$	$+$	$+$	$+$
x	$-$	$-$	0	$+$	$+$	$+$
$Q(x)$	$-$	0	$+$	$-$	0	$+$

$Q(x) \le 0$ då $x<-1$ eller $0<x\le 2$

Df: $x>\sqrt{2}$

Svar: $\sqrt{2}<x\le 2$.

ex) 100000 kr med ränta 5%/år.
Bestäm antal år tills kapitalet har fördubblats?

$100000 \cdot 1{,}05^x = 200000$

$1{,}05^x = \frac{200000}{100000}$

$\ln(1{,}05^x) = \ln(2)$

Logaritmera båda sidor för att best. okänd exponent.

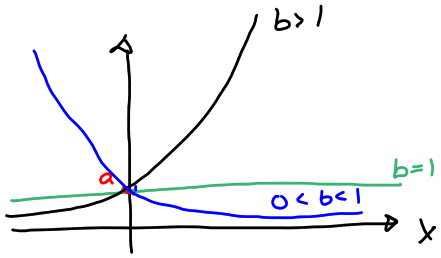
$\frac{x \cdot \ln 1{,}05}{\ln 1{,}05} = \frac{\ln 2}{\ln 1{,}05}$

$x = \frac{\ln 2}{\ln 1{,}05} \approx 14{,}2$ år

Exponentialfunktionen:

$y = a \cdot b^x$

basen > 0



$100000 \cdot 0{,}90^x$ avtagande funktion.

$a \cdot e^x$

$e \approx 2.7182\ldots$

$2^x = e^{\ln 2^x} = e^{x \cdot \ln 2}$

ex) $e \cdot 3^{-x}$ Skriv på formen e^{kx+b}

$= e \cdot e^{\ln 3^{-x}}$

$= e \cdot e^{-x \cdot \ln 3} =$

$= e^{1 + (-x \cdot \ln 3)} = e^{(-\ln 3) \cdot x + 1}$

$\therefore \; k = -\ln 3$
$b = 1$

$10^2 \cdot 10^3 = 10^{2+3} = 10^5$

$10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$

Potensfunktioner

$y = x^q$

t.ex x^2, $x^{1/2} = \sqrt{x}$

$x^x = e^{\ln x^x} = e^{x \cdot \ln x}$

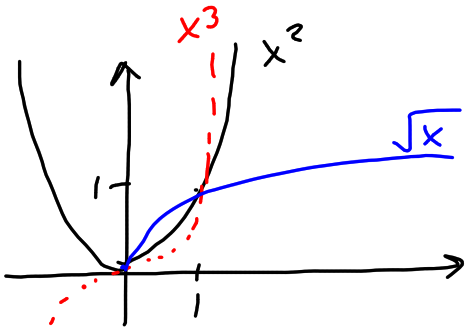
ex) $\left(\frac{x^{\sqrt{2}}}{x^{-\sqrt{2}}}\right)^{\sqrt{2}} =$

$= \left(x^{\sqrt{2}} \cdot x^{\sqrt{2}}\right)^{\sqrt{2}}$

$= \left(x^{\sqrt{2}+\sqrt{2}}\right)^{\sqrt{2}}$

$= \left(x^{2\sqrt{2}}\right)^{\sqrt{2}}$

$= x^{2 \cdot \sqrt{2} \cdot \sqrt{2}} = x^4$



PEMDAS

- $a^{-b} = \frac{1}{a^b}$
- $(a^b)^c = a^{b \cdot c} = (a^c)^b$
- $a^b \cdot a^c = a^{b+c}$

$(10^2)^3 = 10^6$

ex) $\ln(3^{x+1} + 3^x) = 0$ Bestäm x.

$e^{VL} = e^{HL}$

$e^{\ln(3^{x+1}+3^x)} = e^0$ ($e^0 = 1$)

$3^{x+1} + 3^x = 1$

$3^x \cdot 3 + 3^x = 1$ Lägg ihop alla 3^x

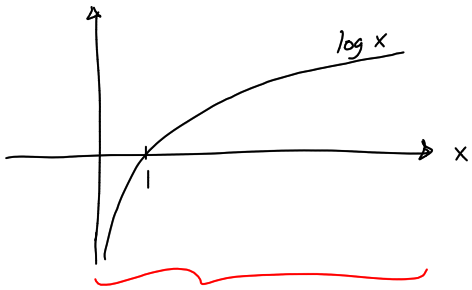
$3^x \cdot 4 = 1$

$3^x = \frac{1}{4}$

$\ln(3^x) = \ln(\frac{1}{4})$ $\ln(\frac{1}{4}) = \ln 1 - \ln 4$ ($\ln 1 = 0$)

$x \cdot \ln 3 = -\ln 4$

$x = -\frac{\ln 4}{\ln 3}$



Vf: $\mathbb{R}$

Df: $x > 0$

Df: $x > 0$

ex) $(\log_2 x)^2 + \log_2(x^2) - 3 = 0$

$(\log_2 x)^2 + 2 \cdot (\log_2 x) - 3 = 0$ Sätt $\log_2 x = t$

$t^2 + 2 \cdot t - 3 = 0$

$t = -1 \pm \sqrt{1+3}$

$t = -1 \pm 2$

$\begin{cases} t_1 = 1 \\ t_2 = -3 \end{cases}$

$t_1 = 1 = \log_2 x$

$2^1 = 2^{\log_2 x}$

$2 = x$

t_2: $-3 = \log_2 x$

$2^{-3} = 2^{\log_2 x}$

$\frac{1}{2^3} = x = \frac{1}{8}$