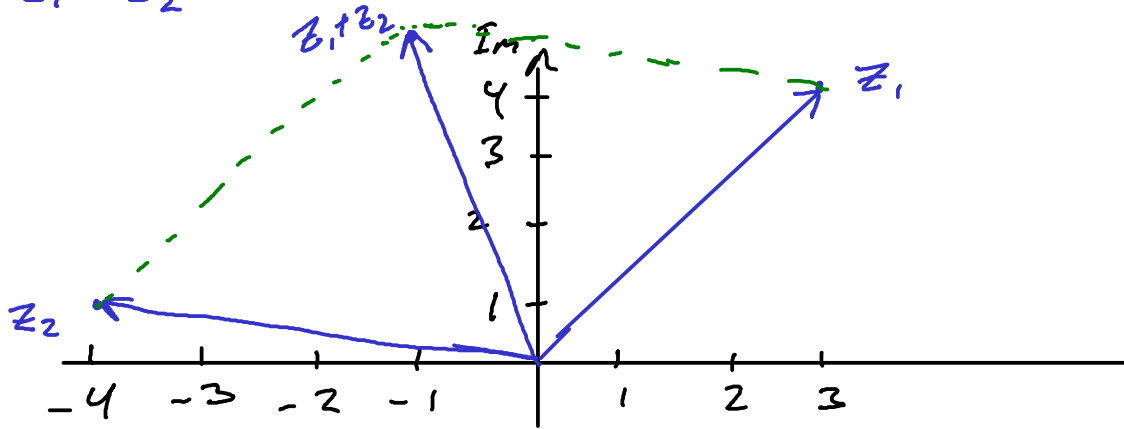


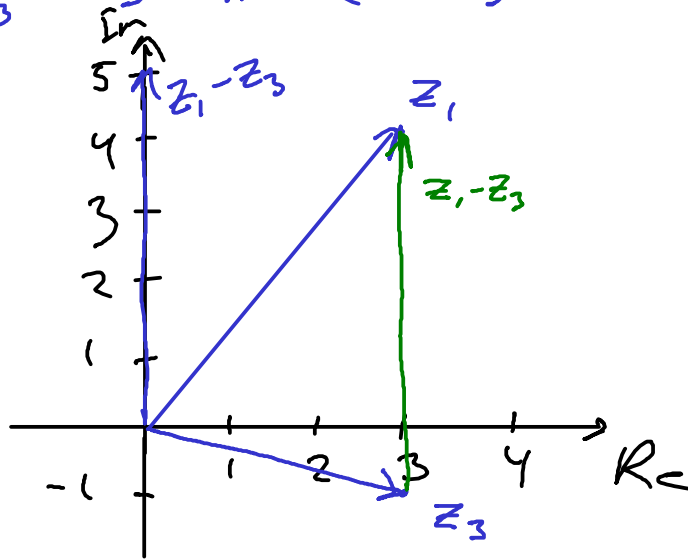
Ex: $z_1 = 3 + 4i$ $z_2 = -4 + i$ $z_3 = 3 - i$ $z_4 = 2 - i$

- $\operatorname{Re} z_3 = 3$ $\operatorname{Im} z_3 = -1$

- $z_1 + z_2 = 3 + 4i + -4 + i = -1 + 5i$



- $z_1 - z_3 = 3 + 4i - (3 - i) = 3 + 4i - 3 + i = 0 + 5i$



- $z_1 \cdot z_2 = (3 + 4i)(-4 + i) = 3(-4) + 3i + 4i(-4) + 4i \cdot i$
 $= -12 + 3i - 16i - 4 = -16 - 13i$

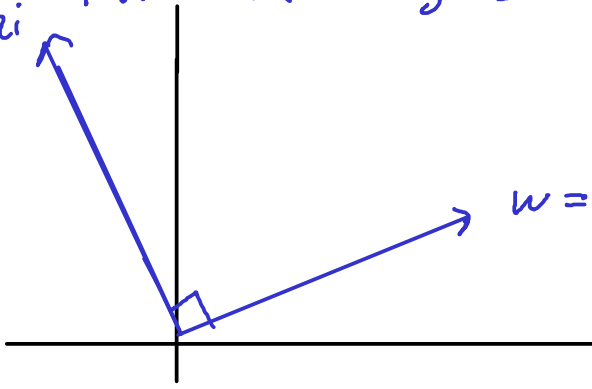
- $z_1 \cdot z_4 - z_3^2 = (3 + 4i)(2 - i) - (3 - i)^2$
 $= 3 \cdot 2 - 3 \cdot i + 4i \cdot 2 - 4i \cdot i - (3^2 - 2 \cdot 3 \cdot i + i^2)$
 $= 6 - 3i + 8i + 4 - 9 + 6i + 1 = 2 + 11i$

Ex:

$$w = x + y \cdot i$$

$$i \cdot w = i(x + y \cdot i) = i \cdot x + y \cdot \underbrace{i^2}_{=-1} = -y + x \cdot i$$

$$i \cdot w = -1 + 2i$$



$$i \cdot w = -1 + 2i$$

Ex:

$$z = 3 - 2i$$

$$(a) |z|^2 = (\sqrt{3^2 + (-2)^2})^2 = 9 + 4 = 13$$

$$z^2 = (3 - 2i)^2 = 3^2 - 2 \cdot 3 \cdot 2i + (2i)^2 = 9 - 12i + 4 \cdot \underbrace{i^2}_{=-1} = 5 - 12i$$

$$|z^2| = \sqrt{5^2 + (-12)^2} = \sqrt{25 + 144} = \sqrt{169} = 13$$

$$(b) \quad z = 2 - 3i \quad w = 1 - 2i$$

$$(i) \quad \bar{z} = 2 + 3i \quad \bar{w} = 1 + 2i$$

$$(ii) \quad \overline{zw} = \overline{(2 - 3i)(1 - 2i)} = \overline{2 - 4i - 3i + 6 \cdot \underbrace{i^2}_{=-1}} = \overline{-4 - 7i} = -4 + 7i$$

$$\overline{z \cdot w} = \bar{z} \cdot \bar{w} = (2 + 3i)(1 + 2i) = 2 + 4i + 3i + 6 \cdot \underbrace{i^2}_{=-1} = -4 + 7i$$

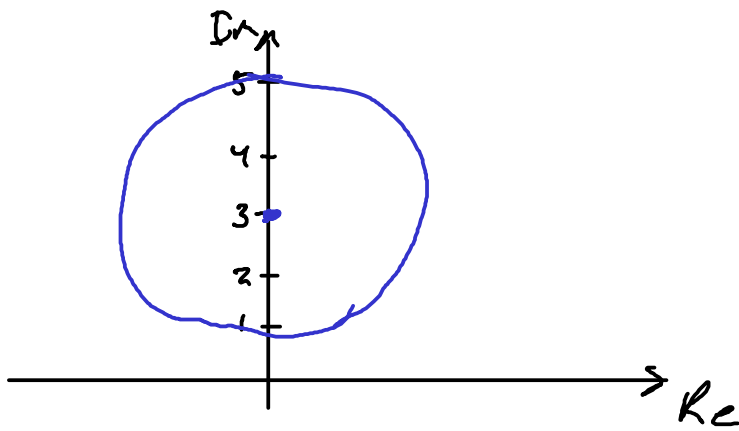
$$(iii) \quad |z| = |2 - 3i| = \sqrt{2^2 + (-3)^2} = \sqrt{13}$$

$$z \bar{z} = (2 - 3i)(2 + 3i) = 2^2 - (3i)^2 = 4 - 9 \cdot \underbrace{i^2}_{=-1} = 13$$

(iv) $|z - w|$ afstand mellan z & w

$$|z - w| = |2 - 3i - (1 - 2i)| = |1 - i| = \sqrt{1^2 + (-1)^2} = \sqrt{2}$$

(c)



$$|u - 3i| = 2$$

Cirkel med centrum i $3i$
och radie 2.

Ex

$$\begin{aligned} \frac{3-7i}{-6+5i} &= \frac{(3-7i)(-6-5i)}{(-6+5i)(-6-5i)} = \frac{-18 - 15i + 42i + 35i^2}{36 + 30i - 30i - 25i^2} \\ &= \frac{-53 + 27i}{61} = -\frac{53}{61} + \frac{27}{61}i \end{aligned}$$

$$z = 2 + 3i$$

$$\begin{aligned} z^2 &= (2+3i)^2 = 2^2 + 2 \cdot 2 \cdot 3i + (3i)^2 \\ &= 4 + 12i + 9 \cdot \underbrace{i^2}_{=-1} = -5 + 12i \end{aligned}$$

$$\operatorname{Re}(z^2) = -5$$

$$\operatorname{Im}(z^2) = 12$$