$\boxed{F11}$ Partikulär lösn. forts

Rep.) $$\frac{d^2y}{dx^2} + 10\frac{dy}{dx} + 25y = 14e^{-5x}$$

kar. ekv. $r^2 + 10r + 25 = 0$

$\therefore$ $r_1 = r_2 = -5$ dubbelrot

$$\underline{y_h = C_1 e^{-5x} + C_2 \cdot x \cdot e^{-5x}} = (C_1 + C_2 x)e^{-5x}$$

Ansats av samma "typ" som HL.

Ansats: $y_p = (A \cdot e^{-5x}) \cdot x^2 =$

$= Ax^2 \cdot e^{-5x}$

mult. med x till dess annan lösning än y_h

iw. i d.e

$y_p' = 2Ax \cdot e^{-5x} + Ax^2(-5) \cdot e^{-5x}$

$= e^{-5x} \cdot (2Ax - 5Ax^2)$

$y_p'' = e^{-5x} \cdot (-5) \cdot (2Ax - 5Ax^2) + e^{-5x}(2A - 10Ax)$

$= e^{-5x}(-10Ax + 25Ax^2 + 2A - 10Ax)$

$$\cancel{e^{-5x}} \cdot \Big[\underbrace{25\cancel{A}x^2 - 20\cancel{A}x + 2A}_{y''} + 10 \cdot \underbrace{(-5\cancel{A}x^2 + 2\cancel{A}x)}_{y'} + 25\underbrace{\cancel{Ax^2}}_{y}\Big] = 14 \cdot \cancel{e^{-5x}}$$

$2A = 14 \iff \underline{A = 7}$

$\therefore$ $\underline{y_p = 7x^2 \cdot e^{-5x}}$

AL: $y(x) = y_h + y_p = \underline{(C_1 + C_2 x + 7x^2) \cdot e^{-5x}}$

Alt. lösning "variation av parameter"
(se bokens ex.)

ex) $y'' + 10y' + 25y = 14e^{-x\cdot 5}$

$y_h = (C_1 + C_2 x)e^{-5x}$

Ansats: $y_p = z(x)\cdot e^{-5x}$

$y_p' = z'\cdot e^{-5x} + z\cdot(-5)\cdot e^{-5x} = e^{-5x}(z' - 5z)$

$y_p'' = e^{-5x}(-5)(z' - 5z) + e^{-5x}(z'' - 5z')$

$= e^{-5x}(-5z' + 25z + z'' - 5z') = e^{-5x}(z'' - 10z' + 25z)$

ins. i d.e.

$\cancel{e^{-5x}}\cdot\left[z'' - 10z' + 25z + 10\cdot(z' - 5z) + 25z\right] = 14\cdot\cancel{e^{-5x}}$

$\boxed{z'' = 14}$ $(*)$ Ny diff.ekv. med polynom i HL.

Sätt: $z_p = A\cdot x^2$

$z_p' = 2Ax$

$z_p'' = 2A$ ins $(*)$

$2A = 14 \Rightarrow A = 7$ ins i ansats till y_p.

$\therefore z_p = 7x^2$

$\Rightarrow y_p = 7x^2\cdot e^{-5x}$

forts. b.v. 1 $y(0)=2$
b.v 2 $y'(0)=0$

Allm. lösn. $\underline{y(x) = (C_1 + C_2 x + 7x^2) \cdot e^{-5x}}$

b.v.1 $y(0) = (C_1 + 0 + 0)\underbrace{e^0}_{1} = 2 \Leftrightarrow \underline{C_1 = 2}$

$y'(x) = (C_2 + 14x) \cdot e^{-5x} + (C_1 + C_2 x + 7x^2) \cdot (-5) e^{-5x}$

b.v.2 $y'(0) = C_2 - 5C_1 = 0$

$C_2 = 5C_1 = 5 \cdot 2 = 10$

$\therefore \underline{y(x) = (2 + 10x + 7x^2)\, e^{-5x}}$

Typ 5) $y'' + 2y' = \cos 2x$

sin/cos i HL

homogenlösn:

karakteristisk ekvation: $r^2 + 2r = 0$

$r(r+2) = 0$

$r = 0$ eller $r = -2$

$$y_h = C_1 \cdot e^{0x} + C_2 \cdot e^{-2x} = \underline{C_1 + C_2 \cdot e^{-2x}}$$

partikulärlösn:

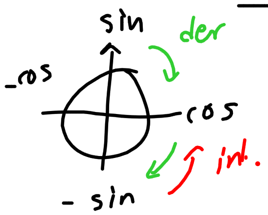
Ansats: $y_p = A \cdot \cos 2x + B \cdot \sin 2x$

samma typ som HL, men alltid både sin och cos.

$y_p' = -2A \sin 2x + 2B \cos 2x$

$y_p'' = -4A \cos 2x - 4B \sin 2x$

ins i d.e



$$\underbrace{-4A\cos 2x - 4B\sin 2x}_{y''} + 2\cdot(\underbrace{-2A\sin 2x + 2B\cos 2x}_{y'}) = \cos 2x$$

$$\left.\begin{aligned} \cos 2x: &\quad -4A + 4B = 1 \\ \sin 2x: &\quad -4B - 4A = 0 \end{aligned}\right\}$$

$$\left(\begin{array}{cc|c} -4 & 4 & 1 \\ -4 & -4 & 0 \end{array}\right) (-1) \sim \left(\begin{array}{cc|c} -4 & 4 & 1 \\ 0 & -8 & -1 \end{array}\right)$$

rad 2: $-8B = -1$

$B = 1/8$

$A = -1/8$

$$\left(\begin{array}{cc|c} -4 & 4 & 1 \\ 0 & -8 & -1 \end{array}\right) \cdot \left(-\tfrac{1}{8}\right) \sim \left(\begin{array}{cc|c} -4 & 4 & 1 \\ 0 & 1 & 1/8 \end{array}\right) (-4)$$

$$\sim \left(\begin{array}{cc|c} -4 & 0 & -\frac{1}{2}+1 \\ 0 & 1 & 1/8 \end{array}\right) \sim \left(\begin{array}{cc|c} -4 & 0 & 1/2 \\ 0 & 1 & 1/8 \end{array}\right) \cdot \left(-\tfrac{1}{4}\right) \sim \begin{array}{cc} A & B \end{array} \left(\begin{array}{cc|c} 1 & 0 & -1/8 \\ 0 & 1 & 1/8 \end{array}\right)$$

$$y_p = -\frac{1}{8}\cos 2x + \frac{1}{8}\sin 2x$$

AL. $y(x) = y_h + y_p = \underline{C_1 + C_2 \cdot e^{-2x} - \frac{1}{8}\cos 2x + \frac{1}{8}\sin 2x}$

Typ 6) $y'' + 4y = 8 \cdot \sin 2x$

kar. ekv. $r^2 + 4 = 0$

$r = \pm 2i$

$$\underline{y_h} = \underbrace{e^{\cancel{0 \cdot x}}}_{1} \cdot \underline{C_1 \cdot \cos 2x + C_2 \cdot \sin 2x}$$

p.g.a y_h

Ansats: $y_p = (A \cdot \sin 2x + B \cos 2x) \cdot x$

(produktregeln) $y_p' = \ldots = \cos 2x \cdot (2Ax + B) + \sin 2x\,[-2Bx + A]$

$y_p'' = \ldots = \cos 2x\,[-4Bx + 4A] + \sin 2x\,[-4Ax - 4B]$

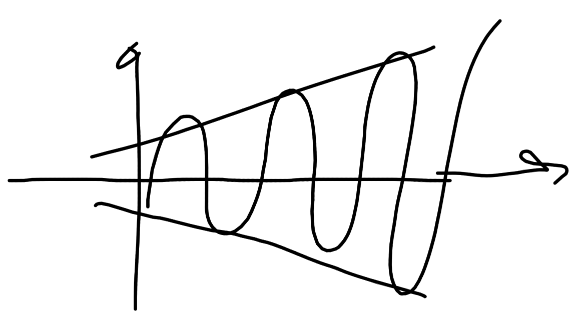
ins. i d.e.

identifiera koeff i VL = HL ......

$$\left.\begin{array}{ll} \cos 2x: & -4Bx + 4A + 4Bx = 0 \\ \sin 2x: & -4Ax - 4B + 4 \cdot Ax = 8 \end{array}\right\} \sim \left\{\begin{array}{l} A = 0 \\ B = -2 \end{array}\right.$$

$\therefore\ y_p = -2x \cdot \cos 2x$

A.L: $\underline{y(x) = y_h + y_p = (C_1 - 2x) \cdot \cos 2x + C_2 \cdot \sin 2x}$



jfr. Tacoma bridge.

Ex 7) $y'' - 2y' + 5y = 8 \cdot \sin 2x$

$\vdots$

$y_h = e^x \cdot (C_1 \cdot \cos 2x + C_2 \sin 2x)$

Ansats: $y_p = A \cdot \cos 2x + B \sin 2x$ OK, ej lika y_h

Kolla på egen hand.

Summor i HL

$y'' + 4y = 8 \cdot \sin 2x + e^x$

Dela upp. summan i HL i flera d.e. med var sin part. lösn.

$$\begin{cases} y'' + 4y = 8 \cdot \sin 2x & \Rightarrow \text{ ger } y_{p1} \\ y'' + 4y = e^x & \Rightarrow \text{ ger } y_{p2} \end{cases}$$

Allmän lösn. $y(x) = y_h + \underbrace{y_{p1} + y_{p2}}_{= y_p}$

$y_h = C_1 \cdot \cos 2x + C_2 \sin 2x$

Ansats $y_{p1} = (A \sin 2x + B \cos 2x) \cdot x$ p.g.a y_h

$\vdots$

Ansats $y_{p2} = D \cdot e^x$

Produkt i HL

ex) $y'' - 2y' + y = (x+1)\cdot e^{-2x}$

kar.ekv. $y_h = C_1 e^{x} + C_2\, x\cdot e^{x}$

Ansats: $y_p = (Ax+B)\cdot e^{-2x}$

Allm. "typ" som i HL. (ej lika y_h)

alt: $y_p = z(x)\cdot e^{-2x}$

$\therefore$ ger ny d.e med z där HL är polynom.

ex) $y'' - 2y' + y = (x+1)\cdot e^{x}$

"som HL"

Ansats $y_p = (Ax+B)\cdot e^{x}\cdot x^2$ p.g.a y_h.

$= (Ax^3 + Bx^2)\, e^{x}$

Ex) $y'' + 2y' + 2y = x \cdot \underbrace{e^{x} \cdot \sin x}$

$$\boxed{e^{ix} = \cos x + i \cdot \sin x}$$

$$Im\{e^{ix}\} = \sin x$$

$$Re\{e^{ix}\} = \cos x$$

$$\boxed{e^{a} \cdot e^{b} = e^{a+b}}$$

$$e^{x} \cdot \sin x = e^{x} \cdot Im(e^{ix}) =$$

$$= Im(e^{x} \cdot e^{ix}) =$$

$$= Im[e^{(1+i)x}]$$