$\boxed{F12}$ Partikulär lösn. slutet

Generaliserade integraler (kap 6.7, 16.2)

forts: F11

$$y'' + 2y' + 2y = x \cdot e^x \cdot \sin x = x \cdot \mathrm{Im}\left[e^{(1+i)x}\right] \quad ①$$

$$\boxed{e^{i\theta} = \cos\theta + i\sin\theta}$$

Lös: hjälp de

$$\boxed{u'' + 2u' + 2u = x \cdot e^{(1+i)x}} \quad ②$$

$$= x \cdot e^x \cdot (\cos x + i\sin x)$$

Sökt: $y_p = \mathrm{Im}\left[u_p\right]$

kar. ekv. $r^2 + 2r + 2 = 0$

$r = -1 \pm \sqrt{1-2}$

$r = -1 \pm i$

$$y_h = e^{-x} \cdot (C_1 \cdot \cos x + C_2 \sin x)$$

Ansats till d.e ②: $u_p = (Ax+B) \cdot e^{(1+i)x}$

Allm. polynom · exp funk. som i HL av ②

$$u_p' = A \cdot e^{(1+i)x} + (Ax+B)(1+i)e^{(1+i)x}$$

$$= e^{(1+i)x} \cdot \left(A + \underbrace{(Ax+B)(1+i)}_{Ax + Axi + B + Bi}\right).$$

$$u_p'' = e^{(1+i)x} \cdot (1+i) \cdot \left(A + (Ax+B)(1+i)\right) +$$

$$+ e^{(1+i)x} \cdot (A + Ai)$$

$(1+i)^2 = 1^2 + 2i - 1 = 2i$

$$= e^{(1+i)x} \cdot \left(A(1+i) + (Ax+B)2i + A + Ai\right)$$

$$= e^{(1+i)x}\left(2A + 2Ai + (Ax+B)2i\right).$$

ins. i d.e ②

$$\cancel{e^{(1+i)x}} \cdot \Big[\underbrace{2A + 2Ai + (Ax+B)2i}_{y''} + 2\big(\underbrace{A + Ax + Axi + B + Bi}_{y'}\big) +$$

$$+ 2 \cdot (Ax+B)\Big] = x \cdot \cancel{e^{(1+i)x}}$$

$$x: \quad 2Ai + 2A + 2Ai + 2A = 1$$

$$x^0: \quad \underline{2A} + \underline{2Ai} + 2Bi + \underline{2A} + 2B + 2Bi + 2B = 0$$

$$\begin{cases} A(4+4i) = 1 \\ A(4+2i) + B(4+4i) = 0 \end{cases}$$

$$A = \frac{1}{4+4i} = \frac{1}{4}\,\frac{1}{(1+i)}\,\frac{(1-i)}{(1-i)} = \underline{\underline{\frac{1}{8}(1-i)}}$$

$$1^2 + 1^2$$

p.s.s. $\quad \underline{\underline{B = \frac{1}{16}(-1+2i)}}$

$$\therefore \quad u_p = (Ax+B)\cdot e^{(1+i)x} = \left(\frac{2\cdot 1}{2\cdot 8}(1-i)x + \frac{1}{16}(-1+2i)\right)e^{(1+i)x}$$

$\underline{S\delta k}$: $y_p = \underline{\mathrm{Im}}\,(u_p)$

$$y_p = \mathrm{Im}\left[\frac{(2x-2xi-1+2i)}{16}\cdot e^{x}\cdot \underbrace{e^{ix}}_{=\cos x + i\sin x}\right] =$$

$$= \mathrm{Im}\left[\frac{e^x}{16}\left((2x-1) + i\cdot(2-2x)\right)\left(\cos x + i\sin x\right)\right]$$

$$= \mathrm{Im}\left[\frac{e^x}{16}\cdot\left((2x-1)\cos x + i\cdot(2x-1)\cdot\sin x + i(2-2x)\cos x - 1\cdot(2-2x)\sin x\right)\right].$$

$$y_p = \frac{e^x}{16}\left((2x-1)\cdot\sin x + (2-2x)\cdot\cos x\right)$$

FN

9.30 c) $\quad y'' - 3y' + 2y = e^{3x} \cdot \underbrace{\sin x}_{\operatorname{Im} e^{ix}}$

kar. ekv. $r_1 = 2$
$r_2 = 1$

$\underline{y_h = C_1 e^{2x} + C_2 e^{1x}}$

$\underbrace{}$
$= \operatorname{Im}(e^{3x} \cdot e^{ix})$

$HL = \operatorname{Im}(e^{(3+i)x})$

Løs: $u'' - 3u' + 2u = e^{(3+i)x}$ (2)

Søk: $\underline{y_p = \operatorname{Im}(u_p)}$

Ansats: $u_p = A \cdot e^{(3+i)x}$

$u_p' = A(3+i) \cdot e^{(3+i)x}$

$(3+i)^2 =$
$9 + 6i - 1$
$= 8 + 6i$

$u_p'' = A(8+6i) \cdot e^{(3+i)x}$

$\underbrace{}_{\text{ins i (2)}}$

$A \cdot \cancel{e^{(3+i)x}} \cdot [(8+6i) - 3(3+i) + 2] = \cancel{e^{(3+i)x}}$

$A(1+3i) = 1$

$A = \frac{1}{(1+3i)} \frac{(1-3i)}{(1-3i)} = \frac{1-3i}{10}$

$$\because \quad u_p = A \cdot e^{(3+i)x} = \frac{1-3i}{10} \cdot e^{3x} \cdot \underbrace{e^{ix}}_{= \cos x + i \sin x}$$

$$y_p = \mathrm{Im}(u_p) =$$

$$= \mathrm{Im}\left[\frac{(1-3i)}{10} \cdot e^{3x} \cdot (\cos x + i \sin x) \right]$$

$$= \mathrm{Im}\left[\frac{e^{3x}}{10} \cdot (1-3i)(\cos x + i \sin x) \right] =$$

$$= \mathrm{Im}\left[\frac{e^{3x}}{10} \cdot \left(\cos x + i \cdot \sin x - 3i \cos x \underbrace{- 3\underbrace{i^2}_{-1} \cdot \sin x}_{+3 \sin x} \right) \right]$$

$$y_p = \frac{e^{3x}}{10} \cdot (\sin x - 3 \cos x)$$

AL.: $y(x) = y_h + y_p = \ldots$

Block 3: Serier och summor

$\boxed{0! = 1}$

MacLaurin polynom

$$e^x \approx \underbrace{1 + x + \frac{x^2}{2} + \frac{x^3}{3!}}_{\text{summa}} = \frac{x^0}{0!} + \frac{x^1}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!}$$

$$= \sum_{k=0}^{3} \frac{x^k}{k!} = \frac{x^0}{0!} + \frac{x^1}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!}$$

ex) Kapital . tillväxt 25% / år.

$$K \cdot 1{,}25^{år} = 2K$$

$$\sum_{k=0}^{år} K \cdot 1{,}25^k = K \cdot \sum_{k=0}^{år} 1{,}25^k =$$

$$= K \cdot \left[1.25^0 + 1{,}25^1 + 1.25^2 + \dots \quad 1.25^{år} \right]$$

$$K \cdot [\, 1 + 1{,}25 + 1{,}25^2 + \dots$$

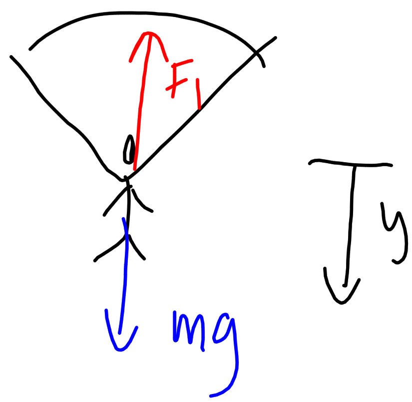
$\cdot\, 1{,}25$ $\quad \cdot\, 1{,}25$

Konstant kvot $\Rightarrow$ geometrisk summa

sid 41. $a + aq + aq^2 + \dots + aq^n =$

$$a(1 + q + q^2 + \dots + q^n) =$$

$$= \sum_{k=0}^{n} a \cdot q^k = \frac{a \cdot (1 - q^{n+1})}{1 - q}$$



$$\boxed{\sum F = m\cdot a}$$

$$mg - F_1 = m\cdot a$$

F_1 = prop mot hast^2

$$F_1 = k\cdot v^2$$

$$mg - k\cdot v^2 = m\cdot v'$$

$$(\qquad) = m\cdot\frac{dv}{dt}$$

$$\sum_{k=1}^{\infty} \frac{1}{k} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots\cdots$$

växande serie

$$\underbrace{1 + \frac{1}{2}}_{\cdot\frac{1}{2}} \underbrace{+ \frac{1}{4}}_{\cdot\frac{1}{2}} + \frac{1}{8} + \cdots\cdots = 2$$

$$\text{summa} = \frac{1\cdot\left(1-\frac{1}{2}^{n}\right)}{1-\frac{1}{2}} \rightarrow \frac{1-0}{\frac{1}{2}} = 2$$

summan <u>konvergerar</u> mot 2.

Generaliserade Integraler

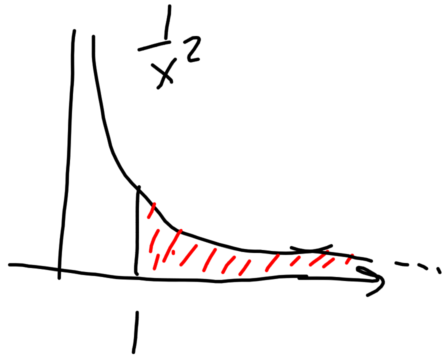
Typ I: Integrationsgränserna inte begränsade (i x-led)

$$\int_1^{\infty} \frac{1}{x^2}\,dx = \lim_{w\to\infty} \int_1^{w} x^{-2}\,dx$$

$$= \lim_{w\to\infty} \left[\frac{x^{-1}}{-1}\right]_1^{w} =$$

$$\frac{x^{-1}}{-1} = \frac{-1}{x}$$

$$= \lim_{w\to\infty} \left[\frac{-1}{x}\right]_1^{w} =$$



$$= \lim_{w\to\infty} \left(\underbrace{-\frac{1}{w}}_{\to 0} - \left(-\frac{1}{1}\right)\right) = \underline{\underline{1}}$$

Den generaliserade integralen har ett gränsvärde och är därför $\boxed{\text{konvergent}}$