$\boxed{\text{F19}}$

- Potensserielösn. till de.
- Laplacetransformer

$$y''+4y = \sin 2x$$

$$r^2+4=0$$

$$r=\pm 2i$$

$$y_h = C_1\cdot \cos 2x + C_2 \cdot \sin 2x$$

Ansats $y_p = \operatorname{Im}\left(e^{2ix}\right)$ eller

$$y_p = x\left(A\cdot \sin 2x + B\cdot \cos 2x\right)$$

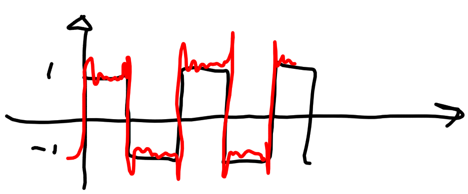
p.g.a $y_h$.

$$y''+4y = \sin 2x \cdot e^x$$

$$y_p = \operatorname{Im}\left(e^{i2x+1x}\right)$$

eller

$$y_p = \left(A\sin 2x + B\cos 2x\right)e^x$$




$$\sum_{k=1}^{\infty} \sin (2k+1)x =$$

$$=$$

Maclaurinserie för x nära 0

$$f(x) = f(0) + \frac{f'(0)}{1!}\cdot x + \frac{f''(0)}{2!}\cdot x^2 + \frac{f'''(0)}{3!}\cdot x^3 + \dots$$

Taylorserie för x nära a

$$f(x) = f(a) + \frac{f'(a)}{1!}\cdot(x-a) + \frac{f''(a)}{2!}\cdot(x-a)^2 + \dots$$

Potensserie lösn ur "Material till Lek18" i Fronter.

2a) $y' = x - y$   b.v $\boxed{y(0)=0} \Rightarrow c_0 = 0$   $\Leftrightarrow$ $\boxed{y' + y = x}$ (*) de

Sätt:

$$\begin{cases} y = \sum_{k=0}^{\infty} c_k \cdot x^k = c_0 + c_1 \cdot x + c_2 x^2 + c_3 x^3 + \dots \\ y' = \sum_{k=1}^{\infty} c_k \cdot k \cdot x^{k-1} = c_1 + c_2 \cdot 2 \cdot x + c_3 3x^2 + \dots \end{cases}$$

ins. i d.e (*)

$$\underbrace{\sum_{k=1}^{\infty} c_k \cdot k \cdot x^{k-1}}_{\text{byte av index.}} + \sum_{k=0}^{\infty} c_k \cdot x^k = x$$

vill ha lika x-termer.

$$\sum_{k=0}^{\infty} c_{k+1}(k+1) \cdot x^k + \sum_{k=0}^{\infty} c_k \cdot x^k = x$$

$$\sum_{k=0}^{\infty} \left(c_{k+1}(k+1) + c_k\right) \cdot x^k = 1 \cdot x$$

$k=0: (x^0): \quad c_1 \cdot 1 + c_0 = 0 \Leftrightarrow \underline{c_1 = -c_0 = 0}$

$k=1: (x^1): \quad c_2 \cdot 2 + c_1 = 1 \Rightarrow \underline{c_2 = \frac{1}{2}}$

$k=2: (x^2): \quad c_3 \cdot 3 + c_2 = 0 \Rightarrow c_3 = -\frac{c_2}{3} = \frac{-1}{3 \cdot 2}$

$k=3\ (x^3): \quad c_4 \cdot 4 + c_3 = 0 \Rightarrow c_4 = -\frac{c_3}{4} = \frac{1}{4 \cdot 3 \cdot 2}$

$\vdots$

$c_5 = \frac{-1}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = \frac{-1}{5!}$

$\therefore$ $c_0 = 0$

$c_1 = 0$

$c_2 = \frac{1}{2}$

$c_3 = -\frac{1}{3!}$

$c_4 = \frac{1}{4!}$

$c_5 = -\frac{1}{5!}$

$$y = c_0 + c_1 \cdot x + c_2 x^2 + c_3 x^3 + \dots$$

$$= 0 + 0 + \frac{1}{2}x^2 - \frac{1}{3!}x^3 + \frac{1}{4!}x^4 - \frac{1}{5!}x^5 + \dots$$

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots$$

$$e^{-x} = 1 - x + \underbrace{\frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} + \dots}_{= y}$$

<u>Svar</u>: $y = e^{-x} - (1 - x)$

## Laplace transformer

Löser differentialekvationer genom att.

$$\begin{tikzcd} \boxed{d.e} \arrow[r, "\mathcal{L}"', "①"] & \boxed{\phantom{x}} \arrow[d, "②"] \\ \boxed{lösn.} & \boxed{\cdot\cdot} \arrow[l, "\mathcal{L}^{-1}"] \end{tikzcd}$$

(funktioner som ofta beror av t.) "tidsidan"

"transformsidan" (frekvenssida) funktioner som beror av s.

① Laplacetransformering (tabeller)

② Omskrivning - partialbråksuppdelning

③ Inverstransformering (tabeller)

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Laplacetransformen av f(t)

L1: tabell-saml.: $$\boxed{\mathcal{L}\{f(t)\} = F(s) = \int_0^{\infty} f(t)\cdot e^{-st}\cdot dt}$$

Existerar bara om den generaliserade integralen konvergerar. $\Rightarrow$ $s>0$ ($Re(s)>0$) och $f(t)$ inte växer snabbare än $e^{kt}$

$f(t) \leq C\cdot e^{kt}$ där $s>k$

Ex) $f(t) = 1$

$$\mathcal{L}\{1\} = \int_0^\infty 1 \cdot e^{-st}\, dt = \left[ \frac{e^{-st}}{-s} \right]_0^\infty =$$

$$= \lim_{\omega \to \infty} \left( \underbrace{\frac{e^{-s\omega}}{-s}}_{\to 0} - \underbrace{\frac{e^{-s \cdot 0}}{-s}}_{\frac{1}{-s}} \right) = \frac{1}{s}$$

L14: $\therefore \mathcal{L}\{1\} = \frac{1}{s}$

L2: $\mathcal{L}\{K\} = K \cdot \mathcal{L}\{1\} = K \cdot \frac{1}{s}$ Transf. av en konstant $K$.

$\mathcal{L}\{-3\} = -3 \cdot \mathcal{L}\{1\} = -3 \cdot \frac{1}{s}$

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Ex) $f(t) = e^t$

$$\mathcal{L}\{e^t\} = \int_0^\infty e^t \cdot e^{-st}\, dt = \int_0^\infty e^{(1-s)t}\, dt =$$

$$= \left[ \frac{e^{(1-s)\cdot t}}{1-s} \right]_0^\infty = \left\{ \begin{array}{l} \text{konv. om} \\ 1-s < 0 \Leftrightarrow \\ \Leftrightarrow s > 1 \end{array} \right\} = 0 - \frac{1}{1-s}$$

$$= \frac{1}{s-1}$$

Det räcker att veta att det finns något $s$ som konvergerar gör att integralen för att Laplace transf. ska existera.

$\mathcal{L}\{e^t\} = \frac{1}{s-1}$

$\mathcal{L}\{e^{-t}\} = \frac{1}{s+1}$

$$\boxed{\mathcal{L}\{e^{-a \cdot t}\} = \frac{1}{s+a}}$$ L.19

$\mathcal{L}\{e^{bt}\} = \frac{1}{s-b}$

Ex) $f(t) = e^{t^2}$

$$\mathcal{L}\{e^{t^2}\} = \int_0^\infty e^{t^2} \cdot e^{-st}\,dt$$ konvergerar ej för något s

$\Rightarrow f(t) = e^{t^2}$ har ingen Laplacetransf.

då $e^{t^2}$ växer snabbare än $e^{-st}$

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Ex) $f(t) = e^{iat} = \cos at + i \cdot \sin at$

$$\mathcal{L}\{e^{iat}\} = \int_0^\infty e^{iat} \cdot e^{-st}\,dt = \int_0^\infty e^{-t(s-ai)}\,dt =$$

$$= \left[\frac{e^{-t(s-ai)}}{-(s-ai)}\right]_0^\infty = 0 - \frac{1}{-(s-ai)}$$

$$= \frac{1}{(s-ai)}\frac{(s+ai)}{(s+ai)} = \frac{s+ai}{s^2+a^2} = \underline{\frac{s}{s^2+a^2} + i\,\frac{a}{s^2+a^2}}$$

$\sin at = \operatorname{Im}[e^{iat}]$

L22 $\boxed{\mathcal{L}\{\sin at\}} = \mathcal{L}\{\operatorname{Im}(e^{iat})\} = \boxed{\frac{a}{s^2+a^2}}$ OBS!

L23 $\boxed{\mathcal{L}\{\cos at\}} = \operatorname{Re}\mathcal{L}\{e^{iat}\} = \boxed{\frac{s}{s^2+a^2}}$ OBS!

ex) $\mathcal{L}\{\cos 8t\} = \frac{s}{s^2+64}$

$\mathcal{L}\{\sin 3t\} = \frac{3}{s^2+9}$

ex) $\mathcal{L}^{-1}\left\{\frac{1}{s^2+4}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{2} \cdot \frac{2}{s^2+4}\right\} = \frac{1}{2}\sin 2t$

Ex) $f(t) = t$

$$\mathcal{L}\{t\} = \int_0^\infty \underset{\text{der}}{t} \cdot \underset{\text{int}}{e^{-st}}\, dt = \quad \text{Part. int.}$$

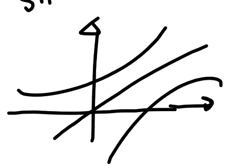
$$= \left[ t \cdot \frac{e^{-st}}{-s} \right]_0^\infty - \int_0^\infty 1 \cdot \frac{e^{-st}}{-s}\, dt$$

$\frac{t}{e^{st}} \to 0$

då $t \to \infty$
då exp växer snabbare mot $\infty$

$$= 0 - 0 + \left[ \frac{e^{-st}}{s \cdot (-s)} \right]_0^\infty =$$

$$= 0 - \frac{1}{-s^2} = \frac{1}{s^2}$$



L16 $\boxed{\therefore \mathcal{L}\{t\} = \frac{1}{s^2}}$

$\mathcal{L}\{t^2\} = \frac{2!}{s^3}$ ty $\int_0^\infty t^2 \cdot e^{-st}\, dt$ P.I 2 ggr.

$\mathcal{L}\{t^3\} = \frac{3!}{s^4}$

$\mathcal{L}\{t^4\} = \frac{4!}{s^5}$

L18 $\boxed{\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}}$

L2: $\boxed{\mathcal{L}\{a \cdot f(t) + b \cdot g(t)\}} = a \cdot F(s) + b \cdot G(s)$

$= \boxed{a \cdot \mathcal{L}\{f(t)\} + b \cdot \mathcal{L}\{g(t)\}}$