$\boxed{F2}$

Ö 2.7. a) $z_1 = \frac{1}{(1-i)}\frac{(1+i)}{(1+i)} = \frac{1+i}{1^2-\underbrace{i^2}_{-1}} = \frac{1+i}{1+1} = \frac{1}{2}+\frac{i}{2}$

Def: $\boxed{i^2=-1}$

$$z\cdot\bar{z} = (x+iy)(x-iy) = x^2-(iy)^2$$
$$= x^2 - i^2y^2$$
$$= x^2 + y^2$$

(ingen imaginär enhet)

Rep. Lös ekvationen:

$$2z - i\bar{z} = 1+2i$$

Sök z

Sätt: $z = x+iy \Rightarrow$

$\bar{z} = x-iy$ ins i ekv.

$$2(x+iy) - i(x-iy) = 1+2i$$
$$2x+2yi - xi + \underbrace{i^2\cdot y}_{-1\cdot y} = 1+2i$$
$$2x+2yi - xi - y = 1+2i$$

Re: $2x - y = 1$
Im: $2y - x = 2$

$$\begin{cases} 2x-y=1 \\ -x+2y=2 \end{cases}$$

$$\left(\begin{array}{cc|c} 2 & -1 & 1 \\ -1 & 2 & 2 \end{array}\right) \sim \left(\begin{array}{cc|c} -1 & 2 & 2 \\ 0 & 3 & 5 \end{array}\right)$$

$y = \frac{5}{3}$

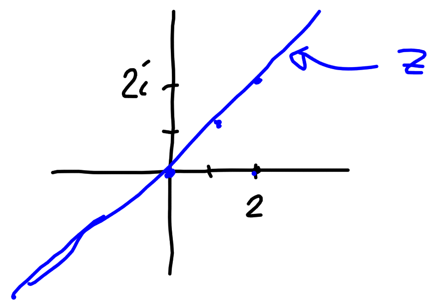
$-x + \frac{2\cdot 5}{3} = 2\cdot\frac{3}{3}$

$\frac{10-6}{3} = x = \frac{4}{3}$

Svar: $z = x+iy = \frac{4}{3}+i\cdot\frac{5}{3}$

Rep. $|z-2i| = |z-2|$

"Avst. mellan z och $2i$ =
avst " " 2".



Sätt: $z = x+iy$

VL: $|x+iy-2i| = |x+i(y-2)| = \sqrt{x^2+(y-2)^2}$.

HL: $|x+iy-2| = |(x-2)+iy| = \sqrt{(x-2)^2+y^2}$

VL = HL $\quad \sqrt{x^2+(y-2)^2} = \sqrt{(x-2)^2+y^2}$ kvadrera

$x^2+y^2-4y+4 = x^2-4x+4+y^2$

$-4y = -4x+\cancel{4}\cancel{-4}$

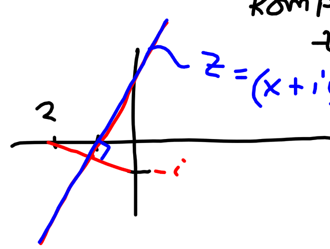
$\underline{y = x}$ ekv. för lösnings-linjen i komplexa talplanet.

Öva själv: $|z+i| = |z+2|$

(Svar: $y = 2x+\frac{3}{2}$)

där $z = x+iy$

$\underline{z = x+i\left(2x+\frac{3}{2}\right)}$



Lösning av algebraiska ekvationer.

ex) $z^3+1=0$ Reell polynomekv av grad 3 => 3 rötter ev kompl. (inga i i ekv.)

Gissa rot: $z=-1$

=> $(z+1)$ är en faktor till polynomet (z^3+1)

$z^3+1 = (z+1)\cdot(\;\ldots\;)$ polynomdiv

$$\begin{array}{r|l} & z^2-z+1 \\ \hline z+1 & z^3 \qquad + 1 \\ & -(z^3+z^2) \\ \hline & -z^2 \\ & -(-z^2-z) \\ \hline & z+1 \\ & z+1 \\ \hline & 0 \end{array}$$

$z^2-z+1=0$

$z = \frac{1}{2} \pm \sqrt{\frac{1}{4} - 1\cdot\frac{4}{4}}$

$= \frac{1}{2} \pm \sqrt{\frac{-3}{4}}$

$\boxed{i^2=-1}$

$z = \frac{1}{2} \pm \sqrt{\frac{3i^2}{4}}$

$z = \frac{1}{2} \pm \frac{i\sqrt{3}}{2}$

Svar: $\begin{cases} z_1 = -1 \\ z_2 = \frac{1}{2} + i\frac{\sqrt{3}}{2} \\ z_3 = \frac{1}{2} - i\frac{\sqrt{3}}{2} \end{cases}$ Varandras konjugat!

Gäller för polynom ekv. med reella koefficienter

DVS. I reella ekv. förekommer ev. komplexa rötter i konjugerande par; Är z_1 rot är även det komplexa konjugatet: $\bar{z}_1$ rot.

ex) Komplext polynom

$z^2 - 2i = 0$

$z^2 = 2i \quad (*)$

Sätt: $\boxed{z = x + iy}$ $\quad x \in \mathbb{R}$, $y \in \mathbb{R}$ $\quad (z \in \mathbb{C})$

$(*)$

$z^2 = (x+iy)^2 = \underbrace{x^2 + 2xyi - y^2 = 2i}$

Re: $x^2 - y^2 = 0 \iff x^2 = y^2 \iff x = (\pm) y$

Im: $2xy = 2 \iff x = \frac{1}{y}$

"samma tecken"

$\left(\frac{1}{y}\right)^2 - y^2 = 0$

$\frac{1}{y^2} = y^2$

$1 = y^4$

$y^2 = (\pm) 1 \qquad y \in \mathbb{R} \quad \because \quad y^2 \neq -1$

$\underline{y = \pm 1} \Rightarrow x = \pm 1$

$\therefore \quad z = x + iy = \underline{1 + i}$ eller $\underline{-1 - i}$

Svar: $\underline{\underline{z = \pm(1+i)}}$

ex) $z^2 + 4iz - 4 - 2i = 0$

Komplex 2:a grads ekv.

Komplext polynom.

$z = \cancel{\ldots \pm \sqrt{2-i}}$

$z = x + iy$

- kvadratkompl.

$(z + 2i)^2 - (2i)^2 - 4 - 2i = 0$

$(\underbrace{z+2i}_{=w})^2 + \cancel{4} - \cancel{4} - 2i = 0$

$w^2 - 2i = 0$

$w^2 = 2i$

⋮

Sätt $\boxed{w = x + iy}$ osv. Se lösn. förra sidan.

⋮

$w = \pm(1+i)$

$w = z + 2i \iff$

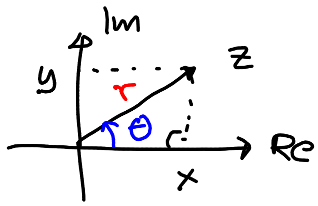
$z = w - 2i \Rightarrow z_1 = 1 + i - 2i$ eller $z_2 = -1 - i - 2i$

$z_1 = 1 - i$ $\qquad$ $z_2 = -1 - 3i$

Polära koordinater Viktigt!

$z = x + iy = a + ib$ Normalform = Rektangulär form = Cartesisk form.

(x, y) kallas rektangulära koord.



Polära koordinater. (r, θ)

där $\boxed{r = |z|}$ beloppet av z $\boxed{= \sqrt{x^2 + y^2}}$

$\theta = \arg(z)$ argumentet till z. (vinkeln).

$\cos\theta = \frac{x}{r} \Rightarrow$ $\begin{cases} x = r \cdot \cos\theta \\ y = r \cdot \sin\theta \end{cases}$

$\sin\theta = \frac{y}{r} \Rightarrow$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{y}{x}$$

$$\theta = \arctan \frac{y}{x}$$

Detta inses enklast ur fig. i. komplexa talplanet

$$\theta = \arg(z) = \begin{cases} \arctan \frac{Im}{Re} & \text{om } Re > 0 & \text{I och IV} \\ \pi + \arctan \frac{Im}{Re} & \text{om } Re < 0 & \text{Kvadrant II och III} \\ \frac{\pi}{2} & Re = 0,\ Im > 0 \\ -\frac{\pi}{2} & Re = 0,\ Im < 0 \end{cases}$$

$x = r \cdot \cos\theta$

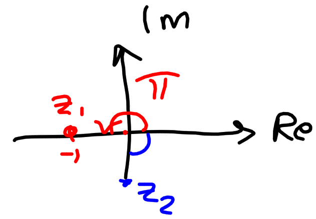
$y = r \cdot \sin\theta$

im i $\quad z = x + iy \quad$ gör

$z = r \cdot \cos\theta + i \cdot r \cdot \sin\theta =$

$$\boxed{z = r \cdot (\cos\theta + i \cdot \sin\theta)}$$

Polär form.



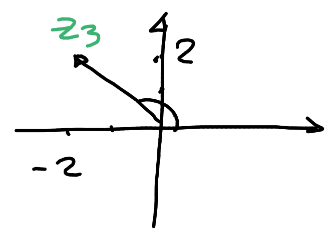
ex) $z_1 = -1$ till polär form.

$r = |z| = 1$

$\theta = \pi$

$\therefore$ $z_1 = 1 \cdot (\cos\pi + i \sin\pi)$

ex) $z_2 = -i = 1 \cdot \left(\cos\left(-\frac{\pi}{2}\right) + i \cdot \sin\left(-\frac{\pi}{2}\right)\right)$



ex) $z_3 = 2i - 2 = r \cdot (\cos\theta + i \sin\theta)$

$r = |z_3| = \sqrt{(-2)^2 + 2^2} = \sqrt{8}$

$\theta = \frac{3\pi}{4}$ (ur fig)

Alt. $\theta = \pi + \underbrace{\arctan\left(\frac{2}{-2}\right)}_{= -\frac{\pi}{4}} = \frac{3\pi}{4}$.

$z_3 = \sqrt{8}\left(\cos\frac{3\pi}{4} + i \cdot \sin\frac{3\pi}{4}\right)$