$\boxed{F\ 25-26}$ Fördröjning, Heaviside-, impuls- och ramp.

b.v.

FN 10.25 a) $(1-x)y' = 2y$ $\quad y=1$ då $x=0$ $\Leftrightarrow$ $y(0)=1$

Sätt: $y = \sum_{k=0}^{\infty} c_k \cdot x^k = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \ldots$

ger: $y' = \sum_{k=1}^{\infty} c_k \cdot k \cdot x^{k-1} = c_1 + 2c_2 x + 3c_3 x^2 + \ldots$

ins i d.e.

$$(1-x)\sum_{k=1}^{\infty} c_k \cdot k \cdot x^{k-1} = 2\sum_{k=0}^{\infty} c_k \cdot x^k \quad \Leftrightarrow$$

$$\sum_{k=1}^{\infty} c_k \cdot k \cdot x^{k-1} - \sum_{k=1}^{\infty} c_k \cdot k \cdot x^k - 2\sum_{k=0}^{\infty} c_k \cdot x^k = 0$$

$\Updownarrow$

$$\sum_{k=0} c_{k+1}(k+1) \cdot x^k \qquad " \qquad "$$

$k=0$: x^0 : $c_1 \cdot 1 \cdot x^0 - 2 c_0 x^0 = 0$

$c_1 - 2c_0 = 0$

$c_1 = +2c_0$

b.v. $y(0)=1$ ger $c_0 = 1$ $\qquad c_1 = 2$

$k \geq 1$: $\sum_{k=1}^{\infty} \left[c_{k+1}(k+1) - c_k \cdot k - 2c_k\right] x^k = 0$

$k=1$: x : $c_2 \cdot 2 - c_1 - 2c_1 = 0$

$c_2 = \frac{3}{2} c_1 = \frac{3}{2} \cdot 2 = 3$

$\vdots$

$c_3 = 4$

$c_n = n+1$

$c_0 = 1 \; ; \; c_2 = 2 \; ; \; c_3 = 3$

$\underline{k \geq 1}$: $\sum_{k=1}^{\infty} \left[c_{k+1}(k+1) - c_k \cdot k - 2c_k \right] x^k = 0$

$\underline{k=3}$: x^3: $c_4 \cdot 4 - c_3 \cdot 3 - 2c_3 = 0$

$c_4 = \frac{5}{4} c_3 = \frac{5}{4}$.

$c_0 = 1$
$c_1 = 2$
$c_2 = 3$

$c_n = n+1$

$$y = \sum_{k=0}^{\infty} c_k \cdot x^k = \underline{\underline{\sum_{k=0}^{\infty} \underbrace{(k+1) \cdot x^k}_{a_k}}}$$

Konvergensradie:

$\underline{\text{kvotkriteriet}}$:

$$\left| \frac{a_{k+1}}{a_k} \right| = \left| \frac{(k+2)\, x^{k+1}}{(k+1)\, x^k} \right| = \left| \frac{k+2}{k+1} \cdot x \right| =$$

$$= \frac{\cancel{k}(1+\frac{2}{k})}{\cancel{k}(1+\frac{1}{k})} \cdot |x| \longrightarrow 1 \cdot |x| < 1 \qquad \text{då } k \to \infty$$

$$\underline{-1 < x < 1}$$

Konvergensradie: $R = 1$.

Laplacetransf.

tidssidan $f(t)$	transformsidan $F(s) = \mathcal{L}\{f(t)\}$
1	$\frac{1}{s}$
t	$\frac{1}{s^2}$
t^3	$\frac{3!}{s^4}$
e^{-at}	$\frac{1}{s+a}$
$\sin(at)$	$\frac{a}{s^2+a^2}$
$\cos(at)$	$\frac{s}{s^2+a^2}$
prod. (LS) $t \cdot f(t)$	$-\frac{d}{ds}\left(\mathcal{L}\{f(t)\}\right) = -F'(s)$
ex) $t \cdot \sin 4t$	$-\frac{d}{ds}\left(\frac{4}{s^2+4^2}\right) = -\frac{d}{ds}\left(4\left(s^2+4^2\right)^{-1}\right)$
	$= -\left(-4\left(s^2+4^2\right)^{-2} \cdot 2s\right) = \frac{+8s}{\left(s^2+16\right)^2}$
$t^2 \cdot f(t)$	$(-1)^2 \cdot \frac{d^2}{ds^2}\left(\mathcal{L}\{f(t)\}\right)$
dämpning - produkt med e^{-at} $e^{-at} \cdot f(t)$	$\mathcal{L}\{f(t)\}_{s \to s+a}$
ex) $e^{-2t} \cdot \cos 5t$	$\mathcal{L}\{\cos 5t\}_{s \to s+2} = \left\{\frac{s}{s^2+5^2}\right\}_{s \to s+2}$
	$= \frac{s+2}{(s+2)^2+5^2}$

5b) $y' + 2y = e^{2t}$ bv: $y(0) = 1$

$\mathcal{L}\{y\} = Y(s) = Y$

$\boxed{L6}$ $\mathcal{L}\{y'\} = s \cdot Y(s) - \underbrace{y(0)}_{=1} = s \cdot Y - 1$

$\mathcal{L}\{e^{2t}\} = \frac{1}{s-2}$

$(sY - 1) + 2 \cdot Y = \frac{1}{s-2}$

$(s+2) \cdot Y = \frac{1}{s-2} + 1 = \frac{1 + s-2}{s-2} = \frac{s-1}{s-2}$

$Y = \frac{s-1}{(s-2)(s+2)} = \frac{1/4}{s-2} + \frac{\frac{-3}{-4}}{s+2}$

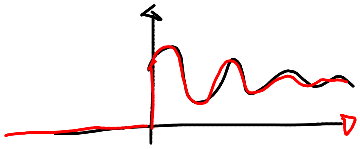
$Y = \frac{1}{4} \cdot \frac{1}{s-2} + \frac{3}{4} \frac{1}{s+2}$

$\sim e^{2t}$ $\sim e^{-2t}$

inverstransform:

$$y(t) = \frac{1}{4} e^{2t} + \frac{3}{4} e^{-2t}$$

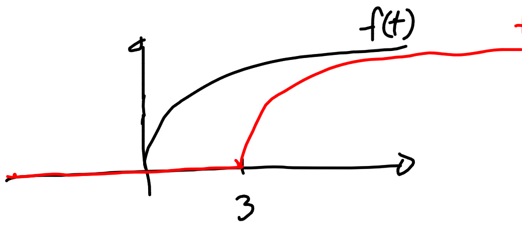
Kausal funktion : är 0 för $t<0$



$$f(t) = \begin{cases} f(t) & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$\Rightarrow$ $f(t)$ kausal funktion.

Fördröjd funktion



$f(t-3)$ fördröjd till $t=3$

$$\mathcal{L}\{f(t-3)\} = \int_0^\infty f(t-3)\cdot e^{-st}\,dt =$$

$$= \underbrace{\int_0^{3} f(t-3)\, e^{-st}\,dt}_{=0} + \underbrace{\int_{3}^\infty f(t-3)\, e^{-st}\,dt}$$

sätt $u = t-3$

$\Leftrightarrow t = u+3$

$dt = du$

$t \to 3$ ger $u \to 0$

$t \to \infty$ ger $u \to \infty$

$$= \int_0^\infty f(u)\cdot e^{-s(u+3)}\,du =$$

$$= \int_0^\infty f(u)\cdot e^{-su}\cdot e^{-3s}\,du = e^{-3s}\cdot \underbrace{\int_0^\infty f(u)\cdot e^{-su}\,du}_{F(s)}$$

L3 $\quad \therefore \boxed{\mathcal{L}\{f(t-3)\} = e^{-3s}\cdot \mathcal{L}\{f(t)\}}$

$$F(s) = \frac{1}{(s+5)^3} = \left.\frac{1}{s^3}\right|_{s \to s+5}$$

$$= \left.\frac{1}{2} \cdot \frac{2}{s^3}\right|_{s \Rightarrow s+5}$$

$\sim t^2$ $\quad\quad$ $\sim e^{-5t}$

$$\underline{f(t) = \frac{1}{2} t^2 \cdot e^{-5t}}$$

ex) $$F(s) = \frac{s \cdot e^{-2s}}{(s+1)^2+1}$$

$$= e^{-2s} \cdot \frac{s+1-1}{(s+1)^2+1}$$

$$= e^{-2s}\left(\frac{s+1}{(s+1)^2+1} - \frac{1}{(s+1)^2+1}\right)$$

$$\left.\frac{s}{s^2+1}\right|_{s \to s+1} - \left.\frac{1}{s^2+1}\right|_{s \to s+1}$$

$$\cos t \cdot e^{-t} \qquad \sin t \cdot e^{-t}$$

fördröjning på tidssidan till $t=2$

$$f(t) = \left[\cos t \cdot e^{-t} - \sin t \cdot e^{-t}\right]_{t \to t-2}$$

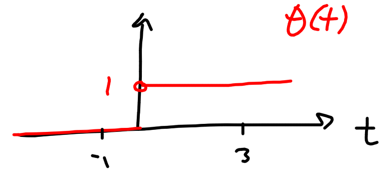
$$f(t) = \begin{cases} \cos(t-2) \cdot e^{-(t-2)} - \sin(t-2) \cdot e^{-(t-2)} & ; t \geq 2 \\ 0 & t \leq 2. \end{cases}$$

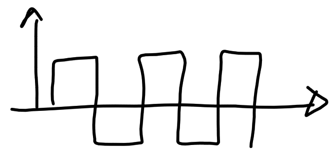
Heaviside stegfunktion

= unit-step function

Betecknas: $\theta(t) = H(t) = u(t)$

$$\boxed{\theta(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}}$$





ex) $\theta(3) = 1$

$\theta(-1) = 0$

$$\boxed{\theta(t-a) = \begin{cases} 1 & t > a \\ 0 & t < a \end{cases}}$$

ex) $f(t) = \begin{cases} 3 & t \geq 4 \\ 0 & t < 4 \end{cases} \quad \Leftrightarrow$

$$f(t) = 3 \cdot \underbrace{\theta(t-4)}_{=1 \text{ då } t > 4}$$

ex) 60 W lampa tänds då $t = 3$

släcks då $t = 7$.

$P(t) = \begin{cases} 60 & 3 \leq t \leq 7 \\ 0 & \text{f.ö.} \end{cases} \quad \Leftrightarrow$

$$= 60 \cdot [\underbrace{\theta(t-3)}_{\text{slå på då } t=3} - \underbrace{\theta(t-7)}_{\text{slå av då } t=7.}]$$

$$\mathcal{L}\{\theta(t)\} = \mathcal{L}\{1\} = \frac{1}{s}$$ $\boxed{L14}$

$$\mathcal{L}\{\theta(t-a)\} = e^{-as} \cdot \mathcal{L}\{1\} = e^{-as} \cdot \frac{1}{s}$$ $\boxed{L15}$

fördröjd
till $t=a$

L3 $$\boxed{\mathcal{L}\{f(t-a)\cdot\theta(t-a)\} = e^{-as} \cdot \mathcal{L}\{f(t)\}}$$

transf. av
ofördröjd.

$$f(t) = \begin{cases} (t-3)^2 & t \geq 3 \\ 0 & t < 3 \end{cases} \iff$$

$$f(t) = (t-3)^2 \cdot \theta(t-3)$$

$$\mathcal{L}\{f(t)\} = \mathcal{L}\{t^2\} \cdot e^{-3s}$$

$$= \underline{\underline{\frac{2}{s^3} \cdot e^{-3s}}}$$

Ex) $f(t) = \begin{cases} t^2 & t \geq 1 \\ 0 & t < 1 \end{cases}$

$= t^2 \cdot \theta(t-1) =$

$= (t-1+1)^2 \cdot \theta(t-1) =$

$= \left[(t-1)^2 + 2(t-1) + 1\right] \cdot \theta(t-1)$

$\mathcal{L}\{t^2\} \quad 2 \cdot \mathcal{L}\{t\} \quad \frac{1}{s} \quad \tilde{e}^{-s}$

$$\mathcal{L}\{f(t)\} = \underline{\left(\frac{2}{s^3} + 2 \cdot \frac{1}{s^2} + \frac{1}{s}\right) \cdot e^{-s}}$$