$\boxed{F3}$ Polär form, de Moivres formel

Rep. P. 1.82) $z^4-2z^3+2z^2-10z+25=0$

rötterna: $z_1 = 2+i$

$z_2 = -1-2i$

$z_3 = \bar{z}_1 = 2-i$

$z_4 = \bar{z}_2 = -1+2i$

Reell ekv. då förekommer ev. komplexa rötter i konjugerade par

koll: $(z-(2+i))\cdot(z-(-1-2i))\cdot(z-(2-i))\cdot(z-(-1+2i))=0$

$$((z-2)^2-i^2)\cdot((z+1)^2-(2i)^2)=0$$

$$(z^2-4z+4+1)\cdot(z^2+2z+1+4)=0$$

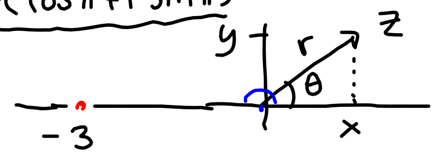
Ö 2.19 b) $z=-3$

$= 3\cdot(\cos\pi + i\sin\pi)$

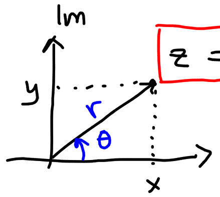
$z = r\cdot(\cos\theta + i\cdot\sin\theta)$

ty $x = r\cdot\cos\theta$

$y = r\cdot\sin\theta$



Komplexa exponentialekvationen : $\boxed{e^{i\theta}}$



$\boxed{z = x + iy}$ Normalform

$\boxed{z = r\cdot(\cos\theta + i\cdot\sin\theta)}$ Polär form

där $r = |z| = \sqrt{x^2+y^2}$

$\theta = \arg(z) = \arctan\left(\frac{Im}{Re}\right)$

Polär form, potensform : $\boxed{z = r\cdot e^{i\theta}}$

Def: $\boxed{e^{i\theta} = \cos\theta + i\sin\theta}$ Eulers formel

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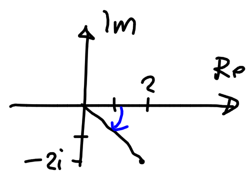
ex) $z = 2 - 2i$ i polär form



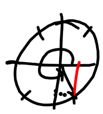
$r = |z| = \sqrt{2^2 + (-2)^2} = \sqrt{8}$

$\theta = -\frac{\pi}{4}$ (ur fig)

$= \arctan\frac{Im}{Re} = \arctan\left(\frac{-2}{2}\right) = -\frac{\pi}{4}$

$\therefore\ z = r\cdot e^{i\theta} = \underline{\sqrt{8}\cdot e^{i\left(-\frac{\pi}{4}\right)}}$

FN. 2.61 a) $z = 3e^{\frac{5\pi}{3}i} = 3\cdot\left(\underbrace{\cos\frac{5\pi}{3}}_{\frac{1}{2}} + i\cdot\underbrace{\sin\frac{5\pi}{3}}_{-\frac{\sqrt{3}}{2}}\right) = 3\left(\frac{1}{2} - \frac{i\sqrt{3}}{2}\right)$



$= \underline{\underline{\frac{3}{2} - \frac{3\sqrt{3}}{2}i}}$

ex) $|e^{i\theta}| = |\cos\theta + i\sin\theta| = \sqrt{\cos^2\theta + \sin^2\theta} = \underline{\underline{1}}$

$|z| = |r\cdot e^{i\theta}| = |r|\cdot\underbrace{|e^{i\theta}|}_{1} = |r| = r\cdot 1$

beloppet av z.

$$z \cdot z = r \cdot e^{i\theta} \cdot r \cdot e^{i\theta} = r^2 \cdot e^{i(\theta+\theta)} = r^2 \cdot e^{i2\theta} \quad *$$

$$\boxed{\begin{aligned} z^n &= (r \cdot e^{i\theta})^n = r^n \cdot e^{in\theta} \\ &= r^n \cdot (\cos(n\theta) + i\sin(n\theta)) \end{aligned}}$$

de Moivres sats

$$z \cdot z = r(\cos\theta + i\sin\theta) \cdot r(\cos\theta + i\sin\theta) =$$
$$= r^2(\cos^2\theta + i\cos\theta \cdot \sin\theta + i\sin\theta \cdot \cos\theta - \sin^2\theta) =$$
$$= r^2((\cos^2\theta - \sin^2\theta) + i(2\sin\theta \cdot \cos\theta)) =$$
$$(*) \quad = r^2(\cos 2\theta + i\sin 2\theta)$$

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- $$z \cdot w = r_1 \cdot e^{i\theta_1} \cdot r_2 \cdot e^{i\theta_2} = r_1 \cdot r_2 \cdot e^{i(\theta_1+\theta_2)}$$

$$|zw| = r_1 \cdot r_2$$
$$\arg(z \cdot w) = \theta_1 + \theta_2$$

Vid multiplikation av komplexa tal mult. beloppen adderas arg.

- $$\frac{z}{w} = \frac{r_1 \cdot e^{i\theta_1}}{r_2 \cdot e^{i\theta_2}} = \frac{r_1}{r_2} \cdot e^{i(\theta_1-\theta_2)}$$

$$\therefore \left|\frac{z}{w}\right| = \frac{r_1}{r_2}$$

beloppen divideras vid div.

$$\arg\left(\frac{z}{w}\right) = \theta_1 - \theta_2$$

argumenten subtrah. —"— .

Binomiska ekv. $\boxed{z^n = w}$

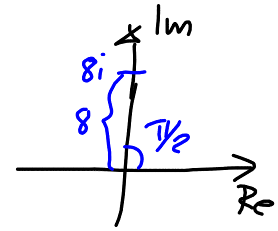
ex) $z^3 = 8i$ Skriv båda leden i polär potensform.

$$r^3 \cdot e^{i3\theta} = 8 \cdot e^{i\frac{\pi}{2}}$$

Sätt: $\boxed{z = r \cdot e^{i\theta}}$

$\Downarrow$

$$z^3 = r^3 \cdot e^{i3\theta}$$

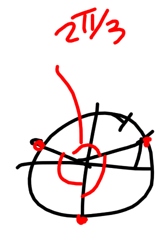


$$\begin{array}{ll} \text{beloppen:} & \left\{ \begin{array}{l} r^3 = 8 \quad \Rightarrow \quad r = 2 \\ 3\theta = \frac{\pi}{2} + 2\pi n \end{array} \right. \\ \text{arg:} & \end{array}$$

om perioden missas erhålls endast en lösning

$$\begin{cases} r = 2 \\ \theta = \frac{\pi}{6} + \frac{2\pi}{3} n \end{cases}$$

$$\therefore \quad z = r e^{i\theta} = \underline{2 \cdot e^{i\left(\frac{\pi}{6} + \frac{2\pi}{3} \cdot n\right)}}$$



i Normalform:

$$n=0: \quad z_1 = 2 \cdot e^{i\frac{\pi}{6}} = 2 \cdot \Big(\underbrace{\cos\tfrac{\pi}{6}}_{\frac{\sqrt{3}}{2}} + i \underbrace{\sin\tfrac{\pi}{6}}_{\frac{1}{2}}\Big) = \underline{\sqrt{3} + i}$$

$$n=1: \quad z_2 = 2 \cdot e^{i\left(\frac{\pi}{6} + \frac{4\pi}{6}\right)} = 2\Big(\underbrace{\cos\tfrac{5\pi}{6}}_{-\frac{\sqrt{3}}{2}} + i \underbrace{\sin\tfrac{\pi \cdot 5}{6}}_{\frac{1}{2}}\Big) = \underline{\underline{-\sqrt{3} + i}}$$

$$n=2: \quad z_3 = 2 \cdot e^{i\left(\frac{\pi}{6} + \frac{8\pi}{6}\right)} = 2 \cdot \Big(\underbrace{\cos\tfrac{\cancel{9}^{3}\pi}{\cancel{6}_{2}}}_{=0} + i \underbrace{\sin\tfrac{9\pi}{6}}_{-1}\Big) = \underline{\underline{-2i}}$$

P. 1.78) $z^2+(1-2i)z-3-i=0$ Komplex polynom ekv.

$$\left(z+\frac{(1-2i)}{2}\right)^2-\left(\frac{1-2i}{2}\right)^2-3-i=0$$

$\sqrt{i}$

$$\left(z+\frac{1}{2}-i\right)^2\underbrace{-\left(\frac{1-4i-4}{4}\right)-3-i}_{+\frac{3}{4}+i-3-i}=0$$

$\left(=\frac{3}{4}-\frac{12}{4}=\frac{-9}{4}\right)$

$$\left(z+\frac{1}{2}-i\right)^2=\frac{9}{4}$$

$$z+\frac{1}{2}-i=\pm\frac{3}{2}$$

$$\begin{cases} z_1=\frac{3}{2}-\frac{1}{2}+i=\underline{1+i} \\ z_2=-\frac{3}{2}-\frac{1}{2}+i=\underline{-2+i} \end{cases}$$

Ö. 2.44) $z^4+6z^2+25=0$ reell ekv.

$z^2 = -3 \pm \sqrt{9-25} = -3 \pm \sqrt{-16} = -3 \pm \sqrt{i^2 4^2}$

$z^2 = -3 \pm 4i$ $\qquad z = \sqrt{-3 \pm 4i}$

Sätt $z = x+iy$

$z^2 = x^2 + 2xyi - y^2 = \begin{cases} -3+4i & \text{I} \\ -3-4i & \text{II} \end{cases}$

I) $\left.\begin{array}{ll} Re: & x^2-y^2 = -3 \\ Im: & 2xy = 4 \end{array}\right\} \Rightarrow x = \frac{2}{y}$

$\frac{4}{y^2} - y^2 = -3$

$4 - y^4 = -3y^2$

$0 = y^4 - 3y^2 - 4$

$y^2 = \frac{3}{2} \pm \sqrt{\frac{9}{4} + \frac{4 \cdot 4}{4}}$

$y^2 = \frac{3 \pm 5}{2}$ $\begin{array}{l} 4 \\ \cancel{-1} \end{array}$ ty $y \in \mathbb{R}$

$y^2 = 4$

$y = \pm 2 \Rightarrow x = \frac{2}{y} \Rightarrow x = \pm 1$

$\therefore z_1 = 1+2i$

$z_2 = -1-2i$

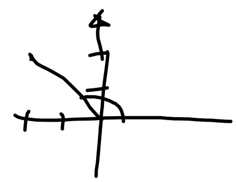
p.p.s. II

Ex) $z^4 = -2+2i$

$r^4 \cdot e^{i4\theta} = \sqrt{8} \cdot e^{i\frac{3\pi}{4}}$

polärsform.

$z = r \cdot e^{i\theta}$



$$\begin{cases} r^4 = \sqrt{8} \\ 4\theta = \frac{3\pi}{4} + 2\pi \cdot n \end{cases}$$

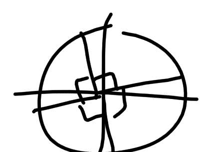
HL: $-2+2i$

$|-2+2i| = \sqrt{(-2)^2 + 2^2} = \sqrt{8}$

$\varphi = \frac{3\pi}{4}$ ur fig.

$$\begin{cases} r = 8^{\frac{1}{2}\cdot\frac{1}{4}} = 8^{\frac{1}{8}} \\ \theta = \frac{3\pi}{16} + \frac{\pi}{2} \cdot n \end{cases}$$

$$z = 8^{1/8} \cdot e^{i\left(\frac{3\pi}{16} + \frac{\pi}{2} \cdot n\right)}$$



Svar: Normalform:

$n=0$: $z_1 = 8^{1/8} \cdot \left(\cos \frac{3\pi}{16} + i \sin \frac{3\pi}{16}\right) \approx 1{,}078 + i \cdot 0{,}72$

$n=1$: $z_2 = 8^{1/8} \cdot \left(\cos \frac{11\pi}{16} + i \sin \frac{11\pi}{16}\right) \approx$ . . .

$n=2$: $z_3 = 8^{1/8} \left(\cos \frac{19\pi}{16} + i \sin \frac{19\pi}{16}\right) \approx$ . . . .