Tenta maj 2012

1a) $z_1 = -2+5i$

$z_2 = 3-i$

$$\overline{z_1 \cdot z_2} = \overline{(-2+5i)\cdot(3-i)} =$$

$$= \overline{-6+2i+15i-5\underbrace{i^2}_{-1}} = \overline{-6+17i+5} =$$

$$= \overline{-1+17i} = \underline{\underline{-1-17i}}$$

1b) $z^3 = 1+i$

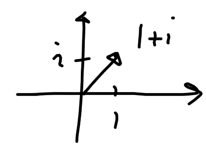
$$r^3 \cdot e^{i3\theta} = \sqrt{2}\cdot e^{i\frac{\pi}{4}}$$

polär form.

$$\boxed{z = r\cdot e^{i\theta}}$$

$r = |z|$

$\theta = \arg z = \arctan \frac{Im}{Re} + \ldots$



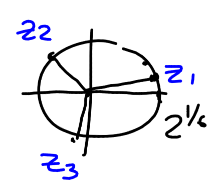
ty $1+i$ har

belopp et $|1+i| = \sqrt{1^2+1^2} = \sqrt{2}$

$\arg(1+i) = \arctan \frac{1}{1} = \frac{\pi}{4}$

$$\begin{cases} r^3 = \sqrt{2} \\ 3\theta = \frac{\pi}{4} + 2\pi n \end{cases} \Rightarrow \begin{cases} r = (2^{\frac{1}{2}})^{\frac{1}{3}} = 2^{\frac{1}{6}} \\ \theta = \frac{\pi}{12} + \frac{2\pi}{3} n \end{cases}$$

$$\therefore \; z = r\cdot e^{i\theta} = \underline{2^{\frac{1}{6}} \cdot e^{i\left(\frac{\pi}{12} + \frac{2\pi}{3}n\right)}}$$



n=0: $z_1 = 2^{\frac{1}{6}} \cdot e^{i\frac{\pi}{12}} =$

$$= 2^{\frac{1}{6}}\left(\underbrace{\cos\frac{\pi}{12}} + i \sin\frac{\pi}{12}\right) = \ldots + i \ldots$$

n=1: $z_2 = 2^{1/6}\left(\underbrace{\cos\frac{3\pi}{4}}_{-\frac{1}{\sqrt{2}}} + i \underbrace{\sin\frac{3\pi}{4}}_{\frac{1}{\sqrt{2}}}\right) = \ldots + i \ldots$

n=2 $z_3 =$

2a) $y'' - 3y' + 2y = 4x$

~~$y'' + 4y = \sin x$~~

~~$r^2 + 4 = 0$~~

kar.ekv: $r^2 - 3r + 2 = 0$

$r = \frac{3}{2} \pm \sqrt{\frac{9}{4} - \frac{2 \cdot 4}{4}}$

$r = \frac{3 \pm 1}{2} <^{2}_{1}$

$y_h = C_1 \cdot e^{2x} + C_2 \cdot e^{x} + \cancel{C_3 \cdot e^{0x}}$

Ansats $y_p = (Ax + B)$ som HL, ok med y_h.

$y_p' = A$

$y_p'' = 0$

ins. i d.e.

$0 - 3A + 2(Ax + B) = 4x$

$x:$ $2A = 4$

$x^0:$ $-3A + 2B = 0$

ger $A = 2$, $B = 3$

$\cancel{2}B = \frac{3A}{2}$

$y_p = 2x + 3$

$y(x) = y_h + y_p = C_1 \cdot e^{2x} + C_2 e^{x} + 2x + 3$

2b) $\frac{dy}{dx} = (x+1)\cdot e^{-x}\cdot y^2$ $y(0)=-1$

ej linjär
prova separera!

$$\int \frac{dy}{y^2} = \int (x+1)\cdot e^{-x}\cdot dx$$

der int.

partiell integration

$$\int y^{-2}\,dy = \int (x+1)\cdot e^{-x}\,dx$$

$$\frac{y^{-1}}{-1} = (x+1)(-e^{-x}) - \int 1\cdot(-e^{-x})\,dx$$

$$\frac{-1}{y} = -(x+1)\cdot e^{-x} - e^{-x} + C \quad (*)$$

$$y = \frac{-1}{-[(x+1)e^{-x}+e^{-x}-C]} = \underline{\frac{1}{(x+1)e^{-x}+e^{-x}-C}}$$

b.v. $y(0)=-1$ ins i

$$\frac{-1}{-1} = -(0+1)\cdot e^{0} - e^{-0} + C$$

$$1 = -1-1+C$$

$$\Leftrightarrow \underline{C=3}$$

$$\therefore\ y = \frac{1}{(x+1)\cdot e^{-x}+e^{-x}+3} = \underline{\frac{1}{(x+2)e^{-x}+3}}$$

aug 2015

3a) $\sum_{k=1}^{\infty} \underbrace{\left(\frac{3}{k}\right)\left(\frac{2}{5}\right)^k \cdot x^k}_{a_k} = \frac{3}{1}\cdot\frac{2}{5}x + \frac{3}{2}\left(\frac{2}{5}\right)^2\cdot x^2 + \ldots\ldots$

Konvergensradie?

Kvotkriteriet:

$$\lim_{k\to\infty}\left|\frac{a_{k+1}}{a_k}\right| = \lim_{k\to\infty}\left|\frac{\left(\frac{3}{k+1}\right)\cdot\left(\frac{2}{5}\right)^{k+1}\cdot x^{k+1}}{\left(\frac{3}{k}\right)\left(\frac{2}{5}\right)^k\cdot x^k}\right| = \lim_{k\to\infty}\left|\frac{\cancel{3}\,k\left(\frac{2}{5}\right)\cdot x}{k+1\cdot\cancel{3}}\right|$$

$$= \lim_{k\to\infty}\frac{\cancel{k}\left(\frac{2}{5}\right)\cdot|x|}{\cancel{k}\left(1+\frac{1}{k}\right)} = \frac{\frac{2}{5}\cdot|x|}{1+0} < 1 \iff \frac{2}{5}|x| < 1$$

$$|x| < \frac{5}{2}$$

$$\underline{\underline{|x| < \frac{5}{2}}}$$

Konvergensradien är $R = \frac{5}{2}$.

Konvergensområde: $-\frac{5}{2} < x < \frac{5}{2}$

$\sum_{k=1}^{\infty} \frac{k^2}{k!}$ konv?

$$\frac{a_{k+1}}{a_k} = \frac{\frac{(k+1)^2}{(k+1)!}}{\frac{k^2}{k!}} = \frac{(k+1)^{\cancel{2}}}{\cancel{(k+1)}\cdot \cancel{k!}} \cdot \frac{\cancel{k!}}{k^2} = \frac{k+1}{k^2} =$$

$$= \frac{1}{k} + \frac{1}{k^2} \longrightarrow \underline{0} < 1 \quad \text{då } k \to \infty$$

∴ Serien konvergent enl. kvotkriteriet.

$$\lim_{x\to 0} \frac{\arctan x - x}{\ln(1+x^3)}$$

$$\frac{x - \frac{x^3}{3} + \frac{x^5}{5} + O(x^6) - x}{x^3 - \frac{x^6}{2} + \frac{x^9}{3} + O(x^{10})} =$$

$$\frac{-\frac{x^3}{3} + \frac{x^5}{5} + O(x^6)}{x^3 - \frac{x^6}{2} + \frac{x^9}{3} + O(x^{10})} =$$

$$\frac{\cancel{x^3}\left(-\frac{1}{3} + \frac{x^2}{5} + O(x^3)\right)}{\cancel{x^3}\left(1 - \frac{x^3}{2} + \frac{x^6}{3} + O(x^7)\right)} \Longrightarrow \frac{-\frac{1}{3} + 0 + 0}{1 - 0 + 0 + 0}$$

går mot $-\frac{1}{3}$ då $x \to 0$

4a) $F(s) = \dfrac{2s+3}{s^2+4s+13} = \dfrac{2s+3}{(s+2)^2+9} =$

$s^2+4s+13=0$

$s=-2\pm\sqrt{4-13}$ <0

Nej

ej faktorisering

$$= 2\cdot\frac{s+2-2}{(s+2)^2+3^2} + \frac{3}{(s+2)^2+3^2}$$

$$= \frac{2(s+2)}{(s+2)^2+3^2} + (-4+3)\left(\frac{1}{(s+2)^2+3^2}\right) =$$

$$= 2\,\frac{(s+2)}{(s+2)^2+3^2} - \frac{1}{3}\cdot\frac{3}{(s+2)^2+3^2}$$

$$= 2\left.\frac{s}{s^2+3^2}\right|_{s\to s+2} - \frac{1}{3}\cdot\left.\frac{3}{s^2+3^2}\right|_{s\to s+2}$$

$\cos 3t \cdot e^{-2t}$ $\quad\sim\quad$ $\sin 3t \cdot e^{-2t}$

$$f(t) = 2\cdot\cos 3t\cdot e^{-2t} - \frac{1}{3}\sin 3t\cdot e^{-2t}$$

4b) $f(t) = e^{-t} * e^{t} = \displaystyle\int_0^t e^{-(t-x)}\cdot e^{x}\,dx$

faltning, har enkel transform. bara produkten av transf.

$\boxed{L10}$

Laplacetransf.

$$\mathcal{L}\{f(t)\} = \mathcal{L}\{e^{-t}\}\cdot\mathcal{L}\{e^{t}\}$$

$$= \frac{1}{(s+1)}\cdot\frac{1}{(s-1)} = \frac{A}{s+1} + \frac{B}{s-1} = \frac{-\frac{1}{2}}{s+1} + \frac{\frac{1}{2}}{s-1}$$

$$\mathcal{L}\{f(t)\} = -\frac{1}{2}\cdot\frac{1}{s+1} + \frac{1}{2}\cdot\frac{1}{s-1}$$

$\sim$

e^{-t} $\quad$ e^{+t}

invers transf.

$$f(t) = -\frac{1}{2}e^{-t} + \frac{1}{2}e^{t}$$

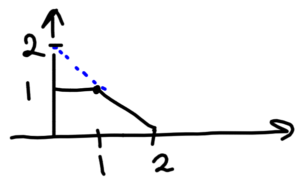
$$\frac{s-5}{(s-1)\,s\,(s+2)} = \frac{A}{s-1} + \frac{B}{s} + \frac{C}{s+2}$$

$$A = \frac{-4}{1 \cdot 3} = -\frac{4}{3}$$

$$B = \frac{-5}{-1 \cdot 1} = 5$$

$$C = \frac{-7}{-3\,(-2)} = \frac{7}{6}$$

$f(t)$



$$f(t) = \begin{cases} 1 & 0 \le t < 1 \\ -t+2 & 1 \le t < 2 \end{cases}$$

$$= 1 \cdot \left(\theta(t-0) - \theta(t-1) \right) +$$
$$+ (-t+2) \cdot \left(\theta(t-1) - \theta(t-2) \right) =$$

$$= \theta(t) \cdot 1 + \theta(t-1) \cdot \underbrace{\left[-1 - t + 2 \right]}_{\substack{1-t = \\ = -(t-1)}} + \theta(t-2)(t-2)$$

Laplace transf.

$$\mathcal{L}\{f(t)\} = \frac{1}{s} + e^{-s} \cdot \mathcal{L}\{-t\} + e^{-2s} \cdot \mathcal{L}\{t\}$$

Svar:

$$= \frac{1}{s} + e^{-s} \left(-\frac{1}{s^2} \right) + e^{-2s} \cdot \frac{1}{s^2}$$

$\sim$ $\qquad$ $\sim -t$ $\qquad$ $\sim t$

$$1 + \theta(t-1) \cdot (-(t-1)) \quad \theta(t-2) \cdot (t-2)$$

6) aug - 2015

$$y(t) = 3t^2 - e^{-t} - \underbrace{\int_0^t y(x) \cdot e^{t-x}\,dx}_{y(t) * e^t}$$

(L10)

Laplacetransf.

$$Y(s) = 3 \cdot \frac{2}{s^3} - \frac{1}{s+1} - \underbrace{\mathcal{L}\{y(t)\} \cdot \mathcal{L}\{e^t\}}_{Y(s) \cdot \frac{1}{s-1}}$$

Løs ut $Y(s) = Y$

$$\underbrace{Y + \frac{1}{s-1} \cdot Y} = \frac{6}{s^3} - \frac{1}{s+1}$$

$$\left(\underbrace{\frac{s-1}{s-1} + \frac{1}{s-1}}\right) \cdot Y = \frac{6}{s^3} - \frac{1}{s+1}$$

$$\left(\frac{s}{s-1}\right) \cdot Y = \frac{(s-1) \cdot 6}{s^4} - \frac{1(s-1)}{(s+1) \cdot s}$$

$$Y = \frac{6}{s^3} - \frac{6}{s^4} - \underbrace{\frac{s-1}{(s+1)(s)}}$$

$$= 3 \cdot \frac{2}{s^3} - \frac{3!}{s^4} - * \quad *$$

$$\sim 3 \cdot t^2 - t^3$$

$$* \quad \frac{s-1}{(s+1) \cdot s} = \frac{A}{s+1} + \frac{B}{s} = \frac{2}{s+1} + \frac{-1}{s}$$

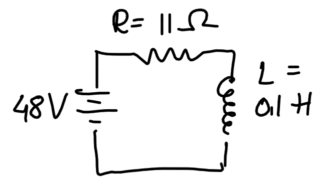
$$Y = 3 \cdot \frac{2}{s^3} - \frac{3!}{s^4} - \frac{2}{s+1} + \frac{1}{s}$$

inverstransf.

$$\underline{y(t) = 3t^2 - t^3 - 2e^{-t} + 1}$$

maj 2012

6)



b.v.

$$L \cdot \frac{dI}{dt} + R \cdot I = E(t) \qquad I(0) = 0$$

<u>Sök</u>: $I(0{,}02)$

$$0{,}1 \cdot I'(t) + 11 \cdot I(t) = 48$$

$$\boxed{I'(t) + 110 \cdot I(t) = 480}$$

- if: $e^{\int 110\,dt} = e^{110t}$
- mult med if.

$$\frac{d}{dt}\left(I(t) \cdot e^{110t}\right) = e^{110t} \cdot 480$$

- integrera

$$I(t) \cdot e^{110t} = \int e^{110t} \cdot 480\,dt$$

$$= 480 \cdot \frac{e^{110t}}{110} + C$$

$$I(t) = \frac{480}{110} + C \cdot e^{-110t}$$

b.v. $I(0) = 0 = \frac{480}{110} + C_1 \cdot e^0$

$$C_1 = -\frac{480}{110}$$

$$\therefore I(t) = \frac{480}{110} - \frac{480}{110} \cdot e^{-110t} = \frac{480}{110}\left(1 - e^{-110t}\right)$$

Sök: $I(0{,}02) = \frac{480}{110}\left(1 - e^{-110 \cdot 0{,}02}\right) \approx \underline{3{,}88 \text{ A}}$