Skisser, M0043M, 110604

```
(%i1)  /* UPPGIFT 1  */
       sol:solve(1-(cos(x))^2=0,[x]);
```

solve : using arc − trig functions to get a solution. Some solutions will be lost.

(%o1) $[x = \pi, x = 0]$

Skärningspunkter

```
(%i2)  sol[2];
```

area = $\int_0^{\pi}(1-\cos^2 x)\,dx = \int_0^{\pi}\left(1-\frac{1+\cos 2x}{2}\right)dx$

(%o2) $x = 0$

"övre − undre"

```
(%i3)  I:integrate(1-(cos(x))^2,x,0,%pi);
```

(%o3) $\frac{\pi}{2}$ $= \left[\frac{x}{2} - \frac{\sin 2x}{4}\right]_0^{\pi}$

```
(%i4)  /* UPPGIFT 2a  */
       I: integrate(2*x/(4+x^2),x,1,2);
```

$\left[\ln\left(4+x^2\right)\right]_1^2$

(%o4) $2\left(\frac{\log(8)}{2} - \frac{\log(5)}{2}\right) = \ln 8 - \ln 5$

(partiell integr.)

```
(%i5)  /* UPPGIFT 2b  */
       integrate(x*sin(x/2),x);
```

$\int x \sin\left(\frac{x}{2}\right)dx = -x\cdot 2\cos\left(\frac{x}{2}\right) + \int 2\cos(x/2)\,dx =$ (inre del)

$= -2x\cos(x/2) + 4\sin(x/2) + C$

(%o5) $4\left(\sin\left(\frac{x}{2}\right) - \frac{x\cos\left(\frac{x}{2}\right)}{2}\right)$

```
(%i6)  /* UPPGIFT 3  */
       A:matrix([1,-1],[2,1],[1,3]);
       B:matrix([2,1],[-1,1],[1,2]);
```

uppg 3a

(%o6) $\begin{pmatrix} 1 & -1 \\ 2 & 1 \\ 1 & 3 \end{pmatrix}$

(%o7) $\begin{pmatrix} 2 & 1 \\ -1 & 1 \\ 1 & 2 \end{pmatrix}$

```
(%i8)  AT:transpose(A);
```

(%o8) $\begin{pmatrix} 1 & 2 & 1 \\ -1 & 1 & 3 \end{pmatrix}$

```
(%i9)  AT.B;
```

(%o9) $\begin{pmatrix} 1 & 5 \\ 0 & 6 \end{pmatrix}$

$(2\times 3)\cdot(3\times 2)$ skalär. prod.

$\begin{pmatrix} 1 & 2 & 1 \\ -1 & 1 & 3 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ -1 & 1 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & \cdots \\ \cdots & \cdots \end{pmatrix}$ OSV

$$\left(\begin{array}{cc|cc} 1 & 5 & 1 & 0 \\ 0 & 6 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{cc|cc} 1 & 5 & 1 & 0 \\ 0 & 1 & 0 & 1/6 \end{array}\right) \sim$$

uppg 3b

$$\sim \left(\begin{array}{cc|cc} 1 & 0 & 1 & -5/6 \\ 0 & 1 & 0 & 1/6 \end{array}\right)$$

invers

```
(%i10) invert(AT.B);
```

(%o10) $\begin{pmatrix} 1 & -\frac{5}{6} \\ 0 & \frac{1}{6} \end{pmatrix}$

```
(%i11) /* UPPGIFT 4 */
       VOL:integrate(2*%pi*x*x*%e^(-x),x,0,4);
```

(%o11) $2\left(2-26\,e^{-4}\right)\pi$

cylindriskt rörelement

$$\int_0^4 2\pi x \cdot x e^{-x}\,dx = 2\pi \int x^2 \cdot e^{-x}\,dx = \left(\begin{matrix}\text{part}\\ \text{int}\end{matrix}\right)$$

```
(%i12) /* UPPGIFT 5 */
       M:matrix([b,1,2],[2,1,b],[b,0,1]);
```

(uppg 5a)

(%o12) $\begin{pmatrix} b & 1 & 2 \\ 2 & 1 & b \\ b & 0 & 1 \end{pmatrix}$

$$\begin{vmatrix} b & 1 & 2 \\ 2 & 1 & b \\ b & 0 & 1 \end{vmatrix} = \begin{vmatrix} b & 1 & 2 \\ 2-b & 0 & b-2 \\ b & 0 & 1 \end{vmatrix} =$$

$$= (-1)^{1+2} \cdot \begin{vmatrix} 2-b & b-2 \\ b & 1 \end{vmatrix} = -\left(2-b-b(b-2)\right)$$

```
(%i13) det:determinant(M);
```

(%o13) b^2-b-2

```
(%i14) solve(det=0);
```

(%o14) $[b=2, b=-1]$

$$= \boxed{-(2-b)(1+b)}$$

(uppg 5b)

```
(%i15) e1:b*x+y+2*z=1;
       e2:2*x+y+b*z=-b;
       e3:b*x+z=0;
```

(%o15) $2z+y+bx=1$

(%o16) $bz+y+2x=-b$

(%o17) $z+bx=0$

$$\left(\begin{array}{ccc|c} 2 & 1 & 2 & 1 \\ 2 & 1 & 2 & -2 \\ 2 & 0 & 1 & 1 \end{array}\right)$$ ← olöslig

```
(%i18) b:2;
```

(%o18) 2

```
(%i19) solve([e1,e2,e3],[x,y,z]);
```

(%o19) [] saknar lösn.

```
(%i20) b:-1;
```

(%o20) -1

$$\left(\begin{array}{ccc|c} -1 & 1 & 2 & 1 \\ 2 & 1 & -1 & 1 \\ -1 & 0 & 1 & 0 \end{array}\right)$$ (2), (−1)

```
(%i21) solve([e1,e2,e3],[x,y,z]);
```

solve : dependentequationseliminated : (1)

$$\sim \left(\begin{array}{ccc|c} -1 & 1 & 2 & 1 \\ 0 & 3 & 3 & 3 \\ 0 & -1 & -1 & -1 \end{array}\right) \sim \left(\begin{array}{ccc|c} -1 & 1 & 2 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

→

$y = 1-t$, $z = t$

$x = -1+1-t+2t = t$

$\bar{X} = \begin{pmatrix} t \\ 1-t \\ t \end{pmatrix}$

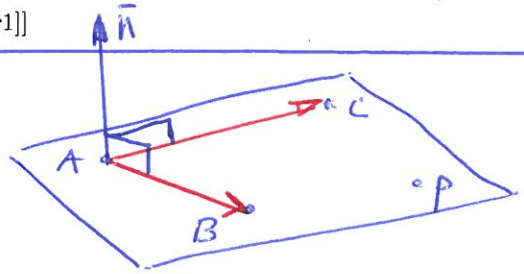
$=t$

(%o21) $[[x = \%r1, y = 1 - \%r1, z = \%r1]]$

(%i22) `/* UPPGIFT 6.1 */`
`A:[0,6,0];`

(uppg a)

(%o22) $[0, 6, 0]$



(%i23) `B:[3,0,0];`

(%o23) $[3, 0, 0]$

(%i24) `C:[0,1,-1];`

(%o24) $[0, 1, -1]$

(%i25) `P:[x,y,z];`

(%o25) $[x, y, z]$

(%i26) `AB:B-A;`

(%o26) $[3, -6, 0]$

(%i27) `AC:C-A;`

(%o27) $[0, -5, -1]$

(%i28) `load(vect);`

(%o28) */Applications/Maxima.app/Contents/Resources/maxima/share/maxima/5.24.0/share/vector/vec*

(%i29) `n:express(AB~AC);`

$\bar{n} = \bar{AB} \times \bar{AC} = \begin{pmatrix} 3 \\ -6 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ -5 \\ -1 \end{pmatrix}$

(%o29) $[6, 3, -15]$

(%i30) `AP:P-A;`

(%o30) $[x, y-6, z]$

(%i31) `load(eigen);`

(%o31) */Applications/Maxima.app/Contents/Resources/maxima/share/maxima/5.24.0/share/matrix/ei*

Planets ekv. →

(%i32) `inprod(n,AP)=0;`

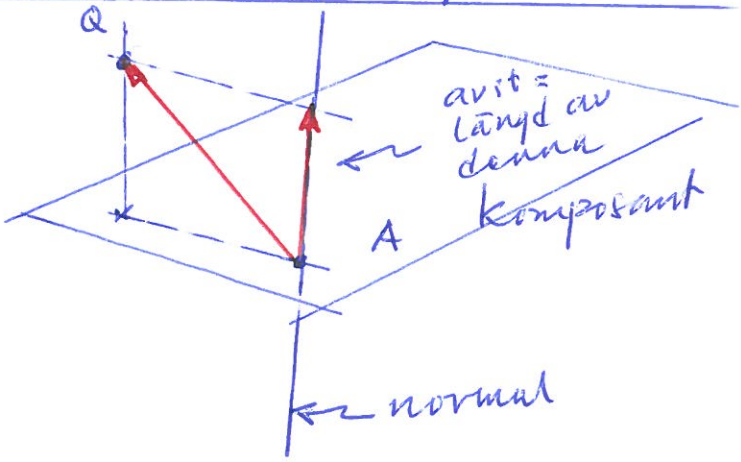
$\bar{n} \cdot \bar{AP} = 0 \quad \begin{pmatrix} 6 \\ 3 \\ -15 \end{pmatrix} \cdot \begin{pmatrix} x \\ y-6 \\ z \end{pmatrix} = 0$

(%o32) $-15z + 3y + 6x - 18 = 0$

(%i33) `Q:[4,1,0];`

(%o33) $[4, 1, 0]$

(uppg b)



avstånd $= |\overrightarrow{AQ} \cdot \bar{e}|$ (egenligen)

```
(%i34) AQ:Q-A;
```
(%o34) $[4,-5,0]$

```
(%i35) e:unitvector(n);
```
enhets normalvektor

(%o35) $[\frac{2}{\sqrt{30}},\frac{1}{\sqrt{30}},-\frac{5}{\sqrt{30}}]$

```
(%i36) dist:inprod(AQ,e);
```
(%o36) $\frac{3}{\sqrt{30}} = \sqrt{\frac{3}{10}}$

```
(%i37) /* UPPGIFT 6.2 */
       integrate((2*x^3-4*x^2-x-3)/(x^2-2*x-3),x);
```
(%o37) $2\log(x+1)+3\log(x-3)+x^2+C$

```
(%i38) T:2*x^3-4*x^2-x-3;
```
(%o38) $2x^3-4x^2-x-3$

```
(%i39) N:x^2-2*x-3;
```
(%o39) x^2-2x-3

```
(%i40) partfrac(T/N,x);
```
$\int(2x+\frac{2}{x+1}+\frac{3}{x-3})\,dx =$

(%o40) $\frac{2}{x+1}+2x+\frac{3}{x-3}$

```
(%i41) quotient(T,N);
```
(%o41) $2x$

```
(%i42) remainder(T,N);
```
(%o42) $5x-3$

gradtals kontroll kräver polynomdivision

```
(%i43) kill(all);
```
(%o0) *done*

$$\frac{2x^3-4x^2-x-3}{x^2-2x-3} = 2x+\frac{5x-3}{x^2-2x-3}$$

partialbråksansats

Ansats

$$\frac{5x-3}{x^2-2x-3} = \frac{A}{x+1}+\frac{B}{x-3}$$

faktorisera: $(x+1)(x-3)$