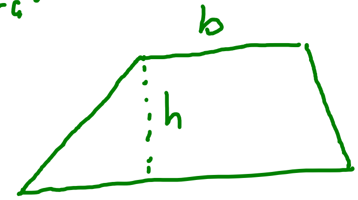
Ex "Föreläsning 1"

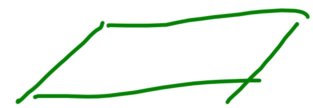
$$A(x) = \int_0^x f(x)\,dx$$

Parallelltrapets



$$A = \frac{h\cdot(a+b)}{2}$$

Parallellogram



$$\int u'v\,dx = uv - \int uv'\,dx$$

Ex:

$$\int \overset{=u'}{e^x}\cdot \overset{=v}{x}\,dx = \overset{=u}{e^x}\,\overset{=v}{x} - \int \overset{=u}{e^x}\,\overset{=v'}{1}\,dx$$

$$= e^x\cdot x - e^x + C$$

$$= e^x(x-1) + C$$

Stämmer det?

$$\frac{d}{dx}\left(e^x(x-1) + C\right) = e^x(x-1) + e^x\cdot 1 + 0$$

$$= e^x\cdot x - e^x + e^x = e^x\cdot x \quad \text{Ja!}$$

Ex:

$$\int \ln x\,dx = \int 1\cdot \ln x\,dx$$

$$= x\cdot \ln x - \int \overbrace{x\cdot\frac{1}{x}}^{=1}\,dx = x\cdot\ln x - x + C$$

Stämmer det?

$$\frac{d}{dx}\left(x\cdot\ln x - x + C\right) = 1\cdot\ln x + x\cdot\frac{1}{x} - 1 + 0$$

$$= \ln x + 1 - 1 = \ln x$$

Sista steget i exemplet

$$\ldots = x^2 \cdot \sin x - \left(2x(-\cos x) - \int 2\cdot(-\cos x)\,dx\right)$$

$$= x^2 \cdot \sin x + 2x\cos x - \int 2\cos x\,dx$$

$$= x^2 \cdot \sin x + 2x\cos x - 2\sin x + C$$

$$= (x^2 - 2)\sin x + 2x\cos x + C$$

Ex:

$$I = \int e^x \cdot \cos x\,dx = e^x \cdot \cos x - \int e^x(-\sin x)\,dx$$

$$= e^x \cdot \cos x + \int e^x \sin x\,dx$$

$$= e^x \cdot \cos x + e^x \cdot \sin x - \underbrace{\int e^x \cdot \cos x\,dx}_{= I}$$

$$I = e^x \cdot \cos x + e^x \cdot \sin x - I$$

$$2I = e^x \cdot \cos x + e^x \cdot \sin x + 2C$$

$$I = \frac{e^x \cdot \cos x + e^x \cdot \sin x}{2} + C$$

$$= \frac{1}{2} e^x(\cos x + \sin x) + C$$

Stämmer det?

$$\frac{d}{dx}\left(\frac{1}{2} e^x(\cos x + \sin x) + C\right) = \frac{1}{2} e^x(\cos x + \sin x) + \frac{1}{2} e^x(\overbrace{-\sin x}^{+\cos x})$$

$$= e^x \cos x$$

Ex:

$$\int x \cdot \arctan x \, dx = \frac{1}{2}x^2 \arctan x - \int \frac{1}{2}x^2 \cdot \frac{1}{1+x^2}\, dx$$

$$= \frac{1}{2}x^2 \cdot \arctan x - \frac{1}{2}\int \frac{x^2}{1+x^2}\, dx$$

$$= \frac{1}{2}x^2 \cdot \arctan x - \frac{1}{2}\int \frac{1+x^2-1}{1+x^2}\, dx$$

$$= \frac{1}{2}x^2 \arctan x - \frac{1}{2}\int \left(1 - \frac{1}{1+x^2}\right) dx$$

$$= \frac{1}{2}x^2 \arctan x - \frac{1}{2}\left(x - \arctan x\right) + C$$

$$= \frac{1}{2}(x^2+1)\arctan x - \frac{1}{2}x + C$$