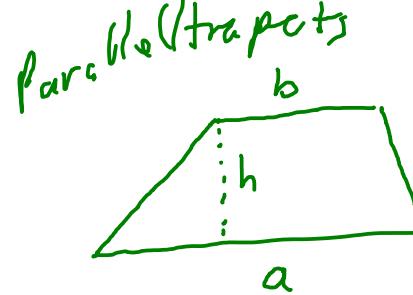


Ex "Föreläsning 1"

$$A(x) = \int_0^x f(x) dx$$



$$A = \frac{n \cdot (a+b)}{2}$$

$$\int u'v dx = uv - \int uv' dx$$

Parallelogram

$$\begin{aligned} \text{Ex: } & \quad \begin{array}{l} u' = v \\ u = v \end{array} \quad \begin{array}{l} u' = v \\ u = v \end{array} \quad \begin{array}{l} u = v \\ u = v \end{array} \\ & \int e^x \cdot x dx = e^x x - \int e^x \cdot 1 dx \\ & = e^x \cdot x - e^x + C \\ & = e^x (x-1) + C \end{aligned}$$



Stämmer det?

$$\begin{aligned} \frac{d}{dx} (e^x (x-1) + C) &= e^x (x-1) + e^x \cdot 1 + 0 \\ &= e^x \cdot x - e^x + e^x = e^x \cdot x \quad \text{Ja!} \end{aligned}$$

$$\text{Ex: } \int \ln x dx = \int 1 \cdot \ln x dx$$

$$\begin{aligned} &= x \cdot \ln x - \int \cancel{x} \cdot \frac{\cancel{1}}{x} dx = x \cdot \ln x - x + C \\ &\quad \downarrow \quad \uparrow \quad \stackrel{=1}{\cancel{x}} \end{aligned}$$

Stämmer det?

$$\begin{aligned} \frac{d}{dx} (x \cdot \ln x - x + C) &= 1 \cdot \ln x + x \cdot \frac{1}{x} - 1 + 0 \\ &= \ln x + 1 - 1 = \ln x \end{aligned}$$

Sista steget i exemplet

$$\begin{aligned}
 & \dots = x^2 \cdot \sin x - \left(2x(-\cos x) - \int 2 \cdot (-\cos x) dx \right) \\
 & = x^2 \cdot \sin x + 2x \cos x - \int 2 \cos x dx \\
 & = x^2 \cdot \sin x + 2x \cos x - 2 \sin x + C \\
 & = (x^2 - 2) \sin x + 2x \cos x + C
 \end{aligned}$$

Ex:

$$\begin{aligned}
 I &= \int e^x \cdot \cos x dx = e^x \cdot \cos x - \int e^x (-\sin x) dx \\
 &= e^x \cdot \cos x + \int e^x \sin x dx \\
 &= e^x \cdot \cos x + e^x \cdot \sin x - \underbrace{\int e^x \cos x dx}_{= I}
 \end{aligned}$$

$$I = e^x \cdot \cos x + e^x \cdot \sin x - I$$

$$2I = e^x \cdot \cos x + e^x \cdot \sin x + 2C$$

$$I = \frac{e^x \cdot \cos x + e^x \cdot \sin x}{2} + C$$

$$= \frac{1}{2} e^x (\cos x + \sin x) + C$$

Sätter in det?

$$\begin{aligned}
 \frac{d}{dx} \left(\frac{1}{2} e^x (\cos x + \sin x) + C \right) &= \frac{1}{2} e^x (\cos x + \sin x) + \frac{1}{2} e^x (\overbrace{-\sin x}^{+\cos x}) \\
 &\approx e^x \cos x
 \end{aligned}$$

$$\begin{aligned}
 & \underset{\substack{\uparrow \\ \rightarrow}}{\underline{x}} : \int x \cdot \arctan x \, dx = \frac{1}{2} x^2 \arctan x - \int \frac{1}{2} x^2 \cdot \frac{1}{1+x^2} \, dx \\
 &= \frac{1}{2} x^2 \arctan x - \frac{1}{2} \int \frac{x^2}{1+x^2} \, dx \\
 &= \frac{1}{2} x^2 \arctan x - \frac{1}{2} \int \frac{1+x^2-1}{1+x^2} \, dx \\
 &= \frac{1}{2} x^2 \arctan x - \frac{1}{2} \int \left(1 - \frac{1}{1+x^2}\right) \, dx \\
 &= \frac{1}{2} x^2 \arctan x - \frac{1}{2} \left(x - \arctan x\right) + C \\
 &= \frac{1}{2}(x^2+1) \arctan x - \frac{1}{2} x + C
 \end{aligned}$$