RH

$$I = \int x \cdot \sin(\ln x)\, dx$$

$$= \frac{x^2}{2}\sin(\ln x) - \int \frac{x^2}{2}\cos(\ln x)\cdot\frac{1}{x}\, dx$$

$$= \frac{x^2}{2}\sin(\ln x) - \frac{1}{2}\int x\cdot\cos(\ln x)\, dx$$

$$= \frac{x^2}{2}\sin(\ln x) - \frac{1}{2}\,\frac{x^2}{2}\cos(\ln x) + \frac{1}{2}\int \frac{x^2}{2}\left(-\sin(\ln x)\right)\frac{1}{x}\, dx$$

$$= \frac{x^2}{2}\sin(\ln x) - \frac{x^2}{4}\cos(\ln x) - \frac{1}{4}\underbrace{\int x\sin(\ln x)\, dx}_{I}$$

$$I = \frac{x^2}{2}\sin(\ln x) - \frac{x^2}{4}\cos(\ln x) - \frac{1}{4}I$$

$$\frac{5}{4}I = \frac{x^2}{2}\sin(\ln x) - \frac{x^2}{4}\cos(\ln x) + C$$

$$I = \frac{x^2}{5}\left(2\sin(\ln x) - \cos(\ln x)\right) + D$$

Ex:

$$\int (x^2-1)^2 \cdot 2x\, dx = \left[\begin{aligned} u &= x^2-1 \\ \frac{du}{dx} &= 2x \\ du &= 2x\, dx \end{aligned}\right]$$

$$= \int u^2 \cdot du = u^3 \cdot \frac{1}{3} + C$$

$$= \frac{1}{3}(x^2-1)^3 + C$$

Ex:

$$\int \sin x \cos x\, dx = \left[\begin{aligned} u &= \sin x \\ \frac{du}{dx} &= \cos x \\ du &= \cos x \cdot dx \end{aligned}\right]$$

$$= \int u\, du = \frac{u^2}{2} + C = \frac{\sin^2 x}{2} + C$$

Ex:

$$\int x^2 \sqrt{x^3+4}\, dx = \left[\begin{aligned} u &= x^3+4 \\ \frac{du}{dx} &= 3x^2 \\ \frac{1}{3} du &= x^2 dx \end{aligned}\right]$$

$$= \int \sqrt{u}\, \frac{1}{3} du = \int \frac{1}{3} u^{1/2} du$$

$$= \frac{1}{3} u^{3/2} \frac{1}{3/2} + C = \frac{2}{9} u^{3/2} + C$$

$$= \frac{2}{9}(x^3+4)^{3/2} + C$$

Ex:

$$\int \frac{x}{3+4x^2}\,dx = \int x \cdot \frac{1}{3+4x^2}\,dx = \begin{bmatrix} u = 3+4x^2 \\ \frac{du}{dx} = 8x \\ \frac{1}{8}du = x \cdot dx \end{bmatrix}$$

$$= \int \frac{1}{u} \cdot \frac{1}{8}\,du = \ln|u| \cdot \frac{1}{8} + C$$

$$= \ln|3+4x^2| \cdot \frac{1}{8} + C = \frac{1}{8}\ln(3+4x^2) + C$$

Ex:

$$\int \frac{\ln x}{x}\,dx = \int \frac{1}{x} \cdot \ln x \cdot dx = \begin{bmatrix} u = \ln x \\ \frac{du}{dx} = \frac{1}{x} \\ du = \frac{1}{x} \cdot dx \end{bmatrix}$$

$$= \int u\,du = \frac{u^2}{2} + C = \frac{(\ln x)^2}{2} + C$$

Ex:

$$\int \tan x\,dx = \int \frac{\sin x}{\cos x}\,dx = \int \sin x \cdot \frac{1}{\cos x}\,dx$$

$$= \begin{bmatrix} u = \cos x \\ \frac{du}{dx} = -\sin x \\ -du = \sin x \cdot dx \end{bmatrix} = \int \frac{1}{u}(-du) = -\ln|u| + C$$

$$= -\ln|\cos x| + C$$

Ex:

$$\int \cos^3 x\,dx = \int \cos x \cdot \cos^2 x\,dx$$

$$= \int \cos x\,(1-\sin^2 x)\,dx = \begin{bmatrix} u = \sin x \\ \frac{du}{dx} = \cos x \\ du = \cos x \cdot dx \end{bmatrix}$$

$$= \int (1-u^2)\,du = u - \frac{u^3}{3} + C$$

$$= \sin x - \frac{1}{3}\sin^3 x + C$$

Ex:

$$\int \frac{1}{x^2+4x+5}\,dx =$$

Kvadrat komplettering

$$x^2+4x+5 = \underbrace{(x+2)^2}_{=x^2+4x+4}-4+5 = (x+2)^2+1$$

$$= \int \frac{1}{(x+2)^2+1}\,dx = \begin{bmatrix} u = x+2 \\ \frac{du}{dx} = 1 \\ du = dx \end{bmatrix} = \int \frac{1}{u^2+1}\,du$$

$$= \arctan u + C = \arctan(x+2) + C$$

Ex:

$$\int x\sqrt{x-4}\,dx = \begin{bmatrix} u = x-4 \\ x = u+4 \\ \frac{dx}{du} = 1 \\ dx = du \end{bmatrix}$$

$$= \int (u+4)\underbrace{\sqrt{u}}_{u^{1/2}}\,du$$

$$= \int \left(u^{3/2}+4u^{1/2}\right)du = u^{5/2}\cdot\frac{2}{5} + 4u^{3/2}\cdot\frac{2}{3} + C$$

$$= \frac{2}{5}(x-4)^{5/2} + \frac{8}{3}(x-4)^{3/2} + C$$