Rep A3

- $\int \sin^3 x\, dx = \int \sin x \cdot \sin^2 x\, dx$

$$= \int \sin x\,(1-\cos^2 x)\, dx = \left[\begin{array}{l} u = \cos x \\ \frac{du}{dx} = -\sin x \\ -du = \sin x \cdot dx \end{array}\right]$$

$$= \int (1-u^2)(-du)$$

$$= -\int (1-u^2)\, du = -\left(u - \frac{u^3}{3}\right) + C$$

$$= -\cos x + \frac{\cos^3 x}{3} + C$$

- $\int e^{2x}\sqrt{1+e^x}\, dx = \int (e^x)^2 \sqrt{1+e^x}\, dx$

$$= \int e^x \cdot e^x \sqrt{1+e^x}\, dx$$

$$= \int e^x (1+e^x-1)\sqrt{1+e^x}\, dx = \left[\begin{array}{l} u = 1+e^x \\ \frac{du}{dx} = e^x \\ du = e^x dx \end{array}\right]$$

$$= \int (u-1)\overbrace{\sqrt{u}}^{u^{1/2}}\, du$$

$$= \int (u^{3/2} - u^{1/2})\, du = u^{5/2}\frac{2}{5} - u^{3/2}\frac{2}{3} + C$$

$$= \frac{2}{5}(1+e^x)^{5/2} - \frac{2}{3}(1+e^x)^{3/2} + C$$

Ex:

$$\int \frac{x^3+3x^2}{1+x^2}\,dx$$

Vi vill ha lägre gradtal i täljaren än i nämnaren $\Rightarrow$ Polynomdivision

$$\begin{array}{r|l} & x + 3 \quad \leftarrow \text{kvot } K(x) \\ \hline x^2+1 & x^3 + 3x^2 \\ & -(x^3 + x) \\ \hline & 3x^2 - x \\ & -(3x^2 + 3) \\ \hline & -x-3 \quad \leftarrow \text{rest } R(x) \end{array}$$

$$\frac{x^3+3x^2}{x^2+1} = x+3+\frac{-x-3}{x^2+1}$$

$$\int \frac{x^3+3x^2}{x^2+1}\,dx = \int \left(x+3+\frac{-x-3}{x^2+1}\right)dx$$

$$= \int \left(x+3-\frac{x}{x^2+1}-3\cdot\frac{1}{1+x^2}\right)dx$$

$$= \frac{x^2}{2}+3x-\frac{1}{2}\ln|x^2+1|-3\cdot\arctan x + C$$

Ex:

$$\frac{x^2+6x+13}{(x-1)(x+1)(x-3)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{x-3}$$

$$= \frac{A(x+1)(x-3)}{(x-1)(x+1)(x-3)} + \frac{B(x-1)(x-3)}{(x-1)(x+1)(x-3)} + \frac{C(x-1)(x+1)}{(x-1)(x+1)(x-3)}$$

$$= \frac{A(x^2-2x-3)+B(x^2-4x+3)+C(x^2-1)}{(x-1)(x+1)(x-3)}$$

$$= \frac{(A+B+C)x^2+(-2A-4B)x+(-3A+3B-C)}{(x-1)(x+1)(x-3)}$$

$$= \frac{x^2+6x+13}{(x-1)(x+1)(x-3)}$$

$$\begin{array}{l} (x^2) \\ (x) \\ (1) \end{array} \begin{cases} A+B+C = 1 \\ -2A-4B = 6 \\ -3A+3B-C = 13 \end{cases} \quad 1x$$

$$1x \begin{cases} A+B+C = 1 \\ -2A-4B = 6 \\ -2A+4B = 14 \end{cases} \qquad \begin{cases} A+B+C = 1 \\ -2A-4B = 6 \\ -4A = 20 \end{cases}$$

$$A=-5 \qquad B=1 \qquad C=5$$

Ex:

$$\frac{x^3+x+1}{x(x-1)(x-2)(x-3)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x-2} + \frac{D}{x-3}$$

$A = \ldots \quad B = \ldots \qquad C = \frac{11}{-2} = -\frac{11}{2} \qquad D = \ldots$

Ex:

$$\int \frac{1}{x^3+2x^2+2x}\,dx$$

Gradtal? Ok!

Faktorisera $\quad x^3+2x^2+2x = x(x^2+2x+2)$

Partialbråk

$$\frac{1}{x(x^2+2x+2)} = \frac{A}{x} + \frac{Bx+C}{x^2+2x+2}$$

$$= \dots = \frac{A(x^2+2x+2) + Bx^2+Cx}{x(x^2+2x+2)}$$

$$= \frac{(A+B)x^2+(2A+C)x+2A}{x(x^2+2x+2)}$$

$$\begin{cases} A+B \qquad\quad = 0 \\ 2A \qquad + C = 0 \\ 2A \qquad\quad\;\; = 1 \end{cases} \qquad\qquad \begin{aligned} A &= \tfrac{1}{2} \\ B &= -\tfrac{1}{2} \\ C &= -1 \end{aligned}$$

$$\int \frac{1}{x^3+2x^2+2x}\,dx = \int\left(\frac{1/2}{x} + \frac{-\frac{1}{2}x-1}{x^2+2x+2}\right)dx$$