Rep A5

$$\int \sin^3 x \cdot \cos^8 x \cdot dx$$

$$= \int \sin x \cdot \underbrace{\sin^2 x}_{=1-\cos^2 x} \cdot \cos^8 x \, dx$$

$$= \int \sin x \left(1-\cos^2 x\right) \cos^8 x \, dx = \left[\begin{array}{l} u = \cos x \\ \frac{du}{dx} = -\sin x \\ -du = \sin x \, dx \end{array}\right.$$

$$= \int (1-u^2)\, u^8 (-du)$$

$$= -\int (u^8 - u^{10})\, du = -\left(\tfrac{1}{9}u^9 - \tfrac{1}{11}u^{11}\right) + C$$

$$= \frac{1}{11}\cos^{11} x - \frac{1}{9}\cos^9 x + C \quad \square$$

Ex: Omöjligt att finna primitiv funktion

$$\int e^{-x^2} dx$$

den finns men kan ej beskrivas med elementära funktioner

Ex

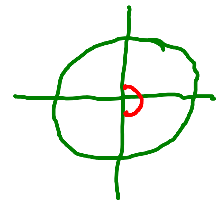
$$\int e^{\sqrt{x}} dx = \left[\begin{array}{l} u = \sqrt{x} \\ x = u^2 \\ \frac{dx}{du} = 2u \\ dx = 2u\, du \end{array}\right] = \int e^u \, 2u \, du$$

$$= e^u \cdot 2u - \int e^u \cdot 2 \, du = e^u \cdot 2 \cdot u - 2e^u + C$$

$$= 2e^u(u-1) + C = 2e^{\sqrt{x}}(\sqrt{x}-1) + C$$

Ex: Substitution med $x = a \sin\theta$

$$\int \frac{1}{(5-x^2)^{3/2}}\,dx = \left[\begin{array}{l} x = \sqrt{5}\sin\theta \\ \frac{dx}{d\theta} = \sqrt{5}\cos\theta \\ dx = \sqrt{5}\cos\theta\, d\theta \end{array}\right] \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$$



$$= \int \frac{1}{(5 - 5\cdot\sin^2\theta)^{3/2}}\sqrt{5}\cos\theta\, d\theta$$

$$\sqrt{x^2} = |x|$$

$$= \int \frac{1}{(5\underbrace{(1-\sin^2\theta)}_{=\cos^2\theta})^{3/2}}\sqrt{5}\cos\theta\, d\theta$$

$$= \int \frac{1}{5^{3/2}\left(\sqrt{\cos^2\theta}\right)^3}\sqrt{5}\cos\theta\, d\theta$$

$$= \int \frac{1}{5^{3/2}(\underbrace{|\cos\theta|}_{>0})^3}\sqrt{5}\cos\theta\, d\theta$$

$$= \int \frac{\sqrt{5}\cos\theta}{5^{3/2}\cos^3\theta}\,d\theta = \int 5^{1/2-3/2}\cdot\frac{1}{\cos^2\theta}\,d\theta$$

$$= 5^{-1}\tan\theta + C = \frac{1}{5}\tan\left(\arcsin\frac{x}{\sqrt{5}}\right) + C$$

$$= \frac{1}{5}\cdot\frac{\sqrt{5}\sin\theta}{\sqrt{5}\cos\theta} + C = \frac{1}{5}\,\frac{\sqrt{5}\sin\theta}{\sqrt{5}\sqrt{1-\sin^2\theta}} + C$$

$$= \frac{1}{5}\,\frac{x}{\sqrt{5}\sqrt{1-\frac{x^2}{5}}} + C = \frac{1}{5}\,\frac{x}{\sqrt{5\left(1-\frac{x^2}{5}\right)}} + C$$

$$= \frac{x}{5\sqrt{5-x^2}} + C$$

<u>Ex</u>:

$$\int x\cdot \arcsin x \, dx = \left[\begin{array}{l} \theta = \arcsin x \\ x = \sin\theta \\ \frac{dx}{d\theta} = \cos\theta \\ dx = \cos\theta \, d\theta \end{array}\right] \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$$

$$= \int \sin\theta \cdot \theta \cos\theta \, d\theta =$$

$$= \int \theta \, \frac{1}{2}\sin 2\theta \, d\theta = \theta\,\frac{1}{2}\left(-\cos 2\theta\right)\cdot\frac{1}{2} - \int \frac{1}{2}\left(-\cos 2\theta\right)\frac{1}{2}\,d\theta$$

$$= -\frac{\theta}{4}\cos 2\theta + \frac{1}{4}\int \cos 2\theta \, d\theta$$

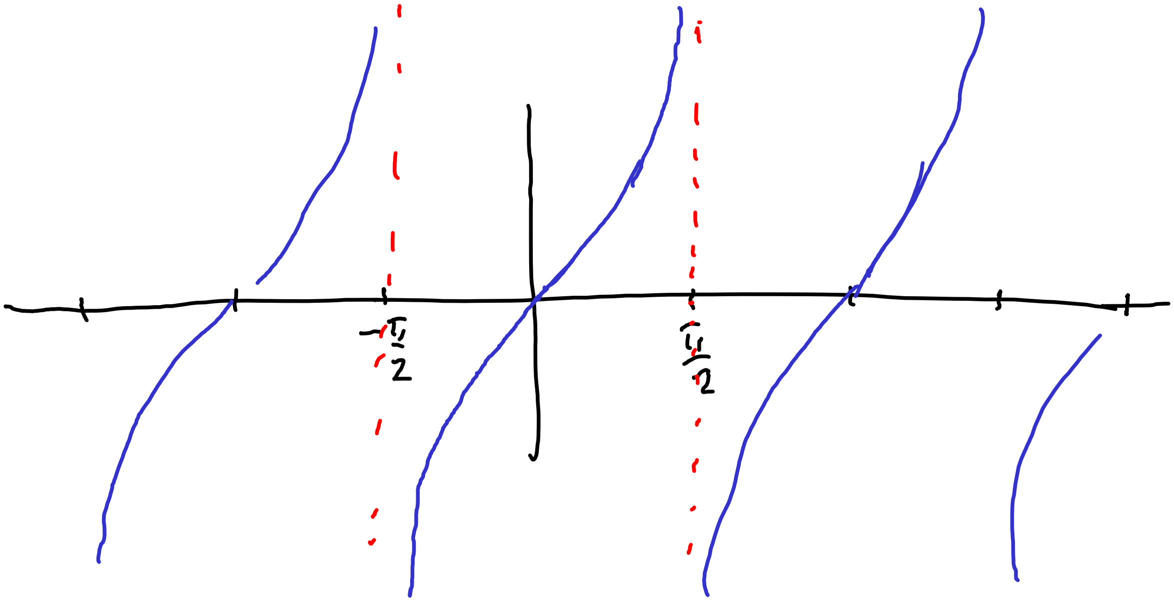
$$= -\frac{\theta}{4}\cos 2\theta + \frac{1}{4}\sin 2\theta \, \frac{1}{2} + C$$

$$= \frac{1}{8}\sin 2\theta - \frac{\theta}{4}\cos 2\theta + C$$

$$= \frac{1}{8}\,2\sin\theta\,\underbrace{\cos\theta}_{=\sqrt{1-\sin^2\theta}} - \frac{\theta}{4}\left(1 - 2\sin^2\theta\right) + C$$

$$= \frac{1}{4}\,x\sqrt{1-x^2} - \frac{\arcsin x}{4}\left(1-2x^2\right) + C$$

tan x — ser ut hur?



Ex:

$$\int \frac{x^2}{(1+x^2)^2}\,dx = \left[\begin{array}{l} x = \tan\theta \\ \dfrac{dx}{d\theta} = \dfrac{1}{\cos^2\theta} \\ dx = \dfrac{1}{\cos^2\theta}\,d\theta \end{array}\right]$$

$$= \int \frac{\tan^2\theta}{(1+\tan^2\theta)^2}\,\frac{1}{\cos^2\theta}\,d\theta$$

$$= \int \frac{\dfrac{\sin^2\theta}{\cos^2\theta}}{\left(\dfrac{1}{\cos^2\theta}\right)^2}\,\frac{1}{\cos^2\theta}\,d\theta$$

$$= \int \frac{\sin^2\theta}{\cos^2\theta\,\dfrac{1}{\cos^4\theta}\,\cos^2\theta}\,d\theta = \int \sin^2\theta\,d\theta$$

$$= \int \frac{1}{2}(1 - \cos 2\theta)\, d\theta = \frac{1}{2}\left(\theta - \sin 2\theta \, \frac{1}{2}\right) + C$$

$$= \frac{1}{2}\theta - \frac{1}{4}\sin 2\theta + C$$

$$\frac{1}{\cos^2\theta} = 1 + \tan^2\theta$$

$$\cos^2\theta = \frac{1}{1+\tan^2\theta}$$

$$= \frac{\theta}{2} - \frac{1}{4}\, 2 \sin\theta \cos\theta + C$$

$$= \frac{\theta}{2} - \frac{1}{2} \overbrace{\frac{\sin\theta}{\cos\theta}}^{= \tan\theta} \overbrace{\cos^2\theta}^{\frac{1}{1+\tan^2\theta}} + C$$

$$= \frac{\theta}{2} - \frac{1}{2}\tan\theta \, \frac{1}{1+\tan^2\theta} + C$$

$$= \frac{1}{2}\arctan x - \frac{1}{2}\, x \, \frac{1}{1+x^2} + C$$