

# Rep A6

$$\sqrt{a^2 - x^2} \rightarrow x = a \cdot \sin \theta$$

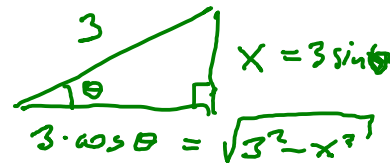
$$\int \frac{\sqrt{9-x^2}}{x^2} dx = \left[ \begin{array}{l} x = 3 \sin \theta \\ \frac{dx}{d\theta} = 3 \cos \theta \\ dx = 3 \cos \theta d\theta \end{array} \right]_{-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}}$$

$$= \int \frac{\sqrt{9 - 3^2 \cdot \sin^2 \theta}}{3^2 \sin^2 \theta} 3 \cos \theta d\theta$$

$$x = 3 \sin \theta$$

$$\frac{x}{3} = \sin \theta$$

$$= \int \frac{\sqrt{9(1 - \sin^2 \theta)}}{3 \sin^2 \theta} \cos \theta d\theta$$



$$= \int \frac{\cos \theta d\theta}{\sin^2 \theta} \quad -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$$

$$= \int \frac{3 |\cos \theta|}{3 \sin^2 \theta} \cos \theta d\theta$$

$$= \int \frac{\cos^2 \theta}{\sin^2 \theta} d\theta = \int \frac{1 - \sin^2 \theta}{\sin^2 \theta} d\theta = \int \left( \frac{1}{\sin^2 \theta} - 1 \right) d\theta$$

$$\frac{d}{d\theta} \cot \theta = \frac{d}{d\theta} \frac{\cos \theta}{\sin \theta} = \frac{(-\sin \theta) \cdot \sin \theta - \cos \theta \cdot \cos \theta}{\sin^2 \theta}$$

$$= (-1) \frac{\sin^2 \theta + \cos^2 \theta}{\sin^2 \theta} = -\frac{1}{\sin^2 \theta}$$

$$= -\cot \theta - \theta + C = -\frac{\cos \theta}{\sin \theta} - \theta + C = -\frac{3 \cos \theta}{3 \sin \theta} - \theta + C$$

$$= -\frac{\sqrt{9-x^2}}{x} - \arcsin \frac{x}{3} + C$$

Medelvärde av funktion

$$\frac{1}{b-a} \int_a^b f(x) dx = f(c)$$

↑  
det finns  $a < c < b$   
så detta gäller

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Ex: Medelvärde av  $f(x) = 2x$  på  $[1, 5]$

$$\begin{aligned} \bar{f} &= \frac{1}{5-1} \int_1^5 f(x) dx = \frac{1}{4} \left( 4 \cdot 2 + \frac{4 \cdot 8}{2} \right) \\ &= \frac{1}{4} 24 = 6 \end{aligned}$$

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Ex: Medelvärde av  $f(x) = e^{-x} + \cos x$   
på  $\left[-\frac{\pi}{2}, 0\right]$

$$\begin{aligned} \bar{f} &= \frac{1}{0 - \left(-\frac{\pi}{2}\right)} \int_{-\frac{\pi}{2}}^0 (e^{-x} + \cos x) dx \\ &= \frac{2}{\pi} \left[ e^{-x} \frac{1}{-1} + \sin x \right]_{-\frac{\pi}{2}}^0 = \end{aligned}$$

$$= \frac{2}{\pi} \left( -e^{-0} + \sin 0 - \left( -e^{-(-\frac{\pi}{2})} + \sin(-\frac{\pi}{2}) \right) \right)$$
$$= \frac{2}{\pi} \left( -1 + 0 + e^{\frac{\pi}{2}} + 1 \right) = \frac{2}{\pi} e^{\frac{\pi}{2}}$$