Rep A6

$\sqrt{a^2 - x^2} \Rightarrow x = a \cdot \sin\theta$

$$\int \frac{\sqrt{9-x^2}}{x^2}\,dx = \left[\begin{array}{l} x = 3\sin\theta \\ \frac{dx}{d\theta} = 3\cos\theta \\ dx = 3\cos\theta\, d\theta \end{array}\right] \quad -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$$

$x = 3\sin\theta$

$\frac{x}{3} = \sin\theta$

$3 \cdot \cos\theta = \sqrt{3^2 - x^2}$

$$= \int \frac{\sqrt{9 - 3^2 \cdot \sin^2\theta}}{3^2 \sin^2\theta}\, 3\cos\theta\, d\theta$$

$$= \int \frac{\sqrt{9\,(1-\sin^2\theta)}}{3\sin^2\theta}\cos\theta\, d\theta$$

$= \cos\theta \quad d\theta \quad -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

$$= \int \frac{3\,|\cos\theta|}{3\sin^2\theta}\cos\theta\, d\theta$$

$$= \int \frac{\cos^2\theta}{\sin^2\theta}\, d\theta = \int \frac{1-\sin^2\theta}{\sin^2\theta}\, d\theta = \int\left(\frac{1}{\sin^2\theta} - 1\right)d\theta$$

$$\boxed{\begin{array}{l} \frac{d}{d\theta}\cot\theta = \frac{d}{d\theta}\frac{\cos\theta}{\sin\theta} = \frac{(-\sin\theta)\cdot\sin\theta - \cos\theta\cdot\cos\theta}{\sin^2\theta} \\ = (-1)\,\frac{\sin^2\theta + \cos^2\theta}{\sin^2\theta} = -\frac{1}{\sin^2\theta} \end{array}}$$

$$= -\cot\theta - \theta \underbrace{=}_{+C} -\frac{\cos\theta}{\sin\theta} - \theta \underbrace{=}_{+C} -\frac{3\cos\theta}{3\sin\theta} - \theta + C$$

$$= -\frac{\sqrt{9-x^2}}{x} - \arcsin\frac{x}{3} + C$$

Medelvärdet av funktion

$$\frac{1}{b-a}\int_a^b f(x)\,dx = f(c)$$

det finns $a < c < b$ så detta gäller

Ex: Medelvärdet av $f(x) = 2x$ på $[1, 5]$

$$\bar{f} = \frac{1}{5-1}\int_1^5 f(x)\,dx = \frac{1}{4}\left(4\cdot 2 + \frac{4\cdot 8}{2}\right)$$

$$= \frac{1}{4}\,24 = 6$$

Ex: Medelvärdet av $f(x) = e^{-x} + \cos x$ på $\left[-\frac{\pi}{2}, 0\right]$

$$\bar{f} = \frac{1}{0-\left(-\frac{\pi}{2}\right)}\int_{-\frac{\pi}{2}}^{0}\left(e^{-x} + \cos x\right)dx$$

$$= \frac{2}{\pi}\left[e^{-x}\frac{1}{-1} + \sin x\right]_{-\frac{\pi}{2}}^{0} =$$

$$= \frac{2}{\pi}\left(-e^{-0} + \sin 0 - \left(-e^{-\left(-\frac{\pi}{2}\right)} + \sin\left(-\frac{\pi}{2}\right)\right)\right)$$

$$= \frac{2}{\pi}\left(-1 + 0 + e^{\frac{\pi}{2}} + 1\right) = \frac{2}{\pi} e^{\frac{\pi}{2}}$$