Rep A8

(a)

$$\int_1^3 x\sqrt[3]{x^2-1}\,dx = \left[\begin{array}{l|l} u = x^2-1 & x=1 \Downarrow u=0 \\ \frac{du}{dx} = 2x & \\ \frac{1}{2}du = x\cdot dx & x=3 \Downarrow u=8 \end{array}\right]$$

$$= \int_0^8 \sqrt[3]{u}\cdot\frac{1}{2}\,du = \int_0^8 \frac{1}{2}u^{1/3}\,du = \left[\frac{1}{2}\,u^{4/3}\,\frac{1}{4/3}\right]_0^8$$

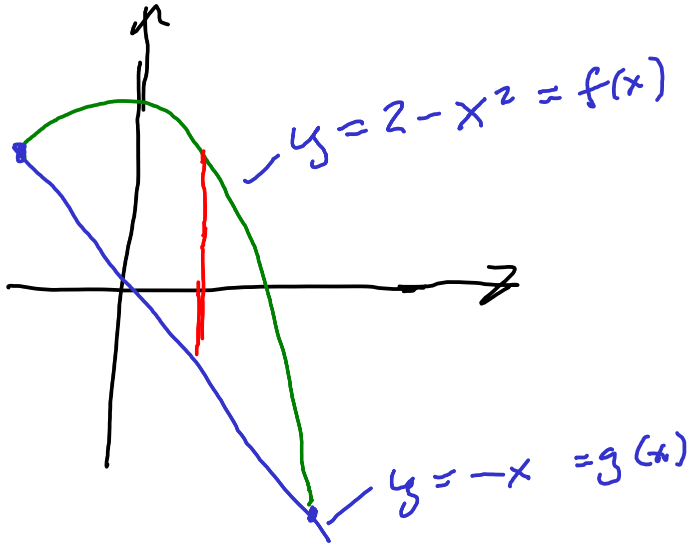
$$= \left[\frac{3}{8}u^{4/3}\right]_0^8 = \frac{3}{8}\,8^{4/3} - \frac{3}{8}\,0^{4/3} = \frac{3}{8}\cdot 16 = 6$$

(b)

$$\int_1^3 x\sqrt[3]{x^2-1}\,dx = \left[\begin{array}{l|l} u^3 = x^2-1 & x=1 \Downarrow u=0 \\ u = \sqrt[3]{x^2-1} & \\ x = \sqrt{u^3+1} & x=3 \Downarrow u=2 \\ \frac{dx}{du} = \frac{1}{2\sqrt{u^3+1}}\,3u^2 & \\ dx = \frac{3}{2}\,\frac{u^2}{\sqrt{u^3+1}}\,du & \end{array}\right]$$

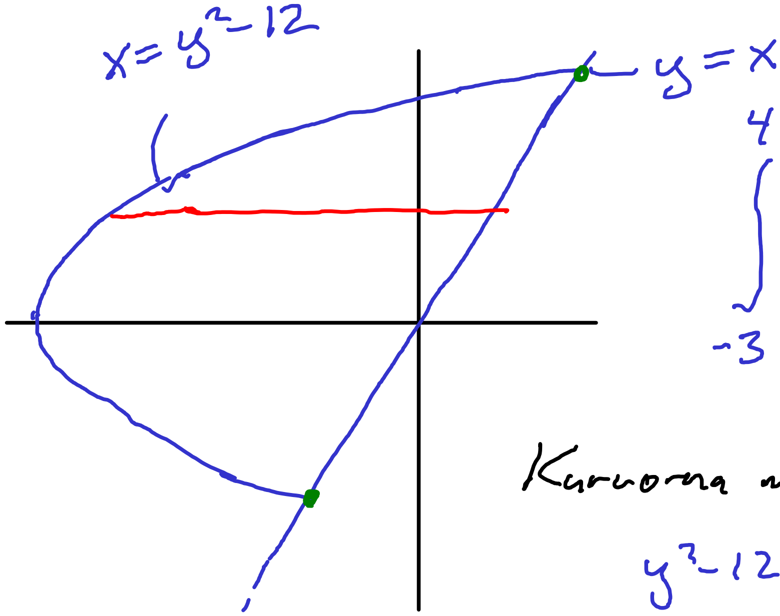
$$= \int_0^2 \cancel{\sqrt{u^3+1}}\cdot u\cdot\frac{3}{2}\,\frac{u^2}{\cancel{\sqrt{u^3+1}}}\,du = \int_0^2 \frac{3}{2}u^3\,du$$

$$= \left[\frac{3}{2}u^4\cdot\frac{1}{4}\right]_0^2 = \left[\frac{3}{8}u^4\right]_0^2 = \frac{3}{8}\underbrace{2^4}_{16} - \frac{3}{8}\,0^4 = 6$$



$$\int_a^b (f(x) - g(x))\,dx$$

Ex:



$$\int_{-3}^{4} (y - (y^2 - 12))\,dy$$

Kurvorna möts i:

$y^2 - 12 = y$

$y^2 - y - 12 = 0$

$(y+3)(y-4) = 0$

$y = -3$ eller $y = 4$

$$\int_{-3}^{4} (y - y^2 + 12)\,dy = \left[\frac{y^2}{2} - \frac{y^3}{3} + 12y\right]_{-3}^{4}$$

$$= \frac{4^2}{2} - \frac{4^3}{3} + 12 \cdot 4 - \frac{(-3)^2}{2} + \frac{(-3)^3}{3} - 12(-3)$$

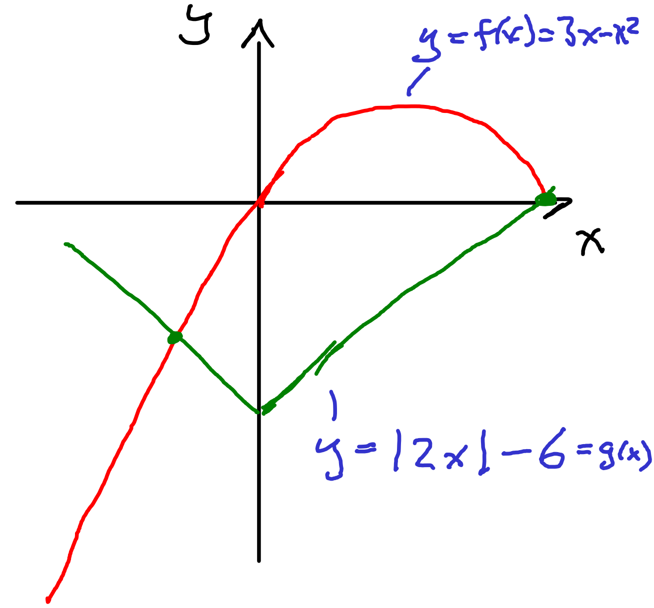
$$= \frac{16}{2} - \frac{64}{3} + 48 - \frac{9}{2} - \frac{27}{3} + 36$$

$$= \frac{7}{2} - \frac{91}{3} + 84 = \frac{21}{6} - \frac{182}{6} + \dots = \frac{343}{6}$$

Ex:

$$f(x) = 3x - x^2$$

$$g(x) = |2x| - 6$$



Skæringspunkter

$x<0$ (0)	$x>0$
$3x - x^2 = -2x - 6$	$3x - x^2 = 2x - 6$
$x^2 - 5x - 6 = 0$	$x^2 - x - 6 = 0$
$(x+1)(x-6) = 0$	$(x+2)(x-3) = 0$
$x = -1$ ~~eller $x = 6$~~	~~$x = -2$~~ eller $x = 3$

$$A = \int_{-1}^{3} \left(f(x) - g(x)\right) dx = \int_{-1}^{3} \left(3x - x^2 - (|2x| - 6)\right) dx$$

$$= \int_{-1}^{0} \left(3x - x^2 - (-2x) + 6\right) dx + \int_{0}^{3} \left(3x - x^2 - 2x + 6\right) dx$$

$$= \int_{-1}^{0} \left(5x - x^2 + 6\right) dx + \int_{0}^{3} \left(x - x^2 + 6\right) dx$$

$$= \left[5\frac{x^2}{2} - \frac{x^3}{3} + 6x\right]_{-1}^{0} + \left[\frac{x^2}{2} - \frac{x^3}{3} + 6x\right]_{0}^{3}$$

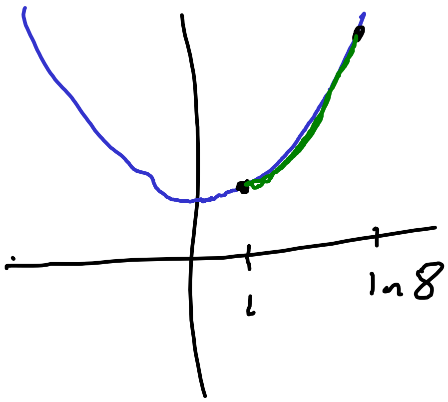
$$= 0 - \frac{5}{2} + \frac{(-1)^3}{3} - 6(-1) + \frac{9}{2} - \frac{27}{3} + 6\cdot 3 - 0$$

$$= -\frac{5}{2} - \frac{1}{3} + 6 + \frac{9}{2} - 9 + 18 = -\frac{1}{3} + 2 + 15 = 16 + \frac{2}{3}$$

Båglängd för $y=f(x)$ då $a \le x \le b$

$$s = \int_a^b ds = \int_a^b \sqrt{1+(f'(x))^2}\,dx$$

Ex: Båglängd för $f(x) = \dfrac{e^x + e^{-x}}{2}$, $1 \le x \le \ln 8$



$$s = \int_1^{\ln 8} \sqrt{1+(f'(x))^2}\,dx$$

$$f'(x) = \frac{1}{2}\left(e^x - e^{-x}\right)$$

$$s = \int_1^{\ln 8} \sqrt{1+\left(\frac{1}{2}\right)^2\left(e^x - e^{-x}\right)^2}\,dx$$

$$= \int_1^{\ln 8} \sqrt{1+\frac{1}{4}\left(e^{2x} - 2\underbrace{e^x \cdot e^{-x}}_{=1} + e^{-2x}\right)}\,dx$$

$$= \int_1^{\ln 8} \sqrt{1+\frac{1}{4}e^{2x} - \frac{1}{2} + \frac{1}{4}e^{-2x}}\,dx$$

$$= \int_1^{\ln 8} \sqrt{\frac{1}{4}e^{2x} + \frac{1}{2} + \frac{1}{4}e^{-2x}}\,dx$$

$$= \int_1^{\ln 8} \sqrt{\frac{1}{4}\left(e^{2x}+2+e^{-2x}\right)}\, dx$$

$$= \int_1^{\ln 8} \sqrt{\left(\frac{1}{2}\right)^2 \left(e^{x}+e^{-x}\right)^2}\, dx$$

$$= \int_1^{\ln 8} \frac{1}{2}\left(e^{x}+e^{-x}\right) dx = \frac{1}{2}\Big[e^{x}-e^{-x}\Big]_1^{\ln 8}$$

$$= \frac{1}{2}\left(\underbrace{e^{\ln 8}}_{8} - \underbrace{\frac{1}{e^{\ln 8}}}_{\frac{1}{8}} - e^{1} + e^{-1}\right) = \frac{1}{2}\left(8 - \frac{1}{8} - e^{1} + e^{-1}\right)$$

= snygga till...