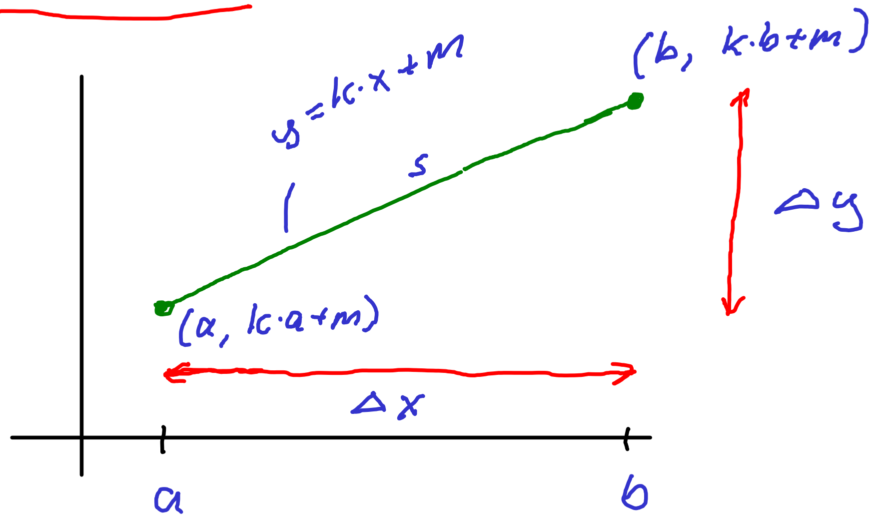
Rep A9



$\Delta x = b - a$

$\Delta y = (k b + m) - (k\cdot a + m)$

$= k\cdot b - k\cdot a = k\cdot(b-a)$

Längd $s = \sqrt{\Delta x^2 + \Delta y^2} = \sqrt{(b-a)^2 + k^2(b-a)^2}$

$= \sqrt{(b-a)^2(1+k^2)} = (b-a)\sqrt{1+k^2}$ $\square$

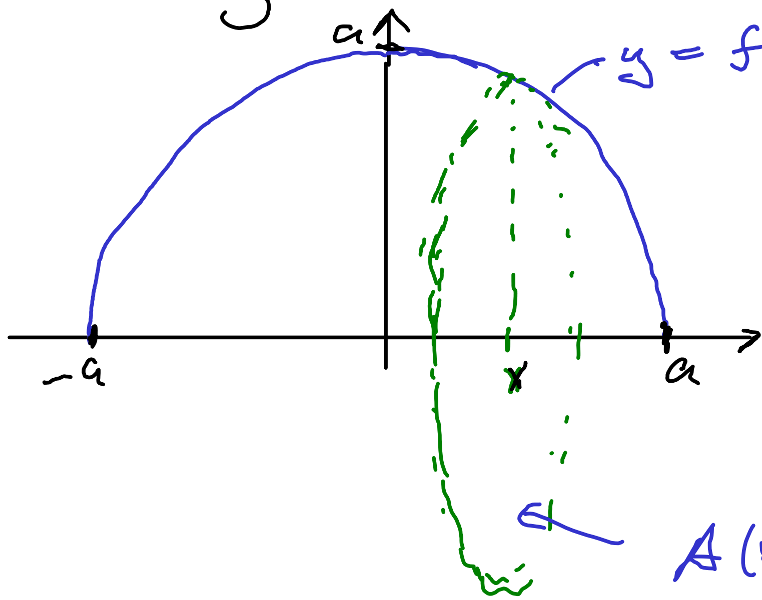
Lösning med båglängdsintegral:

$f(x) = kx + m \qquad f'(x) = k$

$$s = \int_a^b \sqrt{1 + (f'(x))^2}\,dx = \int_a^b \sqrt{1+k^2}\,dx$$

$$= \sqrt{1+k^2}\int_a^b 1\,dx = \sqrt{1+k^2}\,\Big[x\Big]_a^b = \sqrt{1+k^2}\cdot(b-a)$$

Ex: Volymen av ett klot



$y = f(x) = \sqrt{a^2 - x^2}$

$$A(x) = \pi (f(x))^2 = \pi \left(\sqrt{a^2 - x^2}\right)^2$$

$$= \pi (a^2 - x^2)$$

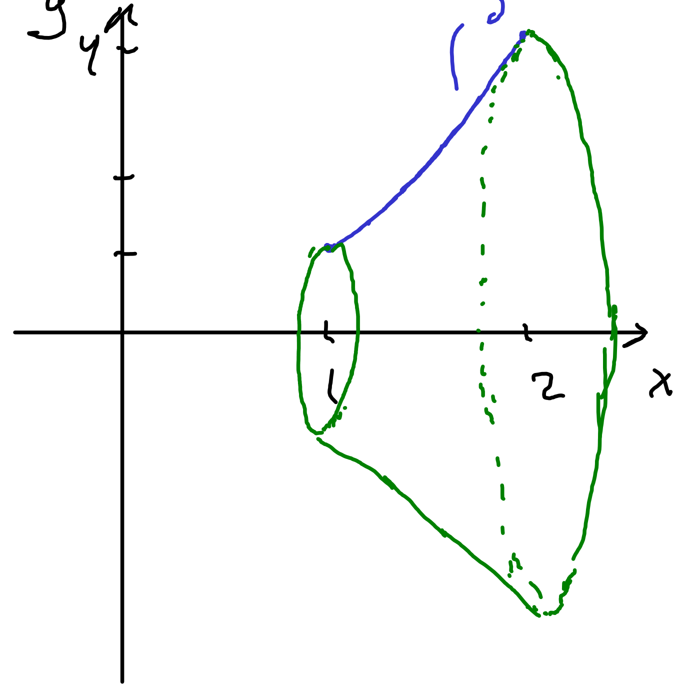
$$V = \int_{-a}^{a} A(x)\,dx = \int_{-a}^{a} \pi (a^2 - x^2)\,dx$$

$$= \pi \left[a^2 \cdot x - \frac{x^3}{3} \right]_{-a}^{a} = \pi \left(a^2 \cdot a - \frac{a^3}{3} - a^2(-a) + \frac{(-a)^3}{3} \right)$$

$$= \pi \left(2a^3 - 2\frac{a^3}{3} \right) = \pi \left(\frac{6a^3}{3} - \frac{2a^3}{3} \right) = \pi \cdot \frac{4a^3}{3}$$

$$= \frac{4}{3} \pi a^3$$

Ex



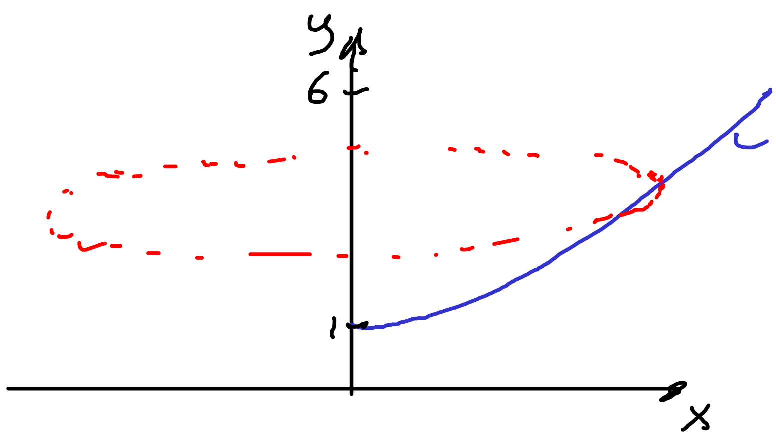
$$V = \int_1^2 \pi (x^2)^2 \, dx$$

$$= \int_1^2 \pi x^4 \, dx$$

$$= \left[\pi \frac{x^5}{5} \right]_1^2 = \pi \frac{2^5}{5} - \pi \frac{1^5}{5}$$

$$= \frac{\pi}{5}\left(2^5 - 1\right) = \frac{31\pi}{5}$$

Ex:



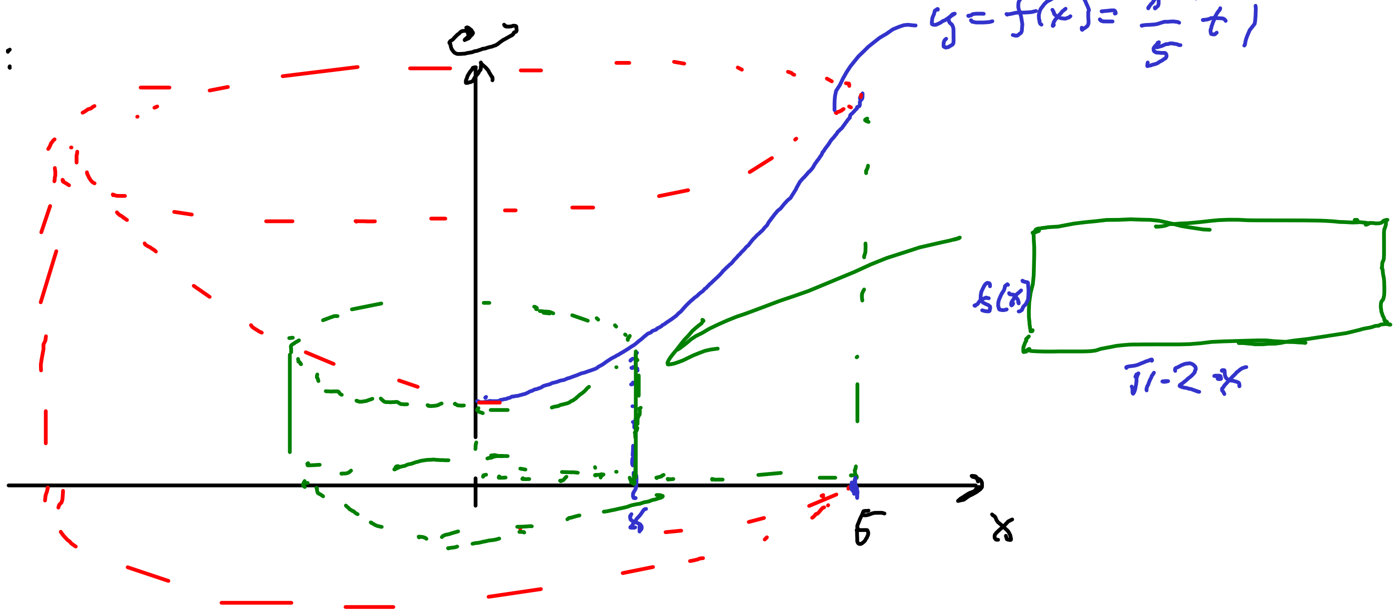
$$y = \frac{x^2}{5} + 1$$

$$y - 1 = \frac{x^2}{5}$$

$$5(y-1) = x^2$$

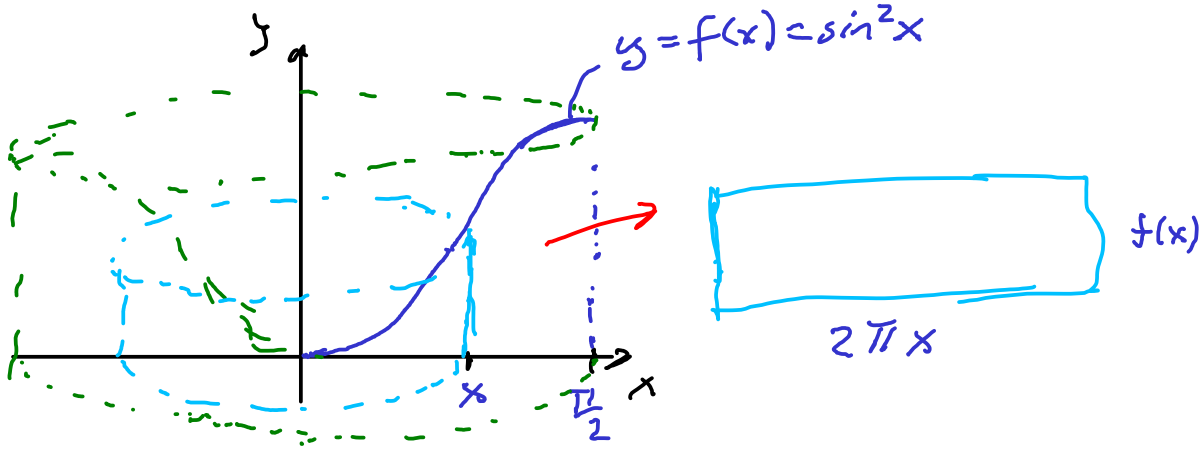
$$A = \int_1^6 \pi x^2 \cdot dy = \int_1^6 \pi \cdot 5(y-1) \, dy$$

Ex:



$$V = \int_0^5 2 \cdot \pi \cdot x \cdot f(x)\, dx = \int_0^5 2\pi x \left(\frac{x^2}{5} + 1\right) dx$$

Ex:



$$\frac{1}{2}(1 - \cos 2x)$$

$$V = \int_0^{\pi/2} 2\pi x \cdot f(x)\, dx = \int_0^{\pi/2} 2\pi x \sin^2 x\, dx$$

$$= \int_0^{\pi/2} \cancel{2}\pi x \, \cancel{\tfrac{1}{2}} [1 - \cos 2x)\, dx$$

$$= \Big[\pi x \left(x - \tfrac{1}{2}\sin 2x\right)\Big]_0^{\pi/2} - \int_0^{\pi/2} \pi \left(x - \tfrac{1}{2}\sin 2x\right) dx$$

$$= \pi \cdot \frac{\pi}{2}\left(\frac{\pi}{2} - \frac{1}{2}\overbrace{\sin \pi}^{=0}\right) - \pi \cdot 0 \cdot (\ldots) - \pi \left[\frac{x^2}{2} + \frac{1}{4}\cos 2x\right]_0^{\pi/2}$$

$$= \frac{\pi^3}{4} - \pi\left(\frac{\left(\frac{\pi}{2}\right)^2}{2} + \frac{1}{4}\overbrace{\cos \pi}^{-1} - \frac{0^2}{2} - \frac{1}{4}\overbrace{\cos 0}^{=1}\right)$$

$$= \frac{\pi^3}{4} - \pi\left(\frac{\pi^2}{8} - \frac{1}{2}\right) = \frac{\pi^3}{4} - \frac{\pi^3}{8} + \frac{\pi}{2} = \frac{\pi^3}{8} + \frac{\pi}{2}$$

v.e.