Uppgift 1

$$\text{Area} = \int_{-1}^{0} (0 - xe^x)\,dx + \int_{0}^{1} (x \cdot e^x - 0)\,dx$$

$$= -\int_{-1}^{0} x e^x dx + \int_{0}^{1} x e^x dx$$

$$= -\left[x e^x\right]_{-1}^{0} + \int_{-1}^{0} 1 \cdot e^x dx + \left[x e^x\right]_0^1 - \int_0^1 1 \cdot e^x dx$$

$$= -0 \cdot e^0 + (-1)e^{-1} + \left[e^x\right]_{-1}^{0} + 1 \cdot e^1 - 0 \cdot e^0 - \left[e^x\right]_0^1$$

$$= -e^{-1} + \underbrace{e^0}_{=1} - e^{-1} + \cancel{e^1} - \cancel{e^1} + \underbrace{e^0}_{=1} = 2 - 2 \cdot e^{-1}$$

$$= 2\left(1 - \frac{1}{e}\right)$$

Uppgift 2a

$$\int \frac{7x - 10}{x^2 - 3x + 2}\,dx$$

1. Lägre gradtal i täljaren? Ja!
2. Faktorisera nämnaren
3. Partialbråksuppdelning
4. Primitiv funktion

2. $x^2 - 3x + 2 = (x-1)(x-2)$

3. $\dfrac{7x-10}{(x-1)(x-2)} = \dfrac{A}{x-1} + \dfrac{B}{x-2}$

Handpåläggning ger

$$\begin{cases} A = 3 \\ B = 4 \end{cases}$$

4. $\displaystyle\int \frac{7x-10}{x^2-3x+2}\,dx = \int\left(\frac{3}{x-1} + \frac{4}{x-2}\right)dx = 3\ln|x-1| + 4\ln|x-2| + C$

Svar

Uppgift 2b

$$\int_0^1 x \cdot \ln(x+1)\,dx = \left[\frac{x^2}{2}\ln(x+1)\right]_0^1 - \int_0^1 \frac{x^2}{2}\,\frac{1}{x+1}\,dx$$

Polynomdivision

$$\begin{array}{r|l} & x - 1 \quad \leftarrow \text{kvot} \\ x+1 & x^2 \\ & -(x^2+x) \\ & -x \\ & -(-x-1) \\ \text{rest} \rightarrow & 1 \end{array}$$

$$= \frac{1^2}{2}\ln 2 - 0 - \frac{1}{2}\int_0^1 \frac{x^2}{x+1}\,dx$$

$\Downarrow$ polynomdivision

$$= \frac{\ln 2}{2} - \frac{1}{2}\int_0^1 \left(\underbrace{x-1}_{\text{kvot}} + \frac{\overset{\leftarrow \text{rest}}{1}}{x+1}\right)dx$$

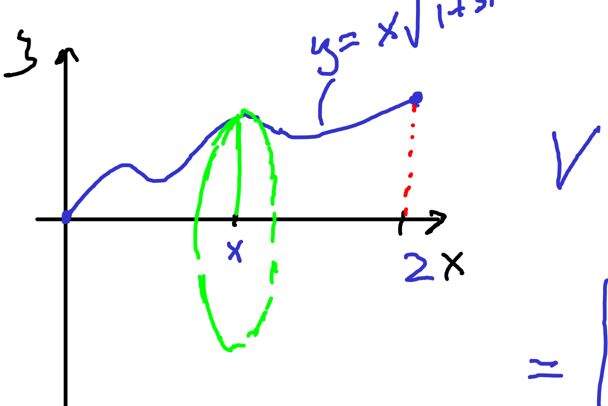
$$= \frac{\ln 2}{2} - \frac{1}{2}\left[\frac{x^2}{2} - x + \ln|x+1|\right]_0^1$$

$$= \frac{\ln 2}{2} - \frac{1}{2}\left(\overbrace{\frac{1}{2} - 1}^{-\frac{1}{2}} + \ln 2 - 0 + 0 - \overbrace{\ln 1}^{=0}\right)$$

$$= \cancel{\frac{\ln 2}{2}} + \frac{1}{4} - \frac{1}{2}\cancel{\ln 2} = \frac{1}{4}$$

Svar: $\frac{1}{4}$

Uppgift 3a



$$A(x) = \pi y^2 = \pi x^2(1+\sin(x^3))$$

$$V = \int_0^2 A(x)\,dx = \int_0^2 \pi x^2(1+\sin(x^3))\,dx$$

$$= \left[\begin{array}{l|l} u = x^3 & x=0 \Rightarrow u=0 \\ \frac{du}{dx} = 3x^2 & \\ \frac{1}{3}du = x^2\,dx & x=2 \Rightarrow u=8 \end{array}\right] = \int_0^8 \frac{\pi}{3}(1+\sin u)\,du$$

$$= \frac{\pi}{3}\left[u - \cos u\right]_0^8 = \frac{\pi}{3}\left(8 - \cos 8 - 0 + \overbrace{\cos 0}^{=1}\right) = \frac{\pi}{3}(9 - \cos 8)$$

Svar

Uppgift 3b

$$y = \frac{2}{3}(x^2+1)^{3/2}$$

$$\frac{dy}{dx} = \cancel{\frac{2}{3}} \cdot \cancel{\frac{3}{2}}(x^2+1)^{1/2} \cdot 2x = 2x\,(x^2+1)^{1/2}$$

Båglängd

$$s = \int_1^2 \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx = \int_1^2 \sqrt{1+4x^2(x^2+1)}\,dx$$

$$= \int_1^2 \sqrt{1+4(x^2)^2+4x^2}\,dx = \int_1^2 \sqrt{(2x^2)^2+2\cdot 2x^2+1}\,dx$$

$$= \int_1^2 \sqrt{(2x^2+1)^2}\,dx = \int_1^2 (2x^2+1)\,dx$$

$$= \left[\frac{2}{3}x^3+x\right]_1^2 = \frac{2}{3}8+2-\frac{2}{3}\cdot 1-1 = \frac{14}{3}+1$$

$$= \frac{17}{3} = 5+\frac{2}{3} \qquad 5\frac{2}{3}$$ ← Svar

---

Uppgift 4

$$\int_0^2 e^{\sin x}\,dx$$

Approximera med Trapetsmetoden då $h = 0.5$

$y_i = e^{\sin x_i}$

i	0	1	2	3	4
x_i	0.0	0.5	1.0	1.5	2.0
y_i	1.0000	1,6151	2,3198	2,7115	2,4826

$$T(0.5) = h\cdot\left(y_1+y_2+y_3+\frac{y_0+y_4}{2}\right)$$

$$= 0.5\left(1.6151+2,3198+2,7115+\frac{1.0+2,4826}{2}\right) = 4,194$$

Svar!