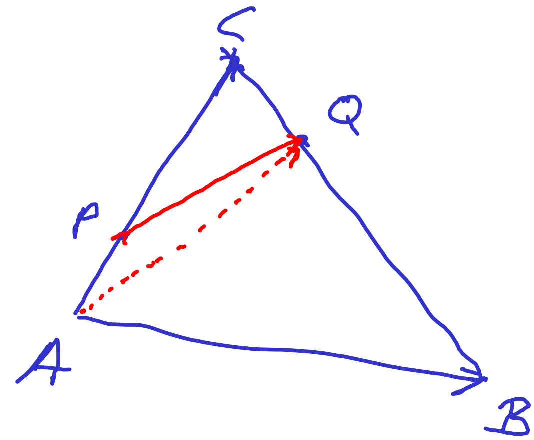
Exempel:

Söks: $\overline{PQ} = ?\cdot\overline{AB} + ?\,\overline{AC}$

Bestäm



Lösning:

$$\overline{PQ} = \overline{AQ} - \overline{AP}$$
$$= (\overline{AC} + \overline{CQ}) - \overline{AP}$$
$$= \overline{AC} + \tfrac{1}{4}\overline{CB} - \overline{AP}$$
$$= \overline{AC} + \tfrac{1}{4}(\overline{AB} - \overline{AC}) - \tfrac{1}{3}\overline{AC}$$
$$= \overline{AC} + \tfrac{1}{4}\overline{AB} - \tfrac{1}{4}\overline{AC} - \tfrac{1}{3}\overline{AC}$$
$$= \tfrac{1}{4}\overline{AB} + \left(1 - \tfrac{1}{4} - \tfrac{1}{3}\right)\overline{AC}$$
$$= \tfrac{1}{4}\overline{AB} + \left(\tfrac{12}{12} - \tfrac{3}{12} - \tfrac{4}{12}\right)AC$$
$$= \tfrac{1}{4}\overline{AB} + \tfrac{5}{12}\overline{AC} \quad \leftarrow \text{Svar!}$$

Ex:

$$\overline{x} = 3\overline{u} + 2\overline{v}$$
$$\overline{y} = 2\overline{u} + a\overline{v}$$

Vinkeln mellan $\overline{u}$ & $\overline{v}$ är $\frac{\pi}{3}$

Välj a så att $\overline{x}$ & $\overline{y}$ är ortogonala dvs välj a så att $\overline{x} \bullet \overline{y} = 0$.

$$0 = \overline{x} \bullet \overline{y} = (3\overline{u} + 2\overline{v}) \bullet (2\overline{u} + a\overline{v})$$
$$= (3\overline{u} + 2\overline{v}) \bullet 2\overline{u} + (3\overline{u} + 2\overline{v}) \bullet a\overline{v}$$

$$= 3\cdot 2\,\bar{u}\bullet\bar{u} + 2\cdot 2\cdot\bar{v}\bullet\bar{u} + 3a\cdot\bar{u}\bullet\bar{v} + 2a\bar{v}\bullet\bar{v}$$

$$= 6\,\underbrace{|\bar{u}|^2}_{1^2} + 4\cdot\underbrace{|\bar{u}|}_{=1}\underbrace{|\bar{v}|}_{2}\cdot\underbrace{\cos\frac{\pi}{3}}_{=\frac{1}{2}} + a\Big(3\,\overbrace{|\bar{u}|}^{1}\overbrace{|\bar{v}|}^{2}\overbrace{\cos\frac{\pi}{3}}^{1/2} + 2\,\underbrace{|\bar{v}|^2}_{2^2}\Big)$$

$$= 6 + 4 + a(3 + 8)$$

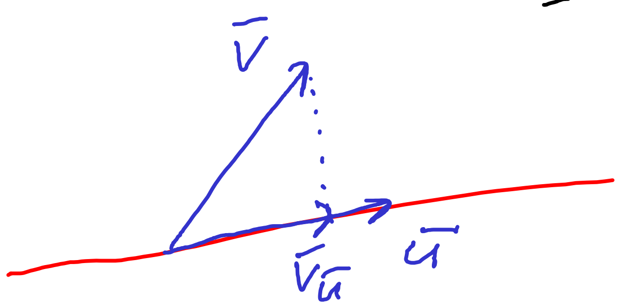
$$= 10 + 11a = 0$$

$$\boxed{a = -\frac{10}{11}}$$

Svar

Exempel

$$\bar{v} = \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix} \qquad \bar{u} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$$



Söks: $\bar{c} = \bar{v}_{\bar{u}}$ $\qquad \bar{d} = \bar{v} - \bar{v}_{\bar{u}}$

$$\bar{c} = \frac{\bar{v}\bullet\bar{u}}{\bar{u}\bullet\bar{u}}\,\bar{u}$$

$$\bar{v}\bullet\bar{u} = \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}\bullet\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$$

$$= 3\cdot 1 + 0\cdot 2 + (-1)\cdot 2$$

$$= 3 + 0 - 2 = 1$$

$$= \frac{1}{9}\cdot\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 1/9 \\ 2/9 \\ 2/9 \end{bmatrix}$$

$$\bar{u}\bullet\bar{u} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}\bullet\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} = 1^2 + 2^2 + 2^2 = 9$$

$$\bar{d} = \bar{v} - \bar{v}_{\bar{u}} = \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix} - \begin{bmatrix} 1/9 \\ 2/9 \\ 2/9 \end{bmatrix}$$

$$= \begin{bmatrix} 26/9 \\ -2/9 \\ -11/9 \end{bmatrix}$$

Exempel:

$$\bar{u} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \qquad \bar{v} = \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}$$

Bestäm vinkeln mellan $\bar{u}$ & $\bar{v}$

vinkeln

$$\bar{u} \bullet \bar{v} = |\bar{u}|\,|\bar{v}| \cdot \cos\theta$$

$$\bar{u} \bullet \bar{v} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \bullet \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix} = 1 \cdot 3 + 2(-1) + 1 \cdot 1$$

$$= 2$$

$$|\bar{u}| = \sqrt{1^2 + 2^2 + 1^2} = \sqrt{6}$$

$$|\bar{v}| = \sqrt{3^2 + (-1)^2 + 1^2} = \sqrt{11}$$

$$\bar{u} \bullet \bar{v} = |\bar{u}|\,|\bar{v}| \cdot \cos\theta$$

$$2 = \underbrace{\sqrt{6} \cdot \sqrt{11}}_{\sqrt{66}} \cdot \cos\theta$$

$$\frac{2}{\sqrt{66}} = \cos\theta$$

$$\theta = 75.75^\circ$$