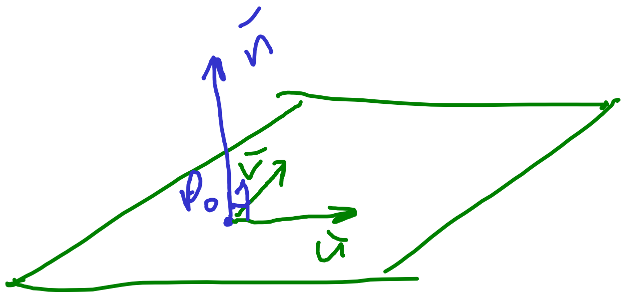
Rep L5

Plan genom $P_0 = (1, 2, -1)$

med riktning $\bar{u} = \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}$ $\quad \bar{v} = \begin{bmatrix} 2 \\ 5 \\ -1 \end{bmatrix}$



$$\bar{n} = \bar{u} \times \bar{v} = \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix} \times \begin{bmatrix} 2 \\ 5 \\ -1 \end{bmatrix} = \begin{bmatrix} 2(-1) - (-2)\cdot 5 \\ -(1\cdot(-1) - (-2)\cdot 2) \\ 1\cdot 5 - 2\cdot 2 \end{bmatrix}$$

$$= \begin{array}{cc} \begin{bmatrix} 8 \\ -3 \\ 1 \end{bmatrix} & \begin{matrix} A \\ B \\ C \end{matrix} \end{array}$$

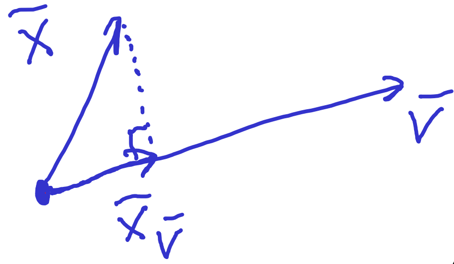
Planets ekv: $A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$

$$8(x-1) + (-3)(y-2) + 1(z - (-1)) = 0$$

$$8x - 8 - 3y + 6 + z + 1 = 0$$

$$\boxed{8x - 3y + z - 1 = 0}$$

Ortogonal projektion



$$\bar x_{\bar v} = \frac{\bar x \bullet \bar v}{\bar v \bullet \bar v}\,\bar v$$

alt.

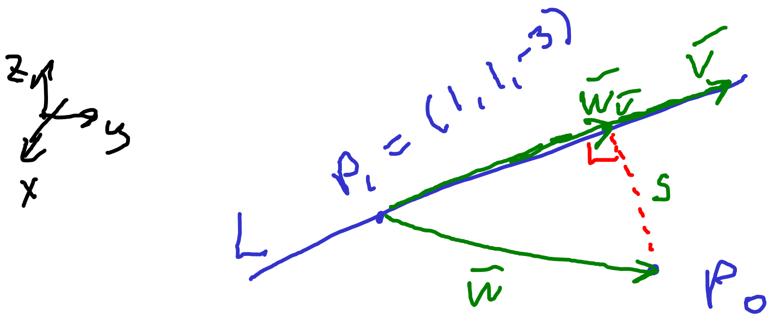
$$\bar v_e = \frac{1}{\|\bar v\|}\,\bar v$$

$$\bar x_{\bar v} = (\bar x \bullet \bar v_e)\,\bar v_e$$

Ex:

$$L: \begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}1\\1\\-3\end{bmatrix} + t\underbrace{\begin{bmatrix}0\\3\\4\end{bmatrix}}_{\bar v}$$

Kortaste avstånd till $P_0 = (2,0,-3)$



$$\bar w = \overline{P_1P_0} = \begin{bmatrix}2-1\\0-1\\-3-(-3)\end{bmatrix} = \begin{bmatrix}1\\-1\\0\end{bmatrix}$$

$$\bar w_{\bar v} = \frac{\bar w \bullet \bar v}{\bar v \bullet \bar v}\,\bar v = \frac{\bar w \bullet \bar v}{\|v\|^2}\,\bar v$$

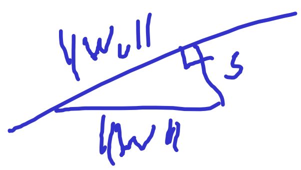
$$\|\bar w_{\bar v}\| = \left\| \frac{\bar w \bullet \bar v}{\|\bar v\|^2}\,\bar v \right\| = \frac{|\bar w \bullet \bar v|}{\|v\|^2}\,\|v\| =$$

$$\boxed{\|\bar w_{\bar v}\| = \frac{|\bar w \bullet \bar v|}{\|v\|}}$$

$$\|\bar w_{\bar v}\| = \frac{\left|\begin{bmatrix}1\\-1\\0\end{bmatrix} \bullet \begin{bmatrix}0\\3\\4\end{bmatrix}\right|}{\sqrt{0^2+3^2+4^2}} = \frac{|1\cdot 0 + (-1)\cdot 3 + 0\cdot 4|}{5}$$

$$= \frac{3}{5}$$

$$\|\bar{w}\| = \sqrt{1^2+(-1)^2+0^2} = \sqrt{2}$$

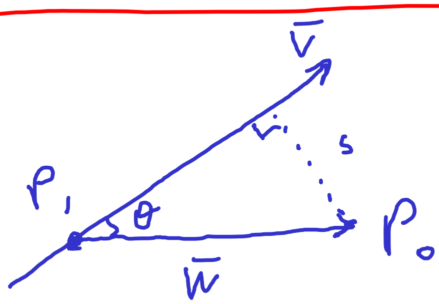


$$s = \sqrt{\|w\|^2 - \|w_v\|^2}$$

$$= \sqrt{2 - \frac{9}{25}} = \sqrt{\frac{41}{25}} = \frac{\sqrt{41}}{5}$$

<u>Svar</u>: Avståndet är $\frac{\sqrt{41}}{5}$ l.e.

<u>Alt</u>:



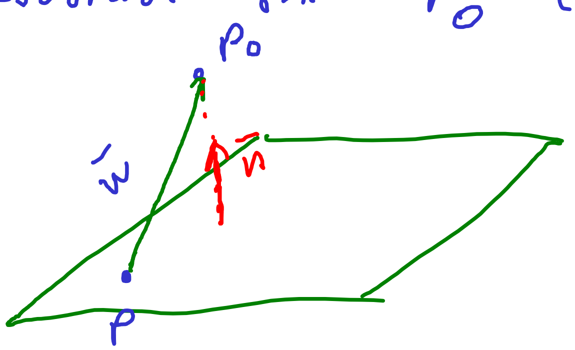
$$s = \|\bar{w}\| \cdot \sin\theta$$

$$= \frac{\|\bar{v}\| \cdot \|\bar{w}\| \cdot \sin\theta}{\|\bar{v}\|}$$

$$= \frac{\|\bar{v} \times \bar{w}\|}{\|\bar{v}\|}$$

<u>Ex</u>: Plan $2x-3y+z-1=0$ $\quad \bar{n} = \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}$

Avstån till $P_0 = (-1, 3, -2)$



Avstånd: $\|\bar{w}_{\bar{n}}\|$

1. Punkt $P = (0, 0, 1)$ $\quad 2\cdot 0 - 3\cdot 0 + z - 1 = 0$, $z = 1$

2. $\bar{w} = \begin{bmatrix} -1-0 \\ 3-0 \\ -2-1 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \\ -3 \end{bmatrix}$

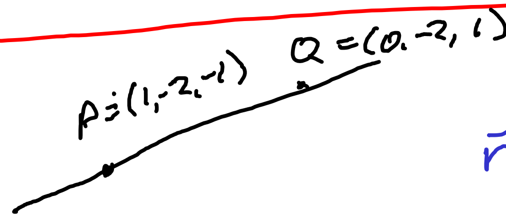
3. Avståndet: $\|\bar{w}_{\bar{n}}\| = \dfrac{|\bar{w} \cdot \bar{n}|}{\|\bar{n}\|}$

$$= \frac{\left| \begin{bmatrix} -1 \\ 3 \\ -3 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix} \right|}{\sqrt{2^2 + (-3)^2 + 1^2}}$$

$$= \frac{|(-1)\cdot 2 + 3(-3) + (-3)\cdot 1|}{\sqrt{14}} = \frac{|-14|}{\sqrt{14}}$$

$$= \frac{14}{\sqrt{14}} = \frac{\sqrt{14}\cdot\sqrt{14}}{\sqrt{14}} = \sqrt{14}$$

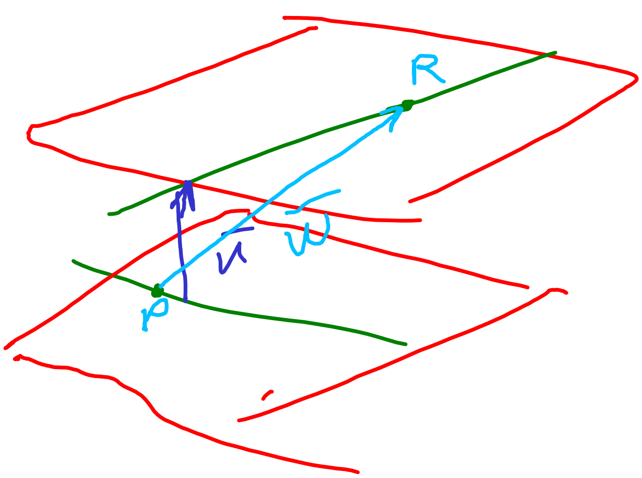
L 1



$$\bar{r} = \begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix} + t \underbrace{\begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}}_{\bar{v}_1}$$

L 2

$$r = \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix} + t \underbrace{\begin{bmatrix} 0 \\ -2 \\ -2 \end{bmatrix}}_{v_2}$$



$$\bar{n} = \bar{v}_1 \times \bar{v}_2$$

$$= \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix} \times \begin{bmatrix} 0 \\ -2 \\ -2 \end{bmatrix}$$

$$= \begin{bmatrix} 4 \\ -2 \\ 2 \end{bmatrix}$$

$$\bar{w} = \begin{bmatrix} -1 & -1 \\ 2 & -(-2) \\ 0 & -(-1) \end{bmatrix} = \begin{bmatrix} -2 \\ 4 \\ 1 \end{bmatrix}$$

Avståndet: $\|\bar{w}_{\bar{n}}\| = \dfrac{|\bar{w} \cdot \bar{n}|}{\|\bar{n}\|}$

$$= \frac{\left| \begin{bmatrix} -2 \\ 4 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ -2 \\ 2 \end{bmatrix} \right|}{\sqrt{4^2+(-2)^2+(2)^2}} = \frac{|-8-8+2|}{\sqrt{24}}$$

$$= \frac{14}{\sqrt{24}} = \frac{14}{2\sqrt{6}} = \frac{7}{\sqrt{6}}$$