$\boxed{1\ a}$

$\|\bar{u}\| = 4 \quad \|\bar{v}\| = 2$

$\bar{w} = 2\bar{u} + t\cdot\bar{v} \qquad \|\bar{w}\| = 13$ då $t = \frac{7}{2}$

Sökes: Det andra t-värdet så $\|\bar{w}\| = 13$ sökes.

$$13^2 = \|w\|^2 = \bar{w}\bullet\bar{w} = (2\bar{u}+t\bar{v})\bullet(2\bar{u}+t\bar{v})$$

$$= 2(2\bar{u}+t\bar{v})\bullet\bar{u} + t(2\bar{u}+t\bar{v})\bullet\bar{v}$$

$$= 4\bar{u}\bullet\bar{u} + 2\cdot t\bar{v}\bullet\bar{u} + t\cdot 2\cdot\bar{u}\bullet\bar{v} + t^2\bar{v}\bullet\bar{v}$$

$$= 4\underbrace{\|\bar{u}\|^2}_{4^2} + 4\cdot t\,\bar{u}\bullet\bar{v} + t^2\underbrace{\|\bar{v}\|^2}_{2^2}$$

d.v.s.

$$13^2 = 64 + 4t\,\bar{u}\bullet\bar{v} + 4t^2$$

gäller då $t = \frac{7}{2}$

$$13^2 = 64 + \cancel{4}^{\,2}\cdot\frac{7}{\cancel{2}_{\,1}}\,\bar{u}\bullet\bar{v} + 4\cdot\overbrace{\left(\frac{7}{2}\right)^2}^{\frac{49}{4}}$$

$$13^2 - 64 - 49 = 14\,\bar{u}\bullet\bar{v}$$

$$\bar{u}\bullet\bar{v} = \frac{56}{14} = \frac{28}{7} = 4$$

Bestäm "t"

$$13^2 = 64 + 4t\cdot 4 + 4t^2$$

$$t^2 + 4t - \frac{13^2 - 64}{4} = 0$$

pq-formeln

$$t = -2 \pm \sqrt{4 + \frac{13^2 - 64}{4}} = -2 \pm \sqrt{\frac{16 + 13^2 - 64}{4}}$$

$$= -2 \pm \sqrt{\frac{121}{4}} = -2 \pm \frac{11}{2}$$

$$t = \frac{7}{2} \quad \text{eller} \quad \boxed{t = -\frac{15}{2}}$$

1b

Vinkeln ges av

$$\bar{u} \bullet \bar{v} = \|\bar{u}\| \, \|\bar{v}\| \cos \theta$$

$$4 = 4 \cdot 2 \cdot \cos \theta$$

$$\frac{1}{2} = \cos \theta \iff \boxed{\theta = \frac{\pi}{3}}$$

2a

$$\begin{aligned} x &= \\ y &= \\ z &= \end{aligned} \begin{cases} -3t + 4 = -2s - 3 \\ 6t - 4 = 4s + 10 \\ -6t + 9 = -6s - 9 \end{cases}$$

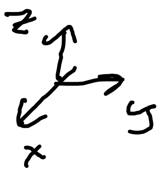
$$\begin{cases} -3t + 4 = -2s - 3 \\ 6t - 4 = 4s + 10 \\ 5 = -2s + 1 \end{cases} \qquad \begin{cases} t = 1 \\ s = -2 \end{cases}$$

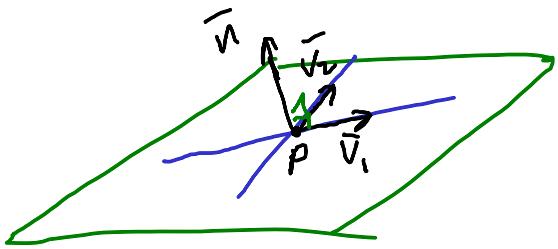
i första systemet
gav detta

Svar:

$$\begin{cases} x = 1 \\ y = 2 \\ z = 3 \end{cases}$$

2b





$$P = (1, 2, 3)$$

$$\bar{v}_1 = \begin{bmatrix} -3 \\ 6 \\ -6 \end{bmatrix}$$

$$\bar{v}_2 = \begin{bmatrix} -2 \\ 4 \\ -6 \end{bmatrix} \qquad \bar{n} = \bar{v}_1 \times \bar{v}_2 = \begin{bmatrix} -3 \\ 6 \\ -6 \end{bmatrix} \times \begin{bmatrix} -2 \\ 4 \\ -6 \end{bmatrix}$$

$$= \begin{bmatrix} 6(-6) - (-6)4 \\ -((-3)(-6) - (-6)(-2)) \\ (-3)4 - 6(-2) \end{bmatrix} = \begin{bmatrix} -12 \\ -6 \\ 0 \end{bmatrix}$$

Planets ekv

$$A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$$

$$(-12)(x-1) + (-6)(y-2) + 0(z-3) = 0$$

$$2(x-1) + 1(y-2) = 0$$

$$2x - 2 + y - 2 = 0$$

$$\boxed{2x + y - 4 = 0}$$

$z = 2$

2c Välj r och a så att

$$\begin{cases} 1 = 2r + a \\ 2 = -2r + 5 \\ 3 = 3 \end{cases} \quad \text{①} \qquad \begin{cases} 1 = 2r + a \\ 3 = 0 + a + 5 \\ 3 = 3 \end{cases}$$

Svar 1: $\boxed{a = -2}$

Vinkeln mellan L_1 & L_3.

$$\bar{v}_1 = \begin{bmatrix} -3 \\ 6 \\ -6 \end{bmatrix} \qquad \bar{v}_3 = \begin{bmatrix} 2 \\ -2 \\ 0 \end{bmatrix}$$

$$\bar{v}_1 \cdot \bar{v}_3 = \|\bar{v}_1\| \|\bar{v}_3\| \cdot \cos\theta$$

$$(-3)\cdot 2 + 6(-2) + (-6)\cdot 0 = \sqrt{(-3)^2 + 6^2 + (-6)^2} \times \dots$$

$$\dots \times \sqrt{2^2 + (-2)^2 + 0^2} \cdot \cos\theta$$

$$-18 = \underbrace{\sqrt{81}}_{=9} \underbrace{\sqrt{8}}_{2\sqrt{2}} \cdot \cos\theta$$

$$\cos\theta = -\frac{18}{9 \cdot 2 \cdot \sqrt{2}} = -\frac{1}{\sqrt{2}} \qquad \boxed{\theta = \frac{3\pi}{4}}$$

$\boxed{6}$

$\bar{u} = \bar{a} + 2\bar{b}$ koordinat $(1, 2)$

(1) $\bar{c} = 2\bar{a} - 2\bar{b}$ $(2, -2)$

(2) $\bar{d} = -\bar{a} + 3\bar{b}$ $(-1, 3)$

(1) + 2 (2) $\bar{c} + 2\bar{d} = -2\bar{b} + 6\bar{b}$

$$\bar{c} + 2\bar{d} = 4\bar{b}$$

$$\boxed{\bar{b} = \frac{1}{4}\bar{c} + \frac{1}{2}\bar{d}}$$

i (2) ger

$$\bar{d} = -\bar{a} + \frac{3}{4}\bar{c} + \frac{3}{2}\bar{d}$$

$$\boxed{\bar{a} = \frac{3}{4}\bar{c} + \frac{1}{2}\bar{d}}$$

$$\bar{u} = \bar{a} + 2b = \frac{3}{4}\bar{c} + \frac{1}{2}\bar{d} + \frac{2}{4}\bar{c} + \frac{2}{2}\bar{d}$$

$$= \frac{5}{4}\bar{c} + \frac{3}{2}\bar{d}$$

Ger koordinat: $\left(\frac{5}{4}, \frac{3}{2}\right)$