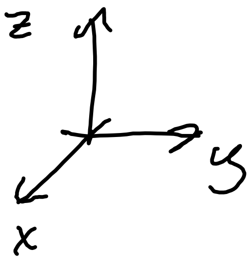
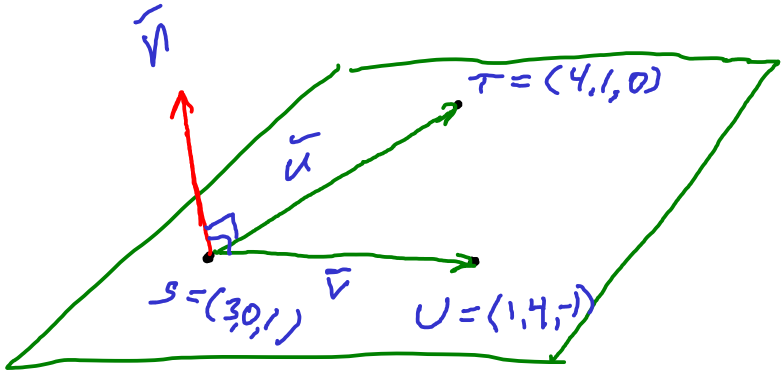
3a





$$\bar{u} = \begin{bmatrix} 4-3 \\ 1-0 \\ 0-1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} \qquad \bar{v} = \begin{bmatrix} 1-3 \\ 4-0 \\ -1-1 \end{bmatrix} = \begin{bmatrix} -2 \\ 4 \\ -2 \end{bmatrix}$$

$$\bar{n} = \bar{u} \times \bar{v} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} \times \begin{bmatrix} -2 \\ 4 \\ -2 \end{bmatrix} = \begin{bmatrix} 1(-2)-(-1)\cdot 4 \\ -(1(-2)-(-1)(-2)) \\ 1\cdot 4 - 1(-2) \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$$

Ger planet

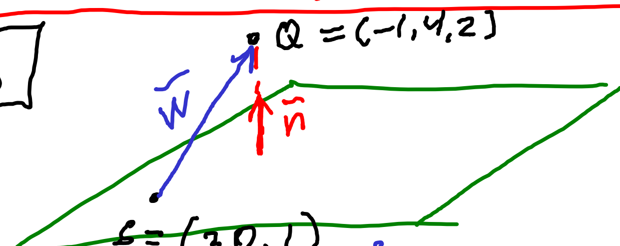
$$2(x-3)+4(y-0)+6(z-1)=0$$

Dela med 2

$$1(x-3)+2(y-0)+3(z-1)=0$$

Svar: $\boxed{x+2y+3z-6=0}$

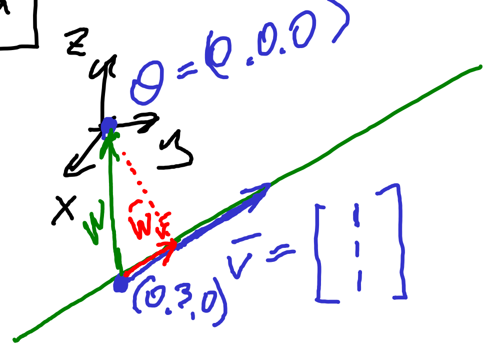
3b



$$\bar{n} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$ från planets ekvation

$$\bar{w} = \begin{bmatrix} -1-3 \\ 4-0 \\ 2-1 \end{bmatrix} = \begin{bmatrix} -4 \\ 4 \\ 1 \end{bmatrix} \quad \text{Avst} = \frac{|\bar{w}\bullet\bar{n}|}{\|\bar{n}\|} = \frac{|(-4)1+4\cdot 2+1\cdot 3|}{\sqrt{1^2+2^2+3^2}} = \frac{7}{\sqrt{14}} = \sqrt{\frac{7}{2}}$$

4a



$$\bar{v} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\bar{w} = \begin{bmatrix} 0-0 \\ 0-3 \\ 0-0 \end{bmatrix} = \begin{bmatrix} 0 \\ -3 \\ 0 \end{bmatrix}$$

$$\bar{w}_{\bar{v}} = \frac{\bar{w} \bullet \bar{v}}{\bar{v} \bullet \bar{v}} \bar{v} = \frac{\begin{bmatrix} 0 \\ -3 \\ 0 \end{bmatrix} \bullet \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}}{\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \bullet \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$= \frac{0 \cdot 1 + (-3) \cdot 1 + 0 \cdot 1}{1 \cdot 1 + 1 \cdot 1 + 1 \cdot 1} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}$$

Punkten närmast origo ges av ortsvektorn

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 0 \end{bmatrix} + \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix}$$

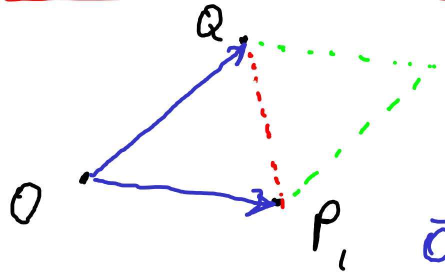
4b

Avståndet ges av

$$\left\| \begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix} \right\| = \sqrt{(-1)^2 + 2^2 + (-1)^2} = \sqrt{1+4+1}$$

$$= \sqrt{6}$$

4c



Triangelns area är hälften av parallellogrammets area.

$$\overline{OP_1} \times \overline{OQ} = \begin{bmatrix} 0 \\ 3 \\ 0 \end{bmatrix} \times \begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 3(-1) - 0 \cdot 2 \\ -(0(-1) - 0(-1)) \\ 0 \cdot 2 - 3(-1) \end{bmatrix} = \begin{bmatrix} -3 \\ 0 \\ 3 \end{bmatrix}$$

$$\text{Area} = \frac{1}{2} \left\| \overline{OP_1} \times \overline{OQ} \right\| = \frac{1}{2} \sqrt{(-3)^2 + 0^2 + 3^2} = \frac{1}{2} \sqrt{2} \cdot 3 = \frac{3}{\sqrt{2}}$$

$\boxed{5a}$

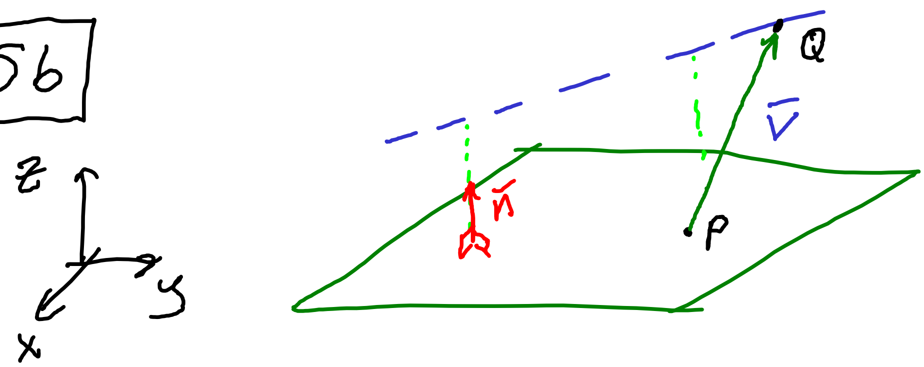
Linjens riktning: $$\bar{v} = \begin{bmatrix} 1 \\ 1/2 \\ -1/2 \end{bmatrix}$$

Planets normal: $$\bar{n} = \begin{bmatrix} 2 \\ 1 \\ 5 \end{bmatrix}$$

Eftersom $\bar{v} \bullet \bar{n} = 1\cdot 2 + \frac{1}{2}\cdot 1 + \left(-\frac{1}{2}\right)\cdot 5 = \frac{4}{2} + \frac{1}{2} - \frac{5}{2} = 0$

så är riktningen vinkelrät mot normalen dvs parallell med planet.

$\boxed{5b}$



P: $(0, 3, 0)$ ← Väljer punkt som uppfyller $2x + y + 5z = 3$

Q: $(1, 1, 1)$ ← Väljer punkt på linjen

$$\bar{v} = \begin{bmatrix} 1-0 \\ 1-3 \\ 1-0 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} \qquad \bar{n} = \begin{bmatrix} 2 \\ 1 \\ 5 \end{bmatrix}$$

Avståndet ges av $$\frac{|\bar{v} \bullet \bar{n}|}{\|\bar{n}\|} = \frac{|1\cdot 2 + (-2)\cdot 1 + 1\cdot 5|}{\sqrt{2^2 + 1^2 + 5^2}} = \frac{5}{\sqrt{30}}$$

$$= \frac{\sqrt{5}\cdot\sqrt{5}}{\sqrt{5}\cdot\sqrt{6}} = \sqrt{\frac{5}{6}}$$