Rep L10

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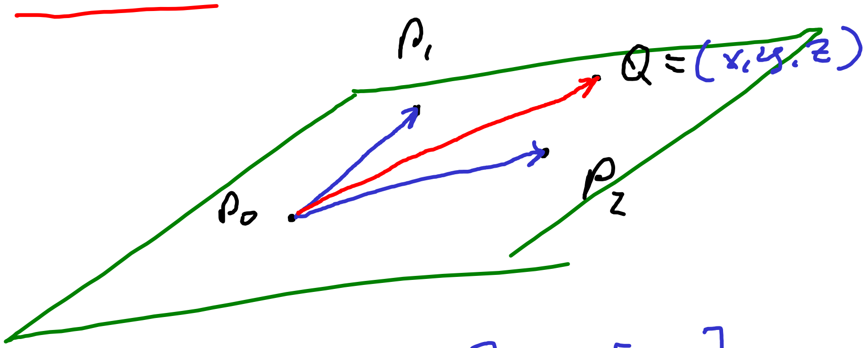

$$\overline{P_0P_1} = \begin{bmatrix} 3-1 \\ 2-2 \\ 1-(-1) \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix} \qquad \overline{P_0P_2} = \begin{bmatrix} -2-1 \\ 0-2 \\ 4-(-1) \end{bmatrix} = \begin{bmatrix} -3 \\ -2 \\ 5 \end{bmatrix}$$

$$\overline{P_0Q} = \begin{bmatrix} x-1 \\ y-2 \\ z-(-1) \end{bmatrix} = \begin{bmatrix} x-1 \\ y-2 \\ z+1 \end{bmatrix}$$

Vektorerna ligger i ett plan om

$$0 = \left| \overline{P_0P_1} \quad \overline{P_0P_2} \quad \overline{P_0Q} \right| = \begin{vmatrix} 2 & -3 & x-1 \\ 0 & -2 & y-2 \\ 2 & 5 & z+1 \end{vmatrix}$$

$$= (x-1)\begin{vmatrix} 0 & -2 \\ 2 & 5 \end{vmatrix} - (y-2)\begin{vmatrix} 2 & -3 \\ 2 & 5 \end{vmatrix} + (z+1)\begin{vmatrix} 2 & -3 \\ 0 & -2 \end{vmatrix}$$

$$= (x-1)\cdot 4 - (y-2)\cdot 16 + (z+1)(-4)$$

$$4(x-1) - 16(y-2) - 4(z+1) = 0$$

$$(x-1) - 4(y-2) - (z+1) = 0$$

$$x - 1 - 4y + 8 - z - 1 = 0$$

$$\boxed{x - 4y - z + 6 = 0}$$

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$$\begin{array}{c}(-1)\\ \begin{vmatrix}1&1&1\\a&b&c\\a^2&b^2&c^2\end{vmatrix}\end{array} = \begin{vmatrix}1&0&0\\a&b-a&c-a\\a^2&b^2-a^2&c^2-a^2\end{vmatrix}$$

$$= 1\cdot\begin{vmatrix}b-a&c-a\\b^2-a^2&c^2-a^2\end{vmatrix} = \begin{vmatrix}b-a&c-a\\(b-a)(b+a)&(c-a)(c+a)\end{vmatrix}$$

$$= (b-a)\begin{vmatrix}1&c-a\\b+a&(c-a)(c+a)\end{vmatrix} = (b-a)(c-a)\begin{array}{c}(-1)\\ \begin{vmatrix}1&1\\b+a&c+a\end{vmatrix}\end{array}$$

$$= (b-a)(c-a)\begin{vmatrix}1&0\\b+a&c-b\end{vmatrix} = (b-a)(c-a)(c-b)$$

Ex:

$$\begin{array}{cc}(-4)&(2)\end{array}\left[\begin{array}{ccc|c}\boxed{1}&3&-1&4\\-2&1&3&9\\4&2&1&11\end{array}\right] \sim \begin{array}{c}\\ \\ \tfrac{1}{5}\times\end{array}\left[\begin{array}{ccc|c}1&3&-1&4\\0&\boxed{7}&1&17\\0&-10&5&-5\end{array}\right]$$

$$\sim \begin{array}{c}\\ \tfrac{2}{7}\\ \end{array}\left[\begin{array}{ccc|c}1&3&-1&4\\0&7&1&17\\0&-2&1&-1\end{array}\right] \sim \left[\begin{array}{ccc|c}1&3&-1&4\\0&7&1&17\\0&0&\frac{9}{7}&\frac{27}{7}\end{array}\right]$$

$$\begin{cases}x+3y-z=4\\ \quad 7y+z=17\\ \quad\quad \frac{9}{7}z=\frac{27}{7}\end{cases}$$

$$x = 4-3\cdot 2+3 = 1$$

$$y = \frac{17-3}{7} = \frac{14}{7} = 2$$

$$z = \frac{\cancel{7}}{9}\cdot\frac{27}{\cancel{7}} = 3$$

Svar: $\begin{cases}x=1\\y=2\\z=3\end{cases}$

Ex:

$$\begin{matrix}\textcircled{-2} & \textcircled{-3}\end{matrix}\left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 3 & 4 & 1 & 3 \\ 2 & -1 & -3 & 2 \end{array}\right] \sim \textcircled{3}\left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 1 & 4 & 3 \\ 0 & -3 & -1 & 2 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 1 & 4 & 3 \\ 0 & 0 & 11 & 11 \end{array}\right]$$

$x = -y+z = 1+1 = 2$

$y = 3-4\cdot 1 = -1$

$11z = 11 \qquad z = 1$

d.v.s. $\begin{cases} x = 2 \\ y = -1 \\ z = 1 \end{cases}$

Ex:

$$\begin{matrix}\textcircled{-1} & \textcircled{-2}\end{matrix}\left[\begin{array}{ccc|c} \boxed{1} & 2 & -1 & 4 \\ 2 & 5 & 2 & 9 \\ 1 & 4 & 7 & 6 \end{array}\right] \sim \textcircled{-2}\left[\begin{array}{ccc|c} 1 & 2 & -1 & 4 \\ 0 & \boxed{1} & 4 & 1 \\ 0 & 2 & 8 & 2 \end{array}\right]$$

$$\sim \begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \left[\begin{array}{ccc|c} \boxed{1} & 2 & -1 & 4 \\ 0 & \boxed{1} & 4 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

Bundna: x, y

Fria: z

$z = t$

$y = 1-4z = 1-4\cdot t$

$x = 4-2y+z = 4-2(1-4t)+t$

$= 2+9t$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2+9t \\ 1-4t \\ t \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} 9 \\ -4 \\ 1 \end{bmatrix}$$