Ex:

$$A=\begin{bmatrix}\boxed{1}&2&3\\4&5&6\\7&8&9\end{bmatrix} \quad (-4)\ (-7) \quad \sim \begin{bmatrix}1&2&3\\0&-3&-6\\0&-6&-12\end{bmatrix} \quad (-2)$$

$$\sim \begin{bmatrix}\boxed{1}&2&3\\0&\boxed{-3}&-6\\0&0&0\end{bmatrix}$$

Två pivotelement

$\Downarrow$

rank $A = 2$

$$\begin{bmatrix}0&\boxed{x}&x&x&x\\0&x&x&x&x\\0&x&x&x&x\end{bmatrix} \sim \begin{bmatrix}0&x&x&x&x\\0&0&0&x&x\\0&0&0&x&x\end{bmatrix}$$

Ex:

$$\left[\begin{array}{ccc|c}1&1&-2&0\\-2&-5&1&0\\1&5&2&0\end{array}\right] \quad (2)\ (-1) \quad \sim \left[\begin{array}{ccc|c}1&1&-2&0\\0&-3&-3&0\\0&4&4&0\end{array}\right] \quad \times -\tfrac{1}{3}$$

$$\sim \left[\begin{array}{ccc|c}1&1&-2&0\\0&1&1&0\\0&4&4&0\end{array}\right] \quad (-4) \quad \sim \left[\begin{array}{ccc|c}1&1&-2&0\\0&1&1&0\\0&0&0&0\end{array}\right]$$

Bundna: x y Fria: z

$z = t \qquad y = -z = -t \qquad x = -y + 2z = t + 2t = 3t$

$$\begin{cases}x = 3t\\ y = -t\\ z = t\end{cases} \quad \text{alt.} \quad \begin{bmatrix}x\\y\\z\end{bmatrix} = t\begin{bmatrix}3\\-1\\1\end{bmatrix}$$

Materialet till L12

Ex:

$$A = \begin{bmatrix} 2 & 2 & -1 \\ 1 & -1 & -2 \\ 1 & 0 & -1 \end{bmatrix}$$ Bestäm A^{-1}

Lösning

$$[A \mid I] = \left[\begin{array}{ccc|ccc} 2 & 2 & -1 & 1 & 0 & 0 \\ 1 & -1 & -2 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|ccc} \boxed{1} & 0 & -1 & 0 & 0 & 1 \\ 1 & -1 & -2 & 0 & 1 & 0 \\ 2 & 2 & -1 & 1 & 0 & 0 \end{array}\right] \begin{array}{c} -1 \quad -2 \end{array}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 0 & 0 & 1 \\ 0 & -1 & -1 & 0 & 1 & -1 \\ 0 & 2 & 1 & 1 & 0 & -2 \end{array}\right] \begin{array}{c} 2 \end{array}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 0 & 0 & 1 \\ 0 & -1 & -1 & 0 & 1 & -1 \\ 0 & 0 & -1 & 1 & 2 & -4 \end{array}\right] \begin{array}{l} \\ \times -1 \\ \times -1 \end{array}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & -1 & 1 \\ 0 & 0 & 1 & -1 & -2 & 4 \end{array}\right] \begin{array}{c} -1 \quad 1 \end{array}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & -2 & 5 \\ 0 & 1 & 0 & 1 & 1 & -3 \\ 0 & 0 & 1 & -1 & -2 & 4 \end{array}\right]$$

Matriselcuationer

$$AXB = C - 2XB$$

Lös ut X.

$$AXB + 2X \cdot B = C$$

$$(AX + 2X) \cdot B = C$$

$$(A + 2I)X \cdot B = C$$

$$(A+2I)X \cdot \underbrace{BB^{-1}}_{=I} = CB^{-1}$$

$$\underbrace{(A+2I)^{-1}(A+2I)}_{=I} X = (A+2I)^{-1} C B^{-1}$$

$$X = (A+2I)^{-1} C B^{-1}$$