Rep 412

$$AXB^{-1} = A + B^T$$

$$\underbrace{A^{-1}A}_{I}XB^{-1} = A^{-1}(A + B^T)$$

$$X\underbrace{B^{-1}B}_{I} = A^{-1}(A + B^T)B$$

$$X = \underbrace{A^{-1}A}_{I}B + A^{-1}B^TB$$

$$X = B + A^{-1}B^TB$$

$$(-1)\ (-2)\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -3 & -5 & -2 & 1 & 0 \\ 0 & -1 & -2 & -1 & 0 & 1 \end{array}\right]$$

$$\sim (-3)\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -1 & -2 & -1 & 0 & 1 \\ 0 & -3 & -5 & -2 & 1 & 0 \end{array}\right] \sim (-3)\ (2)\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -1 & -2 & -1 & 0 & 1 \\ 0 & 0 & \boxed{1} & 1 & 1 & -3 \end{array}\right]$$

$$\sim -1\times\left[\begin{array}{ccc|ccc} 1 & 2 & 0 & -2 & -3 & 9 \\ 0 & -1 & 0 & 1 & 2 & -5 \\ 0 & 0 & 1 & 1 & 1 & -3 \end{array}\right] \sim (-2)\left[\begin{array}{ccc|ccc} 1 & 2 & 0 & -2 & -3 & 9 \\ 0 & 1 & 0 & -1 & -2 & 5 \\ 0 & 0 & 1 & 1 & 1 & -3 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & -1 & -2 & 5 \\ 0 & 0 & 1 & 1 & 1 & -3 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 0 & 1 & -1 \\ -1 & -2 & 5 \\ 1 & 1 & -3 \end{bmatrix}$$

$$X = B + A^{-1}B^TB =$$

$$= \begin{bmatrix} 2 & -1 & 1 \\ 1 & 0 & 1 \\ -1 & 1 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 1 & -1 \\ -1 & -2 & 5 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} 2 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & -1 & 1 \\ 1 & 0 & 1 \\ -1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -1 & 1 \\ 1 & 0 & 1 \\ -1 & 1 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 1 & -1 \\ -1 & -2 & 5 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} 6 & -3 & 2 \\ -3 & 2 & 0 \\ 2 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -1 & 1 \\ 1 & 0 & 1 \\ -1 & 1 & 1 \end{bmatrix} + \begin{bmatrix} -5 & 2 & -3 \\ 10 & -1 & 13 \\ -3 & -1 & -7 \end{bmatrix} = \begin{bmatrix} -3 & 1 & -2 \\ 11 & -1 & 14 \\ -4 & 0 & -6 \end{bmatrix}$$

Egenvärden & Egenvektorer

Ex: $A = \begin{bmatrix} -3 & 2 \\ -10 & 6 \end{bmatrix}$

$$\begin{bmatrix} -3 & 2 \\ -10 & 6 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ -4 \end{bmatrix}$$

$$\begin{bmatrix} -3 & 2 \\ -10 & 6 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$ ← Egenvärde 1

Samma

$$\begin{bmatrix} -3 & 2 \\ -10 & 6 \end{bmatrix} \begin{bmatrix} 2 \\ 5 \end{bmatrix} = \begin{bmatrix} 4 \\ 10 \end{bmatrix} = \overset{\text{Egenvärde 2}}{\overset{\downarrow}{2}} \cdot \begin{bmatrix} 2 \\ 5 \end{bmatrix}$$

proportionella

Definition av egenvärde λ och egenvektor $\bar{v}$

$$\boxed{A\bar{v} = \lambda \cdot \bar{v}}$$

$$A\bar{v} - \lambda\bar{v} = \bar{0}$$

$$(A - \lambda I)\bar{v} = \bar{0}$$

Välj λ så att $(A - \lambda I)$ ej har invers

dvs så $\det(A - \lambda I) = 0$

Ex: 1. Egenvärden

Välj λ så att

$$0 = \det(A - \lambda I) = \begin{vmatrix} 9-\lambda & 5 \\ 1 & 5-\lambda \end{vmatrix} = (9-\lambda)(5-\lambda) - 1\cdot 5$$

$$= 45 - 14\lambda + \lambda^2 - 5 = \lambda^2 - 14\lambda + 40$$

$$\lambda = 7 \pm \sqrt{49-40} = 7 \pm \sqrt{9} = 7 \pm 3$$

$\boxed{\lambda = 10}$ eller $\boxed{\lambda = 4}$

2. Egenvektorer

$\boxed{\lambda = 10}$ $\qquad (A - 10\cdot I)\bar{v} = \bar{0}$

$$\left[\begin{array}{cc|c} 9-10 & 5 & 0 \\ 1 & 5-10 & 0 \end{array}\right] = \left[\begin{array}{cc|c} -1 & 5 & 0 \\ 1 & -5 & 0 \end{array}\right] \overset{1}{\sim} \left[\begin{array}{cc|c} \boxed{-1} & 5 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

Bunden v_1 $\qquad$ Fri v_2

$v_2 = t \quad v_1 = 5t \qquad \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 5t \\ t \end{bmatrix} = t\begin{bmatrix} 5 \\ 1 \end{bmatrix}$

$\boxed{\lambda = 4}$

$$\left[\begin{array}{cc|c} 9-4 & 5 & 0 \\ 1 & 5-4 & 0 \end{array}\right] = \left[\begin{array}{cc|c} 5 & 5 & 0 \\ 1 & 1 & 0 \end{array}\right] \overset{-\frac{1}{5}}{\sim} \left[\begin{array}{cc|c} \boxed{5} & 5 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$v_2 = t \quad v_1 = -t \qquad \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} -t \\ t \end{bmatrix} = t\begin{bmatrix} -1 \\ 1 \end{bmatrix}$

Ex: 1. Egenvärden

$$0 = \det(A-\lambda I) = \begin{vmatrix} 5-\lambda & 4 & -2 \\ 4 & 5-\lambda & 2 \\ -2 & 2 & 8-\lambda \end{vmatrix} = \begin{vmatrix} 5-\lambda & 4 & -2 \\ 0 & 9-\lambda & 18-2\lambda \\ -2 & 2 & 8-\lambda \end{vmatrix}$$

$$= \begin{vmatrix} 5-\lambda & 4 & -10 \\ 0 & 9-\lambda & 0 \\ -2 & 2 & 4-\lambda \end{vmatrix} = -0\cdot|\ldots| + (9-\lambda)\begin{vmatrix} 5-\lambda & -10 \\ -2 & 4-\lambda \end{vmatrix} - 0\cdot|\ldots|$$

$$= (9-\lambda)\big((5-\lambda)(4-\lambda)-20\big) = (9-\lambda)(\cancel{20}-9\lambda+\lambda^2-\cancel{20})$$

$$= \underbrace{(9-\lambda)}_{-(\lambda-9)}\lambda(\lambda-9) = -\lambda(\lambda-9)^2$$

$\boxed{\lambda=0}$ $\boxed{\lambda=9}$ ← multiplicitet 2

2. Egenvektorer $(A-\lambda I)\bar{v} = \bar{0}$

$\boxed{\lambda=0}$

$$\left[\begin{array}{ccc|c} 5 & 4 & -2 & 0 \\ 4 & 5 & 2 & 0 \\ -2 & 2 & 8 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} -2 & 2 & 8 & 0 \\ 4 & 5 & 2 & 0 \\ 5 & 4 & -2 & 0 \end{array}\right] \sim$$

$$\left[\begin{array}{ccc|c} -2 & 2 & 8 & 0 \\ 0 & 9 & 18 & 0 \\ 0 & 9 & 18 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} \boxed{-2} & 2 & 8 & 0 \\ 0 & \boxed{9} & 18 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Bundna: v_1, v_2 Fri: $v_3 = t$ $v_2 = -\frac{18}{9}v_3 = -2t$

$v_1 = \frac{1}{-2}(-2v_2 - 8v_3) = -2t+4t = 2t$

$$\bar{v} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 2t \\ -2t \\ t \end{bmatrix} = t\begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$$

$\boxed{\lambda = 9}$

$$\left[\begin{array}{ccc|c} 5-9 & 4 & -2 & 0 \\ 4 & 5-9 & 2 & 0 \\ -2 & 2 & 8-9 & 0 \end{array}\right] = \left[\begin{array}{ccc|c} -4 & 4 & -2 & 0 \\ 4 & -4 & 2 & 0 \\ -2 & 2 & -1 & 0 \end{array}\right] \quad (1) \quad \left(-\frac{1}{2}\right)$$

$$\sim \left[\begin{array}{ccc|c} \boxed{-4} & 4 & -2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Bunden: v_1

Fria: $v_2 \quad v_3$

$v_2 = s \qquad v_3 = t \qquad v_1 = \frac{1}{-4}(-4v_2 + 2v_3) = s - \frac{1}{2}t$

$$\bar{v} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} s - 1/2\,t \\ s \\ t \end{bmatrix} = s\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} -1/2 \\ 0 \\ 1 \end{bmatrix}$$