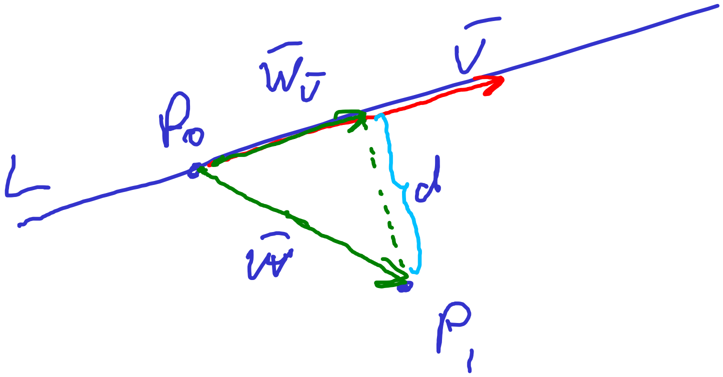
$\boxed{1}$



$$d = \sqrt{\|\bar{w}\|^2 - \|\bar{w}_{\bar{v}}\|^2}$$

$$P_0 : (5, -13, 1)$$

$$\bar{w} = \begin{bmatrix} 1-5 \\ -3-(-13) \\ 2-1 \end{bmatrix} = \begin{bmatrix} -4 \\ 10 \\ 1 \end{bmatrix} \qquad \|\bar{w}\|^2 = (-4)^2 + 10^2 + 1^2 = 117$$

$$\bar{v} = \begin{bmatrix} -3 \\ 2 \\ 2 \end{bmatrix} \qquad \|\bar{w}_{\bar{v}}\| = \frac{|\bar{w} \cdot \bar{v}|}{\|\bar{v}\|} = \frac{|(-4)(-3) + 10\cdot 2 + 1\cdot 2|}{\sqrt{(-3)^2 + 2^2 + 2^2}}$$

$$= \frac{34}{\sqrt{17}} = 2 \cdot \frac{17}{\sqrt{17}} = 2 \cdot \sqrt{17}$$

Avståndet $d = \sqrt{117 - (2\cdot\sqrt{17})^2} = \sqrt{117 - 4\cdot 17}$

$= \sqrt{49} = 7$ l.e.

$\boxed{2}$

$$A = \int_{-1}^{0} (0 - x \cdot e^x)\,dx + \int_{0}^{1} (x \cdot e^x - 0)\,dx$$

$$= -\int_{-1}^{0} x \cdot e^x\,dx + \int_{0}^{1} x \cdot e^x\,dx$$

$$= -\Big[x \cdot e^x\Big]_{-1}^{0} + \int_{-1}^{0} e^x\,dx + \Big[x\,e^x\Big]_{0}^{1} - \int_{0}^{1} e^x\,dx$$

$$= -0 \cdot e^0 + (-1)\,e^{-1} + \Big[e^x\Big]_{-1}^{0} + 1 \cdot e^1 - 0 \cdot e^0 - \Big[e^x\Big]_{0}^{1}$$

$$= -e^{-1} + \underbrace{e^0}_{=1} - e^{-1} + \cancel{e^1} - \cancel{e^1} + \underbrace{e^0}_{=1} = \underbrace{2 - 2e^{-1}}_{\text{Svar}}$$

$\boxed{3}$

$$X \cdot B = X + C$$

$$X \cdot B - X = C$$

$$X \cdot (B - I) = C$$

$$X \cdot \underbrace{(B-I)(B-I)^{-1}}_{= I} = C(B-I)^{-1}$$

$$X = C(B-I)^{-1}$$

$$B - I = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} \qquad (B-I)^{-1} = \frac{1}{1\cdot 2 - 1\cdot 1} \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$$

$$X = C(B-I)^{-1} = \begin{bmatrix} 1 & 4 \\ -2 & 2 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} = \underbrace{\begin{bmatrix} -2 & 3 \\ -6 & 4 \end{bmatrix}}_{\text{svar}}$$

4a

$$\int \frac{7x-10}{x^2-3x+2}\,dx$$

1. Lägre gradtal i täljaren   Ok!
2. Faktorisera nämnaren

$$x^2-3x+2 = (x-1)(x-2)$$

3. Partialbråksuppdela

$$\frac{7x-10}{(x-1)(x-2)} = \frac{A}{x-1} + \frac{B}{x-2}$$

Handpåläggning ger $A = 3$ $\quad B = 4$

4. Lös integralen

$$\int \frac{7x-10}{x^2-3x+2}\,dx = \int \left(\frac{3}{x-1} + \frac{4}{x-2}\right)dx$$

$$= 3\cdot \ln|x-1| + 4\cdot \ln|x-2| + C$$

4b

$$\int_1^e \ln(\sqrt{x})\,dx = \left[\begin{array}{l|l} u=\sqrt{x} & x=1 \\ x=u^2 & \Downarrow \\ \frac{dx}{du}=2u & u=1 \\ & x=e \\ & \Downarrow \\ dx = 2u\,du & u=\sqrt{e}=e^{1/2} \end{array}\right]$$

$$= \int_1^{e^{1/2}} 2u\cdot \ln u \cdot du = \left[u^2 \ln u\right]_1^{e^{1/2}} - \int_1^{e^{1/2}} \underbrace{u^2\cdot\frac{1}{u}}_{=u}\,du$$

$$= \underbrace{(e^{1/2})^2}_{e}\cdot \underbrace{\ln e^{1/2}}_{1/2} - 1^2\cdot \underbrace{\ln 1}_{0} - \left[\tfrac{1}{2}u^2\right]_1^{e^{1/2}}$$

$$= \cancel{e\cdot\tfrac{1}{2}} - \cancel{\tfrac{1}{2}\underbrace{(e^{1/2})^2}_{e}} + \tfrac{1}{2}\,1^2 = \tfrac{1}{2} \leftarrow \text{Svar}$$

$\boxed{5}$

$$A = \begin{bmatrix} 1 & 2 & a \\ 2 & a & 8 \\ a & -2 & 0 \end{bmatrix}$$

Oändligt många eller inga lösningar om $\det A = 0$

$(-a)$ (-2)

$$0 = \det(A) = \begin{vmatrix} 1 & 2 & a \\ 2 & a & 8 \\ a & -2 & 0 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 2 & a-4 & -2a+8 \\ a & -2a-2 & -a^2 \end{vmatrix}$$

$$= 1 \cdot C_{11} + 0 \cdot C_{12} + 0 \cdot C_{13} = 1 \cdot \begin{vmatrix} a-4 & \overbrace{-2a+8}^{(-2)(a-4)} \\ -2a-2 & -a^2 \end{vmatrix}$$

$$= (a-4) \begin{vmatrix} 1 & -2 \\ -2a-2 & -a^2 \end{vmatrix} = (a-4)\left(\overbrace{-a^2-4a-4}^{(-1)(a^2+4a+4)}\right)$$

$$= -(a-4)(a+2)^2 \Leftrightarrow \underbrace{a = 4 \text{ eller } a = -2}$$

Oändligt många lösningar för bägge dessa pga homogent system.

$\boxed{a = 4}$

(-4) (-2)

$$\left[\begin{array}{ccc|c} 1 & 2 & 4 & 0 \\ 2 & 4 & 8 & 0 \\ 4 & -2 & 0 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 2 & 4 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -10 & -16 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & 4 & 0 \\ 0 & -10 & -16 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Bundna $x \quad y$

Fria z

$z = t \qquad -10y - 16t = 0 \qquad x + 2\left(-\frac{8}{5}t\right) + 4t = 0$

$y = -\frac{16}{10}t = -\frac{8}{5}t \qquad x = t\left(\frac{16}{5} - 4\right) = -\frac{4}{5}t$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -4/5 \cdot t \\ -8/5\ t \\ t \end{bmatrix} = \frac{t}{5}\begin{bmatrix} -4 \\ -8 \\ 5 \end{bmatrix}$$

$$\boxed{a = -2}$$

$(2)\ (-2)$

$$\left[\begin{array}{ccc|c} 1 & 2 & -2 & 0 \\ 2 & -2 & 8 & 0 \\ -2 & -2 & 0 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 2 & -2 & 0 \\ 0 & -6 & 12 & 0 \\ 0 & 2 & -4 & 0 \end{array}\right] \quad \frac{1}{3}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & -2 & 0 \\ 0 & -6 & 12 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Bundne: x, y

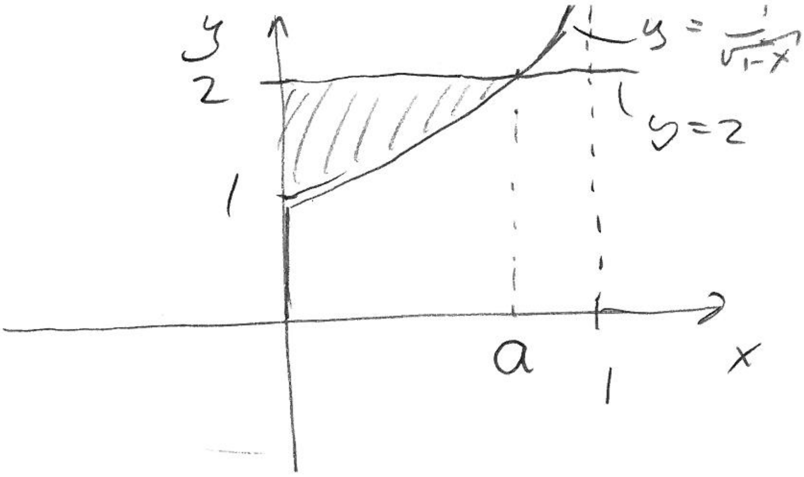
Frie: z

$z = t \qquad (-6)y + 12t = 0 \qquad x + 2\cdot 2t - 2\cdot t = 0$

$y = 2t \qquad x = -2t$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2t \\ 2t \\ t \end{bmatrix} = t\begin{bmatrix} -2 \\ 2 \\ 1 \end{bmatrix}$$

6.1 (a)



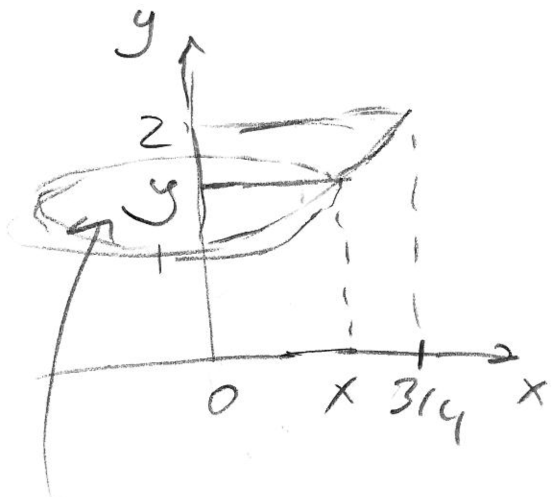
$$\frac{1}{\sqrt{1-a}}=2$$

$$\frac{1}{2}=\sqrt{1-a}$$

$$\frac{1}{4}=1-a$$

$$\boxed{a=1-\frac{1}{4}=\frac{3}{4}}$$

(b)



$$y=\frac{1}{\sqrt{1-x}}$$

$$\sqrt{1-x}=\frac{1}{y}$$

$$1-x=\frac{1}{y^2}$$

$$x=1-\frac{1}{y^2}$$

$$A(y)=\pi x^2=\pi\left(1-\frac{1}{y^2}\right)^2=\pi\left(1-\frac{2}{y^2}+\frac{1}{y^4}\right)$$

$$V=\int_1^2 A(y)\,dy=\pi\int_1^2\left(1-2y^{-2}+y^{-4}\right)dy$$

$$=\pi\left[y+2y^{-1}-\frac{1}{3}y^{-3}\right]_1^2=\pi\left(\cancel{2}+\cancel{2\cdot\frac{1}{2}}-\frac{1}{3}\frac{1}{2^3}-\cancel{1}-\cancel{2}+\frac{1}{3}\right)$$

$$=\frac{\pi}{3}\left(1-\frac{1}{8}\right)=\frac{7\pi}{24}$$

$\boxed{6.2}$ $$f'(x) = 3x^2 - \frac{1}{12x^2}$$

$$s = \int_{1/2}^{2} \sqrt{1 + (f'(x))^2}\, dx = \int_{1/2}^{2} \sqrt{1 + \left(3x^2 - \frac{1}{12x^2}\right)^2}\, dx$$

$$= \int_{1/2}^{2} \sqrt{1 + 9x^4 - \frac{2 \cdot 3x^2}{12x^2} + \frac{1}{144x^4}}\, dx$$

$$= \int_{1/2}^{2} \sqrt{9x^4 + \frac{1}{2} + \frac{1}{144x^4}}\, dx = \int_{1/2}^{2} \sqrt{\left(3x^2 + \frac{1}{12x^2}\right)^2}\, dx$$

$$= \int_{1/2}^{2} \left(3x^2 + \frac{1}{12}x^{-2}\right) dx = \left[x^3 - \frac{1}{12}x^{-1} \right]_{1/2}^{2}$$

$$= 2^3 - \frac{1}{12}2^{-1} - \frac{1}{2^3} + \frac{1}{12}2 = 8 - \frac{1}{24} - \frac{1}{8} + \frac{1}{6}$$

$$= 8 - \frac{1}{24} - \frac{3}{24} + \frac{4}{24} = 8$$

$\boxed{6.3}$

$h = 0{,}25$

ger

k	0	1	2	3	4
x_k	0	0.25	0.50	0,75	1.0
y_k	1,0000	0,9923	0,9428	0,8356	0,7071

$$T = h\left(y_1 + y_2 + y_3 + \frac{1}{2}(y_0 + y_4)\right) = 0{,}90682$$