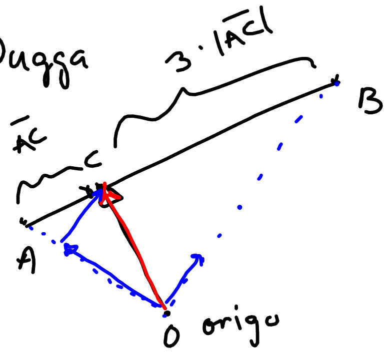
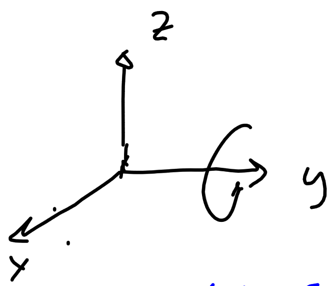
F11 Integralkalkyl - intro (FN kap 5–7)

Rep. Dugga



$$\vec{OC} = \vec{OA} + \vec{AC}$$
$$= \vec{OA} + \frac{1}{4}\vec{AB}$$
$$= \vec{OA} + \frac{1}{4}(\vec{AO} + \vec{OB})$$
$$= \vec{OA} - \frac{1}{4}\vec{OA} + \frac{1}{4}\vec{OB}$$
$$= \frac{3}{4}\vec{OA} + \frac{1}{4}\vec{OB} = \begin{bmatrix} 3/4 \\ 1/4 \end{bmatrix}_{\{\vec{OA},\vec{OB}\}}$$



A beskriver rotation kring y-axeln. vinkeln v

$$A^2 = \begin{pmatrix} \cos v & 0 & -\sin v \\ 0 & 1 & 0 \\ \sin v & 0 & \cos v \end{pmatrix}\begin{pmatrix} \cos v & 0 & -\sin v \\ 0 & 1 & 0 \\ \sin v & 0 & \cos v \end{pmatrix} =$$

$$= \begin{pmatrix} \cos^2 v - \sin^2 v & 0 & -\cos v \sin v \cdot 2 \\ 0 & 1 & 0 \\ +2 \sin v \cos v & 0 & \cos^2 v - \sin^2 v \end{pmatrix} =$$

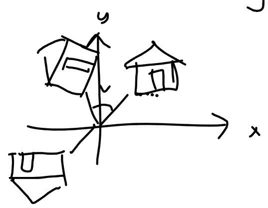
$$= \begin{pmatrix} \cos 2v & 0 & -\sin 2v \\ 0 & 1 & 0 \\ +\sin 2v & 0 & \cos 2v \end{pmatrix}$$

A^2 beskriver rotation kring y-axeln vinkeln $2v$.

Rep: Egenvärden och egenvektorer.

$$A\bar{x} = \lambda \cdot \bar{x} \qquad A = \begin{pmatrix} \cos v & -\sin v \\ \sin v & \cos v \end{pmatrix}$$

roterar kring z-axeln.



$v = 180°$: $A = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

$$\underbrace{A\bar{x}}_{} = \underbrace{\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}}_{A}\underbrace{\begin{pmatrix} x \\ y \end{pmatrix}}_{\bar{x}} = \begin{pmatrix} -x \\ -y \end{pmatrix} = \underbrace{-1}_{\lambda=-1} \cdot \underbrace{\begin{pmatrix} x \\ y \end{pmatrix}}_{\bar{x}}$$

Alla vektorer i $\mathbb{R}^2$ är egenvektorer till A med egenvärdet -1.

Ex) $A = \begin{pmatrix} 11 & 6 \\ -12 & -7 \end{pmatrix}$ Bestäm egenvärden egenvektorer till A.

$$A\bar{x} = \lambda \cdot \bar{x}$$
$$\underbrace{(A - \lambda \cdot I)}_{\det = 0} \cdot \bar{x} = 0$$

- $\begin{vmatrix} 11-\lambda & 6 \\ -12 & -7-\lambda \end{vmatrix} = 0 \iff (11-\lambda)(-7-\lambda) - (-12)\cdot 6 = 0$

$$\lambda^2 - 4\lambda - 5 = 0$$
$$\vdots$$
$$\begin{cases} \lambda_1 = 5 \\ \lambda_2 = -1 \end{cases} \quad \text{egenvärden till } A.$$

- $\underline{\lambda = 5}$

$$\left(\begin{array}{cc|c} 11-5 & 6 & 0 \\ -12 & -7-5 & 0 \end{array}\right) \sim \left(\begin{array}{cc|c} 6 & 6 & 0 \\ -12 & -12 & 0 \end{array}\right) \begin{matrix} 2 \cdot \frac{1}{6} \\ \leftarrow \end{matrix} \sim \left(\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right)$$

Sätt $y = t$ ger $x + t = 0$, $x = -t$ $\qquad \begin{pmatrix} x \\ y \end{pmatrix} = t \cdot \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

$\bar{v}_1 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$ är en egenvektor till A med egenvärdet 5

$$A \cdot \bar{v}_1 = 5 \cdot \bar{v}_1$$

- $\underline{\lambda = -1}$

$$\left(\begin{array}{cc|c} 11-(-1) & 6 & 0 \\ -12 & -7-(-1) & 0 \end{array}\right) \sim \dots \sim \left(\begin{array}{cc|c} 2 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right)$$

Sätt $y = t$, $x = -\frac{t}{2}$ $\qquad \begin{pmatrix} x \\ y \end{pmatrix} = t\begin{pmatrix} -1/2 \\ 1 \end{pmatrix} = \frac{t}{2}\begin{pmatrix} -1 \\ 2 \end{pmatrix}$

$\bar{v}_2 = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$ är egenvektor till A med egenvärdet -1.

även $\begin{pmatrix} 1 \\ -2 \end{pmatrix}, \begin{pmatrix} 10 \\ -20 \end{pmatrix}$ – " – – " – – " –

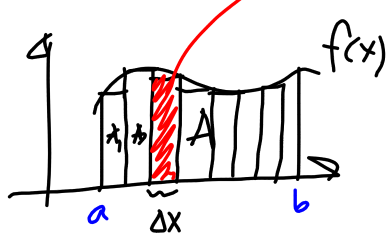
$$|\bar{v}_2| = \sqrt{(-1)^2 + 2^2} = \sqrt{5} \qquad \hat{v}_2 = \frac{1}{\sqrt{5}}\begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

Normerad egenvektor till A.

Primitiva funktioner. kap 5

Riemansumma (kap 6.1)

Antag $f(x) > 0$ $\qquad \Delta A = (\text{Liten rektangel}) = \overset{\text{höjd}}{f(x_i)} \cdot \overset{\text{bas}}{\Delta x}$



Area ?

$$\Delta x = x_i - x_{i-1}$$

$$A \approx A_1 + A_2 + \dots = \sum \Delta A = \sum_{i=1}^{n} f(x_i) \cdot \Delta x$$

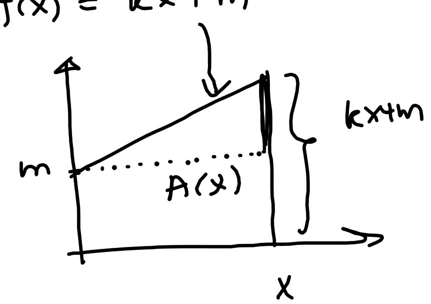
Om $\Delta x \to 0$

$$\sum_{i=1}^{n} f(x_i) \cdot \Delta x \longrightarrow \int_a^b f(x)\, dx$$

$$A = \int_a^b f(x)\, dx$$

– – –

$$f(x) = kx + m$$



$$A(x) = m \cdot x + \frac{kx \cdot x}{2}$$

$$= k \cdot \frac{x^2}{2} + m \cdot x$$

$$A'(x) = \frac{k}{2} \cdot 2 \cdot x + m \cdot 1 = kx + m$$

$$A'(x) = f(x)$$

$$f(x) \xrightarrow{\text{derivera}} f'(x)$$

$$f(x)+C \xleftarrow{\text{integrera}} f'(x)$$

$$F(x)+C \xleftarrow[\text{"antiderivera"}]{\text{integrera}} f(x)$$

- Obestämd integral = primitivfunktion (har inga integrationsgränser)

$$\int f(x)\,dx = F(x)+C$$

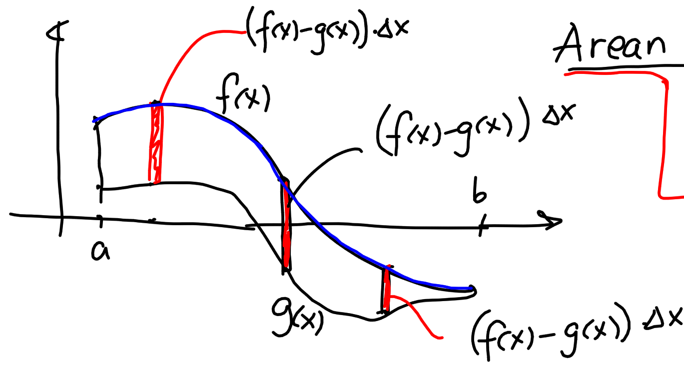
$$F'(x) = f(x)$$

- Bestämd integral : $\int_a^b f(x)\,dx = \left[F(x)\right]_a^b = F(b)-F(a)$

ger ett tal.

$f(x), g(x)$ kan både vara positiva el. neg. för olika x

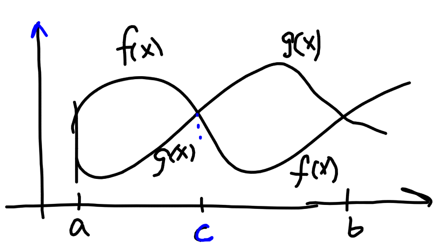
Antag $f(x) > g(x)$



Arean mellan $f(x)$ och $g(x)$:

$$A = \int_a^b \left(f(x)-g(x)\right)\cdot dx$$

där $f(x) > g(x)$



$$\text{Arean} = \int_a^c (f(x)-g(x))\,dx + \int_c^b (g(x)-f(x))\,dx$$

ty $g(x) > f(x)$

Elementära Primitiva funktioner : (Se "nyttiga samband" i Fronter.)

$f(x) = 2x \qquad F(x) = x^2 + C$

$f(x) = e^{3x} \qquad F(x) = \frac{e^{(3x)}}{3} + C \qquad F'(x) = \frac{1}{\cancel{3}} \cdot e^{3x} \cancel{3} + 0$

$f(x) = \frac{1}{x} \qquad F(x) = \ln|x| + C$

- $\boxed{\int \frac{1}{x}\,dx} = \ln|x| + C$

$F(x) = \ln|x|$

$x > 0: \quad F(x) = \ln(x) \qquad F'(x) = \frac{1}{x}$

$x < 0: \quad F(x) = \ln(-x) \qquad F'(x) = \frac{1}{-x} \cdot (-1) = \frac{1}{x}$

- $\int \sin x\,dx = -\cos x + C$
- $\int \cos x\,dx = \sin x + C$
- $\int x^p\,dx = \frac{x^{p+1}}{p+1} + C$ om $p \neq -1$ (om $p = -1$)

$\int \sqrt{x}\,dx = \int x^{1/2} = \frac{x^{3/2}}{3/2} + C$

- $\int \frac{1}{1+x^2}\,dx = \arctan(x) + C$
- $\int (1 + \tan^2 x)\,dx = \int \frac{dx}{\cos^2 x} = \tan x + C$

$\int \frac{1}{2+x^2}\,dx = \frac{1}{2}\int \frac{dx}{1 + (\frac{x^2}{2})} = \frac{1}{2}\int \frac{dx}{1 + (\frac{x}{\sqrt{2}})^2} =$

$= \frac{1}{2} \frac{\arctan\left(\frac{x}{\sqrt{2}}\right)}{\frac{1}{\sqrt{2}}} + C = \frac{\sqrt{2}}{2} \cdot \arctan\left(\frac{x}{\sqrt{2}}\right) + C$

$= \frac{1}{\sqrt{2}} \cdot \arctan\left(\frac{x}{\sqrt{2}}\right) + C$

$\int \sin^2 x \, dx = \int \sin x \cdot \sin x \, dx$

Omskrivning m.h.a "dubbla vinkeln" – vid jämn exponent
trig. ettan – vid udda – " –

I $\cos^2 x + \sin^2 x = 1$

II $\cos^2 x - \sin^2 x = \cos 2x$

I-II $2\sin^2 x = 1 - \cos 2x$

$$\sin^2 x = \frac{1-\cos 2x}{2}$$

$$\int \sin^2 x \, dx = \int \frac{1-\cos 2x}{2} \, dx = \int \left(\frac{1}{2} - \frac{\cos 2x}{2}\right) dx =$$

$$= \frac{x}{2} - \frac{1}{2} \cdot \frac{\sin 2x}{2} + C$$