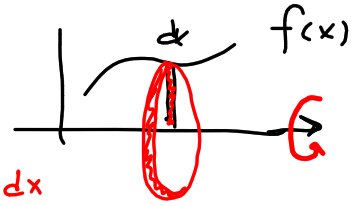
F19 Numerisk integration - Trapetsmetoden

Rep. Rotationsvolym



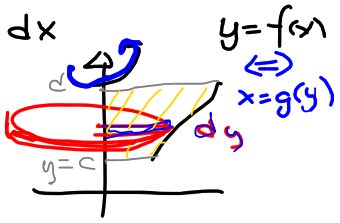
$dV = \pi r^2 \cdot dx$

Skivmetoden (cirkelskivmetoden):

$dV = \pi r^2 \cdot dx$

$= \pi f(x)^2 \, dx$

- dx - integral, rotation kring x-axeln
- dy - integral, rotation kring y-axeln.



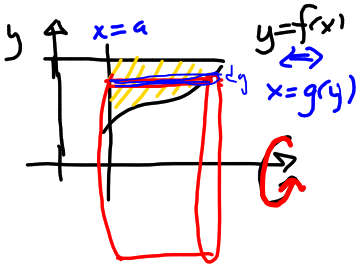
$dV = \pi \cdot g(y)^2 \cdot dy$

Rörmetoden: (cylinderskalmetoden)

$dV = 2\pi r \cdot h \cdot dx$

- dx - integral, rot. kring y-axeln
- dy - integral, rot. kring x-axeln

$dV = 2\pi x \cdot f(x) \, dx$



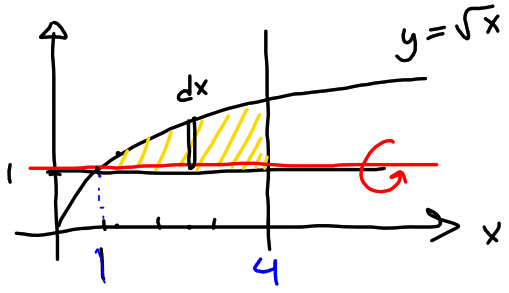
$dV = 2\pi y \cdot (g(y) - a) \, dy$

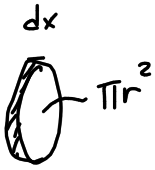
ex) $y=\sqrt{x}$, $y=1$, $x=4$ } Området roterar kring $y=1$. Bestäm rotationsvolymen.

- skärningspunkter: $y=y$

$1=\sqrt{x} \iff x=1 \Rightarrow y=1$

$x=4 \Rightarrow y=2$





skivmetoden:

$dV = \pi r^2 \cdot dx$

$= \pi \cdot (y-1)^2 \, dx$

$= \pi (\sqrt{x}-1)^2 \, dx$

$$V = \int_1^4 \pi (x - 2\sqrt{x} + 1)\, dx =$$

$$= \int_1^4 \pi \left(x - 2x^{1/2} + 1\right) dx = \pi \left[\frac{x^2}{2} - 2\frac{x^{3/2}}{3/2} + x\right]_1^4 =$$

$$= \pi \left[\frac{4^2}{2} - \frac{4}{3} \cdot 4^{3/2} + 4 - \left(\frac{1}{2} - \frac{4}{3} \cdot 1^{3/2} + 1\right)\right] =$$

$(4^{\frac{1}{2}})^3 = 2^3 = 8$

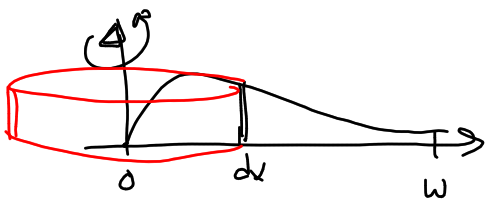
$$= \pi \left[8 - \frac{32}{3} + 4 - \frac{1}{2} + \frac{4}{3} - 1\right] = \ldots = \frac{7}{6}\pi$$

$$= \pi \left[\frac{11 \cdot 6}{6} - \frac{64}{6} - \frac{3}{6} + \frac{8}{6}\right] = \frac{7\pi}{6}.$$

Ex) $y = x^2 \cdot e^{-x^2}$ roterar kring y-axeln.

positiva x-axeln

$x = w \quad ; \quad w > 0$



x-axeln $\Leftrightarrow y = 0$

- skärningspunkter: $0 = x^2 \cdot e^{-x^2} \Leftrightarrow$ $x = 0$

$p > 0$

$e^x > x^p > \ln x$ då $x \to \infty$

$y = \frac{x^2}{e^{x^2}} \to 0$ då $x \to \infty$

- dx-integral, rot. kring y-axeln $\Rightarrow$ rörmetoden:

$dV = 2\pi r \cdot h \cdot dx = 2\pi \cdot x \cdot (x^2 \cdot e^{-x^2})\, dx$

$$V = \int_0^w \pi \cdot 2x \cdot x^2 \cdot e^{-x^2}\, dx = \left\{ \begin{array}{l} t = -x^2 \Leftrightarrow x^2 = -t \\ dt = -2x \cdot dx \\ -dt = 2x\, dx \\ x = [0, w] \Rightarrow t = [0, -w^2] \end{array} \right\} =$$

$= -dt$

$$= \int_0^{-w^2} \pi (-t) \cdot e^t (-dt) = \int_0^{-w^2} \pi \cdot t \cdot e^t\, dt = \left\{ \text{Partiell integration} \right\}$$

der int $\to$ $\uparrow$

$$= \left[\pi \cdot t \cdot e^t \right]_0^{-w^2} - \int_0^{-w^2} \pi \cdot e^t\, dt =$$

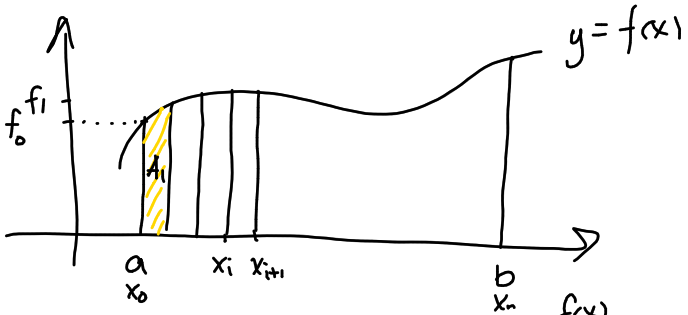
$$= \pi(-w^2) \cdot e^{-w^2} - 0 - \left[\pi e^t \right]_0^{-w^2} =$$

$$= -\frac{\pi w^2}{e^{w^2}} - \left(\pi e^{-w^2} - \pi \cdot e^0 \right) = -\frac{\pi \cdot w^2}{e^{w^2}} - \frac{\pi}{e^{w^2}} + \pi$$

b) $\lim_{w \to \infty} V = \lim_{w \to \infty} -\frac{\pi w^2}{e^{w^2}} - \frac{\pi}{e^{w^2}} + \pi = \pi$

$\to 0$ $\to 0$

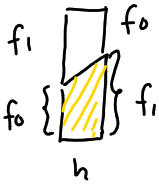
Numerisk integration - Trapetsmetoden



$f_0 = f(x_0)$

$f_1 = f(x_1)$

Dela in i trapetser



$h = \frac{(b-a)}{n} = \Delta x$

n = antal indelningar (partitioner)

Area av trapets $= \frac{(f_0+f_1)\cdot h}{2} = A_1$

$$I \approx A_1 + A_2 + \ldots + A_n$$

$$= \frac{(f_0+f_1)\cdot h}{2} + \frac{(f_1+f_2)\cdot h}{2} + \frac{(f_2+f_3)\cdot h}{2} + \ldots + \frac{(f_{n-1}+f_n)\cdot h}{2}$$

$$= h\cdot\left(\frac{f_0+f_n}{2} + f_1+f_2+\ldots+f_{n-1}\right)$$

Trapets metoden: steglängd h

$$I \approx T(h) = h\cdot\left(\frac{f(x_0)+f(x_n)}{2} + \sum_{i=1}^{n-1} f(x_i)\right)$$

$a = x_0$

$x_1 = a+h$

$\vdots$

$x_i = a + i\cdot h$

$\vdots$

$x_n = b$

där h är steglängden.

ex) $\int_0^{\pi} \underbrace{\cos(\sin x)}_{f(x)}\, dx$

Trapetsregeln m. steglängd $h = \pi/6$. Svara m. 2 decimaler.

- Tabell:

x	0	$\frac{\pi}{6}$	$\frac{2\pi}{6}$	$\frac{3\pi}{6}$	$\frac{4\pi}{6}$	$\frac{5\pi}{6}$	π
$f(x) = \cos(\sin x)$	1	0,87758	0,6479	..	..	0,87758	1

(+h, +h)

sum: 3,5913

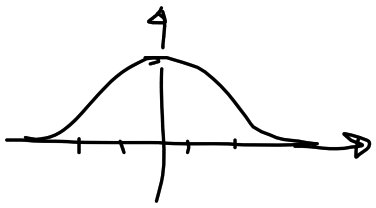
- $I \approx T(h) = h \cdot \left(\frac{f(0)+f(\pi)}{2} + \underbrace{\sum_{i=1}^{n-1} f(x_i)}_{=\text{sum}} \right)$

$= \frac{\pi}{6}\left(\frac{1+1}{2} + 3{,}5913 \right) \approx 2{,}4040$

Svar: $I \approx 2{,}40$

Felet proportionellt mot h^2.
Om steget h halveras, minskas felet av $T(h)$ till en fjärdedel.

$\int e^{-x^2}\,dx$



Tenta ex.

ex) $\int_0^2 \underbrace{e^{\sin x}}_{f(x)}\,dx$

Trapetsmetoden.
Steglängd $h = 0{,}5$
Svara m. 3 decimaler.

· Tabell:

x	0	0,5	1	1,5	2
$f(x) = e^{\sin x}$	1	1,61515	2,31978	2,71148	2,48258

h h h h

sum: = 6,6464

$$I \approx T(h) = h \cdot \left(\frac{f(0) + f(2)}{2} + \text{sum} \right)$$

$$= 0{,}5 \cdot \left(\frac{1 + 2{,}48258}{2} + 6{,}6464 \right) \approx \underline{4{,}194}$$

Glöm ej steglängden!

OBS! Ställ in på radianer.