$\boxed{\text{F20}}$ Repetition (Staffans slide Lek 20)

Se även repetion: Inspelade föreläsningar Luleå....

A12	Rep.	integraler
L07	Rep	vektorer
L14	Rep	matriser

Läs uppg. i presentationen på egen hand först.

tenta aug-14.

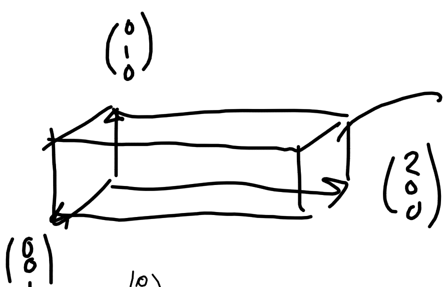
ex) $\det B = 2$ där B är 3×3 matris.

a) $\det(B^2) = \det(BB) = \det B \cdot \det B = 2\cdot 2 = 4$

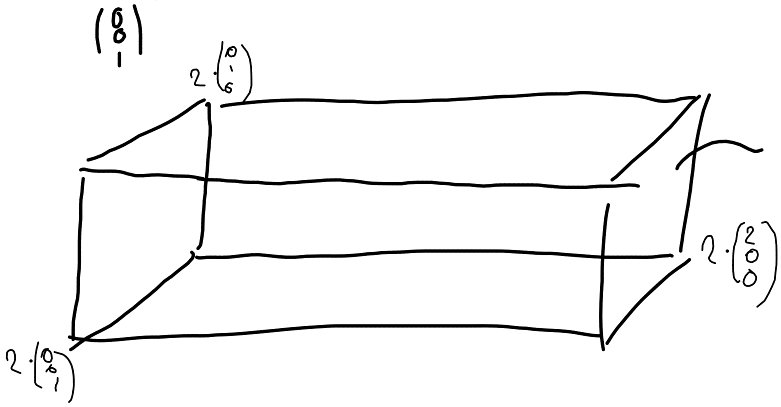
b) $\det(2B) =$

t.ex $B = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ $\quad 2B = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$

$\det(2B) = 4\cdot2\cdot2 = 16$



Volymen $= |\det B| = 2$



Volym $= |\det 2B| =$

$= 8\cdot\det B$

$= 2^3\cdot\det B$

Rep.) Trapetsmetoden.

$h = 0{,}125$

$$\int_0^{1/2} \underbrace{e^x}_{f(x)} \, dx$$

+ 0,125

x	0	0,125	0,25	0,375	0,5
$f(x) = e^x$	1	1,1331	1.284	1,455	1,6487

sum

$$I \approx T(h) = h \cdot \left(\frac{f(a) + f(b)}{2} + \sum f(x_i) \right)$$

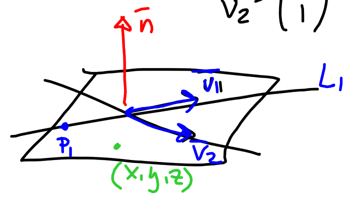
$$T(0{,}125) = 0{,}125 \cdot \left(\frac{1 + 1{,}6487}{2} + \text{sum} \right)$$

$$= \ldots$$

Rep. Staffans slide L20

$$L_1: \begin{cases} x = 4 + t \\ y = -2 - 2t \\ z = 5 + 2t \end{cases} \qquad L_2: \begin{cases} x = 2t \\ y = 2t \\ z = t \end{cases}$$

riktn. vektor: $\bar{v}_1 = \begin{pmatrix} 1 \\ -2 \\ +2 \end{pmatrix}$ $\qquad \bar{v}_2 = \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$



$$\bar{n} = \bar{v}_1 \times \bar{v}_2 = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} \times \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} \times \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} -2-4 \\ 2\cdot 2 - 1\cdot 1 \\ 1\cdot 2 - (-2)\cdot 2 \end{pmatrix} = \begin{pmatrix} -6 \\ 3 \\ 6 \end{pmatrix} = 3 \cdot \underbrace{\begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix}}_{\bar{n}_1}$$

$$\begin{matrix} 1 & 2 \\ -2 & 2 \end{matrix}$$

koll: $\bar{n} \perp \bar{v}_1 \iff \bar{n}_1 \cdot \bar{v}_1 = 0$

$$\begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} = -2\cdot 1 + 1\cdot(-2) + 2\cdot 2 = 0 \text{ Ja!}$$

$$\boxed{Ax + By + Cz + D = 0 \quad \text{där } \bar{n} = \begin{pmatrix} A \\ B \\ C \end{pmatrix}}$$

$\therefore \; -2x + y + 2z + D = 0$

ins. $P_1 = (4, -2, 5)$ ger D

$$-2\cdot 4 - 2 + 2\cdot 5 + D = 0$$
$$D = 0$$

$\therefore \; -2x + y + 2z = 0$

$$\underline{2x - y - 2z = 0}$$

alt. $P_0 = (x, y, z)$ är valfri punkt i planet om

$\overline{P_0P_1} \perp \bar{n} \iff$

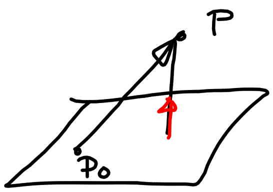
$$\begin{pmatrix} 4-x \\ -2-y \\ 5-z \end{pmatrix} \cdot \begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix} = 0 \qquad -2(4-x) + (-2-y) + 2(5-z) = 0$$

$$\cancel{-8} + 2x \cancel{-2} - y \cancel{+10} - 2z = 0$$

$$\underline{2x - y - 2z = 0}$$

Avst. $P = (3,-1,-1)$ till planet $2x-y-2z=0$

$$\bar{n} = \begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix}$$



$P_0 = (0,0,0) \in \Pi$ • Välj en valfri punkt i planet

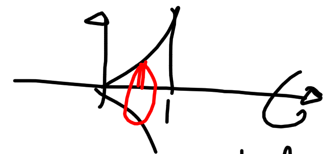
$$\overrightarrow{P_0P} = \begin{pmatrix} 3 \\ -1 \\ -1 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \\ -1 \end{pmatrix}$$ • Vektor mellan punkten

• stl. av ortogonal proj av $\overline{P_0P}$ på $\bar{n}$ ger sökt avst.

$$d = \left| \frac{\overline{P_0P} \cdot \bar{n}}{|\bar{n}|} \right| = \left| \frac{\begin{pmatrix} 3 \\ -1 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix}}{\sqrt{(-2)^2 + 1^2 + 2^2}} \right| = \left| \frac{3(-2) - 1 \cdot 1 - 2}{3} \right| = \left| \frac{-9}{3} \right| = 3 \text{ l.e.}$$

Inte Pythagorassats vid avstånd med plan och...

2) $y = \sqrt{x} \cdot e^{x} \qquad 0 \leq x \leq 1$



dx - integral kring x-axeln => skiv metoden

$$dV = \pi r^2 \cdot dx$$

$$= \pi (f(x))^2 \cdot dx = \pi \left(\sqrt{x} \cdot e^{x}\right)^2 dx \overset{(*)}{=} \pi x \cdot e^{2x}\, dx$$

der int

$$V = \int_0^1 \pi x \cdot e^{2x}\, dx = \left[\pi x \cdot \frac{e^{2x}}{2}\right]_0^1 - \underbrace{\int_0^1 \frac{e^{2x}}{2} \cdot \pi \, dx}_{= -\frac{\pi}{2}\left[\frac{e^{2x}}{2}\right]_0^1}$$

(*) $e^{x} \cdot e^{x} = e^{x+x} = e^{2x}$

jfr. $10^2 \cdot 10^3 = 10^{2+3}$

$$= \left[\pi x \cdot \frac{e^{2x}}{2} - \frac{\pi}{2} \cdot \frac{e^{2x}}{2}\right]_0^1 = \pi \cdot \frac{e^2}{2} - \frac{\pi}{2} \cdot \frac{e^2}{2} - \left(0 - \frac{\pi}{4}\right)$$

$$= \underbrace{\frac{\pi}{2} e^2 - \frac{\pi}{4} e^2 + \frac{\pi}{4}}_{} = \frac{2\pi e^2 - \pi e^2 + \pi}{4} = \underline{\underline{\frac{\pi e^2 + \pi}{4}}}$$

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & 0 \\ 3 & 6 & 4 \end{bmatrix}$$

A inverterbar?

$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$$A^{-1} = \frac{1}{\det A} \cdot \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

om $\det A \neq 0$.

$$\det A = \left|\begin{matrix} 1 & 2 & 1 \\ 0 & 2 & 0 \\ 3 & 6 & 4 \end{matrix}\right| \begin{matrix} 1 & 2 \\ 0 & 2 \\ 3 & 6 \end{matrix} = \begin{Bmatrix} \text{Sarrus} \\ \text{regel} \end{Bmatrix} =$$

$$= 1\cdot 2\cdot 4 + \cancel{2\cdot 0\cdot 3} + \cancel{1\cdot 0\cdot 6} - 3\cdot 2\cdot 1 - \cancel{6\cdot 0\cdot 1} - \cancel{4\cdot 0\cdot 2} = \underline{\underline{2}}$$

alt:

$$\det A = \begin{vmatrix} 1 & 2 & 1 \\ 0 & 2 & 0 \\ 3 & 6 & 4 \end{vmatrix} \begin{Bmatrix} \text{utv. efter} \\ \text{rad 2} \\ \text{glöm ej} \\ \text{tecken.} \end{Bmatrix} =$$

$$= -0 \cdot \begin{vmatrix} 2 & 1 \\ 6 & 4 \end{vmatrix} + 2 \cdot \begin{vmatrix} 1 & 1 \\ 3 & 4 \end{vmatrix} - 0 \cdot \begin{vmatrix} 1 & 2 \\ 3 & 6 \end{vmatrix}$$

$$0 \quad + 2(1\cdot 4 - 3\cdot 1) - 0 \quad = \underline{\underline{2}}$$

alt:

$$\det A = \begin{vmatrix} 1 & 2 & 1 \\ 0 & 2 & 0 \\ 3 & 6 & 4 \end{vmatrix} \xrightarrow{-3} = \begin{vmatrix} 1 & 2 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1\cdot 2\cdot 1 = 2$$

triangulär determinant

(mult. diagonal-elementen.)

alt

$$\det A = \begin{vmatrix} 1 & 2 & 1 \\ 0 & 2 & 0 \\ 3 & 6 & 4 \end{vmatrix} = 2 \cdot \begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 3 & 3 & 4 \end{vmatrix} = 2 \cdot \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{vmatrix} =$$

↑ brytat 2

-1

triang.

$$= 2\cdot(1\cdot 1\cdot 1) = 2$$

Svar: determinanten är 2, dvs ej noll och därför är matris A inverterbar.

forts. $A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & 0 \\ 3 & 6 & 4 \end{bmatrix}$ $A^{-1} = ?$

$(A \mid I) \sim \sim (I \mid A^{-1})$

reducerad trappstegsform (=rref)
= Gauss-Jordan.

(Se mini-räknaren, MATRIX 3×6 rref)

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 & 1 & 0 \\ 3 & 6 & 4 & 0 & 0 & 1 \end{array}\right) \cdot (-3) \sim \left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & -3 & 0 & 1 \end{array}\right) \cdot (-1) \cdot \left(\tfrac{1}{2}\right) \sim$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 2 & 0 & 4 & 0 & -1 \\ 0 & 1 & 0 & 0 & 0{,}5 & 0 \\ 0 & 0 & 1 & -3 & 0 & 1 \end{array}\right) (-2) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 4 & -1 & -1 \\ 0 & 1 & 0 & 0 & 0{,}5 & 0 \\ 0 & 0 & 1 & -3 & 0 & 1 \end{array}\right)$$

$$\underbrace{}_{I} \quad \underbrace{}_{A^{-1}}$$

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koll $A \cdot A^{-1} = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 2 & 0 \\ 3 & 6 & 4 \end{pmatrix} \begin{pmatrix} 4 & -1 & -1 \\ 0 & 0{,}5 & 0 \\ -3 & 0 & 1 \end{pmatrix} =$

$$= \begin{pmatrix} 4+0-3 & -1+1+0 & -1+0+1 \\ 0 & 1 & 0 \\ 12+0-12 & -3+3+0 & -3+0+4 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = I$$

Ja!

$\therefore \quad A^{-1} = \begin{pmatrix} 4 & -1 & -1 \\ 0 & 1/2 & 0 \\ -3 & 0 & 1 \end{pmatrix}$

4) $\int \frac{dx}{x(x-3)} = \int \frac{dx}{x^2-3x}$

a) kvadratkomp.

$\left(x-\frac{3}{2}\right)^2 - \frac{9}{4}$

minus, ej arctan.

b) faktorisering

$x^2-3x=0$

$x(x-3)=0$

Partialbråksuppdelning

Ansats

(*) $\frac{1}{x(x-3)} = \frac{A}{x} + \frac{B}{x-3}$

samma nämnare som i VL

$\frac{1}{x(x-3)} = \frac{A(x-3)+Bx}{x(x-3)}$

Jfr. $\frac{1}{x^2(x-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1} = \frac{Ax(x-1)+B(x-1)+Cx^2}{x^2(x-1)}$

p.g.a 'dubbelrot'.

Täljarna lika: $1 = Ax - 3A + Bx$

$$\left.\begin{aligned} x:&\quad 0 = A+B \\ x^0:&\quad 1 = -3A \end{aligned}\right\} \Rightarrow B = 1/3, \quad A = -1/3$$

Ins. i ansatsen: (*)

$\therefore \frac{1}{x(x-3)} = \frac{-1/3}{x} + \frac{1/3}{x-3}$

forts.

forts. $A = -1/3 \qquad B = 1/3$

$$\frac{1}{x(x-3)} = \frac{-1/3}{x} + \frac{1/3}{x-3}$$

$$\int \frac{-1/3}{x} + \frac{1/3}{x-3}\, dx = -\frac{1}{3}\ln|x| + \frac{1}{3}\ln|x-3| + C$$

$$= \frac{1}{3}\left(\ln|x-3| - \ln|x|\right) + C$$

$$= \frac{1}{3}\ln\left|\frac{x-3}{x}\right| + C$$

$$= \frac{1}{3}\ln\left|1 - \frac{3}{x}\right| + C$$

Ansats:

$$\frac{1}{x(x-3)} = \frac{A}{x} + \frac{B}{x-3}$$

$$\frac{1}{x(x-3)} = \frac{-\frac{1}{3}}{x} + \frac{1/3}{x-3}$$

Handpåläggning:

$A = ?$ håll för x i nämnaren på VL och sätt in hollstället dvs $x = 0$ i VL.

$B = ?$ Håll för $(x-3)$ i nämnaren i VL sätt $x = 3$ i VL.

ger $\frac{1}{3} = B$

$$\int x^3 \cdot \sin(x^2)\, dx$$

$$\int x^2 \cdot \underline{x} \cdot \sin(x^2)\, \underline{dx} = \begin{cases} \boxed{t = x^2} \\ dt = 2x\, dx \\ \frac{dt}{2} = \underline{x \cdot dx} \end{cases}$$

der int

$$= \int t \cdot \sin(t)\, \frac{dt}{2} = P.I \quad \ldots$$

$\rightarrow$ $\uparrow$

5) $\left(\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 3 & a & a & a \\ a & -a & 0 & 1 \end{array}\right)$

Oändligt många lösn. $\Rightarrow$ det $A = 0$.

$$\begin{vmatrix} 1 & 1 & -1 \\ 3 & a & a \\ a & -a & 0 \end{vmatrix} = \begin{vmatrix} 1 & 2 & -1 \\ 3 & a+3 & a \\ a & 0 & 0 \end{vmatrix} = a \cdot \begin{vmatrix} 2 & -1 \\ a+3 & a \end{vmatrix}$$

utv. rad 3

$= a(3a+3) = 3a(a+1) = 0$ $\quad \underline{a=0}$ eller $\underline{a=-1}$

$\underline{a=0}$:

$\left(\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right)$

omöjligt

lösning saknas då $a=0$.

$\underline{a=-1}$

$\left(\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 3 & -1 & -1 & -1 \\ -1 & 1 & 0 & 1 \end{array}\right)$ -3 1 $\sim \left(\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 0 & -4 & 2 & -4 \\ 0 & 2 & -1 & 2 \end{array}\right)$ 2 $\sim$

$\sim \left(\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 0 & 2 & -1 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right)$

OK

oändligt många lösn. då $a=-1$

Sätt t.ex $z=t$

ger $2y = 2+t$

$y = 1 + t/2$

$x = 1 - y + z$

$= 1 - (1 + t/2) + t = t/2$

<u>Svar:</u> $a=-1$ ger $\begin{cases} x = t/2 \\ y = 1 + t/2 \\ z = t \end{cases}$ $t \in \mathbb{R}$.