Hemuppg 1

1. $A = (-1,1)$

   $B = (7,5)$

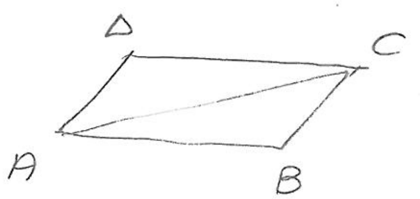
   $$\overrightarrow{AB} = \text{"}B-A\text{"} = \begin{pmatrix} 7 \\ 5 \end{pmatrix} - \begin{pmatrix} -1 \\ 1 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 8 \\ 4 \end{pmatrix}}}$$

2. $A = (1,0,5)$

   $B = (3,-2,6)$

   $$\overrightarrow{AB} = \text{"}B-A\text{"} = \begin{pmatrix} 3 \\ -2 \\ 6 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \\ 5 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}}}$$

   $$\|\overrightarrow{AB}\| = \sqrt{2^2 + (-2)^2 + 1^2} = \sqrt{4+4+1} = \underline{\underline{3}}$$

3. 

   $A = (1,0)$ $C = (4,2)$

   ~~B = (4,2)~~

   $B = (2,-1)$ $D = (x,y)$

   $$\overline{AB} = \text{"}B-A\text{"} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

   $$\overline{BC} = \text{"}C-B\text{"} = \begin{pmatrix} 4 \\ 2 \end{pmatrix} - \begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

   $$\overline{AB} = \overrightarrow{DC} = \text{"}C-D\text{"} = \begin{pmatrix} 4 \\ 2 \end{pmatrix} - \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4-x \\ 2-y \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \overline{AB}$$

   $$\begin{cases} 4-x = 1 \\ 2-y = -1 \end{cases} \Rightarrow \begin{cases} x = 3 \\ y = 3 \end{cases}$$

koll

$$\overline{AD} = \text{"}D-A\text{"} = \begin{pmatrix} x-1 \\ y-0 \end{pmatrix} = \overline{BC} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

$$\begin{cases} x-1 = 2 \\ y = 3 \end{cases} \Rightarrow \begin{cases} x = 3 \\ y = 3 \end{cases}$$

Svar: sista punkten har koord $(3,3)$

4b) $A = (0,-2)$
$B = (4,1)$

$\bar{s} = \bar{AB} = \text{"}B-A\text{"} = \begin{pmatrix}4\\1\end{pmatrix} - \begin{pmatrix}0\\-2\end{pmatrix} = \begin{pmatrix}4\\3\end{pmatrix}$

$\bar{F} = \begin{pmatrix}2\\2\end{pmatrix}$

projektion av F på $\bar{s}$. $\bar{F}_s = (\bar{F} \bullet \bar{e}_s)\bar{e}_s =$

$$\bar{F}_s = \frac{\bar{F} \bullet \bar{s}}{|\bar{s}|\,|\bar{s}|} \cdot \bar{s} = \frac{\begin{pmatrix}2\\2\end{pmatrix} \bullet \begin{pmatrix}4\\3\end{pmatrix}}{\sqrt{4^2+3^2}^{\,2}} \cdot \bar{s} = \frac{2\cdot 4 + 2\cdot 3}{\sqrt{5}\ \sqrt{5}} \cdot \begin{pmatrix}4\\3\end{pmatrix}$$

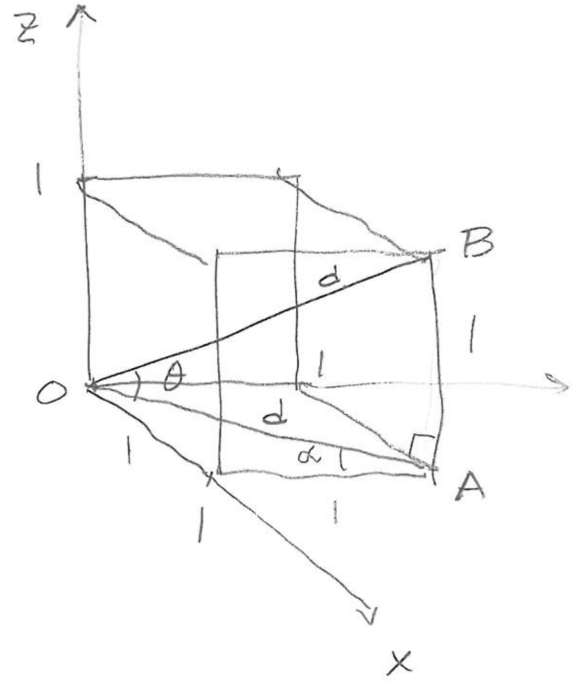
$$= \underline{\frac{14}{5} \cdot \begin{pmatrix}4\\3\end{pmatrix}} = \underline{\begin{pmatrix}56/5\\42/5\end{pmatrix}}$$

$$\begin{array}{r} 14 \\ 4\ 1 \\ \hline 56 \end{array}$$

5)
$$\begin{array}{ccc} \bar{a} & & \bar{b} \\ \begin{pmatrix}-2\\1\\-5\end{pmatrix} & \times & \begin{pmatrix}-3\\3\\4\end{pmatrix} \\ -2 & & -3 \end{array} = \begin{pmatrix}1\cdot 4 - (-5\cdot 3)\\(-5)(-3) - (-2)\cdot 4\\-2\cdot 3 - 1\cdot(-3)\end{pmatrix} = \begin{pmatrix}4+15\\15+8\\-6+3\end{pmatrix} = \underline{\begin{pmatrix}19\\23\\-3\end{pmatrix}}$$

Area av parallellogram: $\| \bar{a} \times \bar{b} \| = \sqrt{19^2 + 23^2 + (-3)^2}$ $\cancel{=}$ $\underline{\sqrt{899}}$

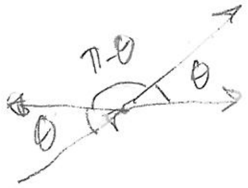
4)



d = rymddiagonal i kub

d_1 = diagonal i sida i kuben.

$\sin\alpha = \frac{1}{\sqrt{2}}$



Välj t.ex sätta in <u>kuben</u> i ett koordinatsystem. med sidlängd 1

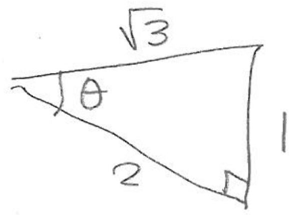
$A = (1,1,0)$ $\quad \vec{OA} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ $\quad |\vec{OA}| = \sqrt{1^2+1^2+0^2} = \sqrt{2}$

$B = (1,1,1)$ $\quad \vec{OB} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ $\quad |\vec{OB}| = \sqrt{1^2+1^2+1^2} = \sqrt{3}$

Vinkeln mellan två vektorer bestäms med skalärprodukt.

$$\vec{OA} \circ \vec{OB} = |\vec{OA}|\,|\vec{OB}| \cdot \cos\theta$$

$$\underbrace{\underbrace{\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \circ \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}}_{1\cdot 1 + 1\cdot 1 + 0\cdot 1}}_{=2} = \sqrt{2} \cdot \sqrt{3} \cdot \cos\theta$$



<u>alt</u>: $\theta = \arctan \frac{1}{\sqrt{2}}$

$\theta = \arcsin \frac{1}{\sqrt{3}}$

$$\Leftrightarrow \quad \cos\theta = \frac{2}{\sqrt{2}\cdot\sqrt{3}} = \frac{\sqrt{2}}{\sqrt{3}}$$

$$\theta = \arccos\left(\frac{\sqrt{2}}{\sqrt{3}}\right) \approx$$

6) Basbyte.

a) $\bar{u} = \frac{1}{3}\bar{a} + \frac{3}{2}\bar{b} = \begin{pmatrix} 1/3 \\ 3/2 \end{pmatrix}_{\bar{a}\bar{b}}$ avläsning ur fig.

b) $\begin{cases} \bar{c} = \frac{4}{3}\bar{a} + \frac{3}{2}\bar{b} & \text{I} \\ \bar{d} = \frac{4}{3}\bar{a} + \frac{5}{2}\bar{b} & \text{II} \end{cases}$

c) $\bar{u}$ i basen $\bar{c}\bar{d}$.

Lös ut $\bar{a}$ och $\bar{b}$ ur ekv. i uppg. b

I-II: $\bar{c} - \bar{d} = -\frac{2}{2}\bar{b} \iff \underline{\bar{b} = -\bar{c} + \bar{d}}$ ins i I

$\bar{c} = \frac{4}{3}\bar{a} + \frac{3}{2}(-\bar{c} + \bar{d})$ Lös ut $\bar{a}$

$\frac{4}{3}\bar{a} = \bar{c} - \frac{3}{2}(-\bar{c} + \bar{d}) = \frac{2\bar{c}}{2} + \frac{3\bar{c}}{2} - \frac{3}{2}\bar{d} = \frac{5\bar{c}}{2} - \frac{3}{2}\bar{d}$

$\underline{\bar{a}} = \frac{3}{4}\cdot\left(\frac{5}{2}\bar{c} - \frac{3}{2}\bar{d}\right) = \underline{\frac{15}{8}\bar{c} - \frac{9}{8}\bar{d}}$

$\therefore$ $\bar{u} = \frac{1}{3}\bar{a} + \frac{3}{2}\bar{b} = \frac{1}{3}\left(\frac{15}{8}\bar{c} - \frac{9}{8}\bar{d}\right) + \frac{3}{2}(-\bar{c} + \bar{d})$

$= \frac{5}{8}\bar{c} - \frac{3}{8}\bar{d} - \frac{3\cdot 4}{2\cdot 4}\bar{c} + \frac{3\cdot 4}{2\cdot 4}\bar{d} = \underline{-\frac{7}{8}\bar{c} + \frac{9}{8}\bar{d}}$

$= \underline{\begin{pmatrix} -7/8 \\ 9/8 \end{pmatrix}_{\bar{c}\bar{d}}}$