Hemuppg. 3:

(Uppgiften är avskriven fel från boken)

1. $\begin{cases} x + 2y - z = 3 \\ 2x + 2y + 2z = 4 \\ 2x + 5y + 2z = 2 \end{cases}$

totalmatris:

$$\left(\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 2 & 2 & 2 & 4 \\ 2 & 5 & 2 & 2 \end{array}\right) -2 \sim$$

$$\sim \left(\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & -2 & 4 & -2 \\ 0 & 1 & 4 & -4 \end{array}\right) \frac{1}{2} \sim \left(\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & -1 & 2 & -1 \\ 0 & 1 & 4 & -4 \end{array}\right) 1 \sim$$

$$\sim \left(\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & 6 & -5 \end{array}\right)$$

r.3 $6z = -5 \qquad z = -5/6$

r.2. $-y + 2 \cdot \left(-\frac{5}{6}\right) = -1$

$y = -\frac{\cancel{10}^{5}}{\cancel{6}_{3}} + \frac{3}{3} = \underline{-\frac{2}{3}}$

r.1 $x + 2 \cdot \left(-\frac{2}{3}\right) - \left(-\frac{5}{6}\right) = 3$

$x = \frac{4 \cdot 2}{3 \cdot 2} - \frac{5}{6} + \frac{3 \cdot 6}{6} = \frac{18 + 8 - 5}{6} = \frac{21}{6}$

$x = \frac{7}{2}$

Svar: $(x, y, z) = \left(\frac{7}{2}, -\frac{2}{3}, -\frac{5}{6}\right)$

enligt boken.

L.S.S.

$\begin{cases} x + 2y - z = 3 \\ 2x + 2y - z = 4 \\ 2x + 5y + 2z = 2 \end{cases}$

$$\left(\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 2 & 2 & -1 & 4 \\ 2 & 5 & 2 & 2 \end{array}\right) -2 \sim \left(\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & -2 & 1 & -2 \\ 0 & 1 & 4 & -4 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & 1 & 4 & -4 \\ 0 & -2 & 1 & -2 \end{array}\right) 2 \sim \left(\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & 1 & 4 & -4 \\ 0 & 0 & 9 & -10 \end{array}\right)$$

$z = -10/9$

$y = -4 - 4 \cdot \left(-\frac{10}{9}\right) =$

$= \frac{-36 + 40}{9} = \frac{4}{9}$

$x = 3 - 2 \cdot \frac{4}{9} + \left(-\frac{10}{9}\right)$

$= \frac{27 - 8 - 10}{9} = \frac{9}{9} = 1$

Svar: $(x, y, z) = \left(1, \frac{4}{9}, -\frac{10}{9}\right)$

Ekv.system & Gausselimination

2.

P5. (3)

$$\begin{cases} x_1 + x_2 = 3 \\ 2x_1 + 4x_2 = 4 \end{cases} \qquad \left(\begin{array}{cc|c} 1 & 1 & 3 \\ 2 & 4 & 4 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 1 & 3 \\ 0 & 2 & -2 \end{array}\right)$$

r2 $\quad 2x_2 = -2 \iff x_2 = -1$

r.1 $\quad x_1 + (-1) = 3$

$x_1 = 4$

Svar: $(x_1, x_2) = (4, -1)$

entydig lösning.

P5. (7)

$$\left(\begin{array}{ccc|c} 0 & 2 & -3 & -1 \\ 1 & 2 & -1 & 1 \\ 2 & 2 & 1 & 2 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 0 & 2 & -3 & -1 \\ 2 & 2 & 1 & 2 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 0 & 2 & -3 & -1 \\ 0 & -2 & 3 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 0 & 2 & -3 & -1 \\ 0 & 0 & 0 & -1 \end{array}\right)$$

orimligt.

$0x + 0y + 0z \neq -1$

lösning saknas till ekv.syst.

P5. (12)

$$\left(\begin{array}{cccc|c} 1 & -1 & 1 & -1 & 0 \\ -1 & 1 & 1 & 1 & 0 \\ 1 & 1 & -1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & -1 & 1 & -1 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 2 & -2 & 2 & 0 \\ 0 & 2 & 0 & 2 & 0 \end{array}\right) \begin{array}{c} \\ \cdot\frac{1}{2} \\ \cdot\frac{1}{2} \\ \cdot\frac{1}{2} \end{array}$$

$$\left(\begin{array}{cccc|c} 1 & -1 & 1 & -1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & -1 & 1 & -1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & -1 & 1 & -1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

sätt $x_4 = t$ (fri)

$x_3 = 0$

$x_2 + t = 0 \quad x_2 = -t$

r.1 $\quad x_1 - (-t) + 0 - t = 0 \quad x_1 = 0$

Svar: $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = t \cdot \begin{pmatrix} 0 \\ -1 \\ 0 \\ 1 \end{pmatrix}$

Determinanter   Hemuppg 3

1)

L 6.32)
$$\begin{vmatrix} -1 & 2 & 3 \\ 4 & -3 & 2 \\ 0 & 1 & 6 \end{vmatrix} \; 4 = \begin{vmatrix} -1 & 2 & 3 \\ 0 & 5 & 14 \\ 0 & 1 & 6 \end{vmatrix} = \text{Utv. kol 1}$$

$$= -1 \cdot \begin{vmatrix} 5 & 14 \\ 1 & 6 \end{vmatrix} = -(5 \cdot 6 - 1 \cdot 14) = \underline{-16}$$

alt Sarrus regel.
$$\begin{vmatrix} -1 & 2 & 3 \\ 4 & -3 & 2 \\ 0 & 1 & 6 \end{vmatrix} \begin{matrix} -1 & 2 \\ 4 & -3 \\ 0 & 1 \end{matrix} = 18 + 0 + 12 - (0) - (-2) - 48 = -\underline{16}$$

2) L. 5.8)
$$\begin{cases} x + 2y - z = 3 \\ 2x + 2y + az = 4 \\ 2x + ay + az = 2 \end{cases} \qquad \underset{A}{\begin{pmatrix} 1 & 2 & -1 \\ 2 & 2 & a \\ 2 & a & a \end{pmatrix}} \cdot \underset{X}{\begin{pmatrix} x \\ y \\ z \end{pmatrix}} = \underset{B}{\begin{pmatrix} 3 \\ 4 \\ 2 \end{pmatrix}}$$

Sök: ej unik lösning $a = ?$

$\Rightarrow$ $\underline{\det = 0}$

alt. Sarrus regel;
$$\begin{vmatrix} 1 & 2 & -1 \\ 2 & 2 & a \\ 2 & a & a \end{vmatrix} \begin{matrix} 1 & 2 \\ 2 & 2 \\ 2 & a \end{matrix} = 2a + 4a - 2a - (-4) - a^2 - 4a = 4 - a^2$$

$\det A = 0 \Leftrightarrow a^2 = 4$, $a = \pm 2$

$$\det A = \begin{vmatrix} 1 & 2 & -1 \\ 2 & 2 & a \\ 2 & a & a \end{vmatrix} \; -2 = \begin{vmatrix} 1 & 2 & -1 \\ 0 & -2 & a+2 \\ 0 & a-4 & a+2 \end{vmatrix} \; -1 =$$

$$= \begin{vmatrix} 1 & 2 & -1 \\ 0 & -2 & a+2 \\ 0 & a-2 & 0 \end{vmatrix} = \text{Utv. kol 1} = +1 \cdot \begin{vmatrix} -2 & a+2 \\ a-2 & 0 \end{vmatrix} = 0 - (a-2)(a+2)$$

$\det A = 0 \Leftrightarrow \underline{a = 2 \text{ eller } a = -2.}$

Om $a = \pm 2$ saknas unik lösning (då finns: • ingen eller • oändligt många lösn)

$\boxed{a=2}$
$$\left(\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 2 & 2 & 2 & 4 \\ 2 & 2 & 2 & 2 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & -2 & 4 & -2 \\ 0 & 0 & 0 & -2 \end{array}\right)$$

går ej

∴ lösning saknas då $a=2$

$\boxed{a=-2}$
$$\left(\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 2 & 2 & -2 & 4 \\ 2 & -2 & -2 & 2 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & -2 & 0 & -2 \\ 0 & -4 & 0 & -2 \end{array}\right) \; (-2) \; (-\tfrac{1}{2}) \left(\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 2 \end{array}\right)$$

ingen lösn.

∴ lösning saknas då $a=-2$

Matriser

L.2.1a) $A = \begin{pmatrix} 1 & -5 \\ 4 & 3 \end{pmatrix} \qquad B = \begin{pmatrix} 2 & -2 \\ 4 & 1 \end{pmatrix}$

$$A+B = \begin{pmatrix} 1 & -5 \\ 4 & 3 \end{pmatrix} + \begin{pmatrix} 2 & -2 \\ 4 & 1 \end{pmatrix} = \begin{pmatrix} 1+2 & -5-2 \\ 4+4 & 3+1 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 3 & -7 \\ 8 & 4 \end{pmatrix}}}$$

$$2A-3B = 2\cdot\begin{pmatrix} 1 & -5 \\ 4 & 3 \end{pmatrix} - 3\cdot\begin{pmatrix} 2 & -2 \\ 4 & 1 \end{pmatrix} =$$

$$= \begin{pmatrix} 2 & -10 \\ 8 & 6 \end{pmatrix} - \begin{pmatrix} 6 & -6 \\ 12 & 3 \end{pmatrix} = \begin{pmatrix} 2-6 & -10-(-6) \\ 8-12 & 6-3 \end{pmatrix} = \underline{\underline{\begin{pmatrix} -4 & -4 \\ -4 & 3 \end{pmatrix}}}$$

$$A-4\cdot I = \begin{pmatrix} 1 & -5 \\ 4 & 3 \end{pmatrix} - 4\cdot\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -5 \\ 4 & 3 \end{pmatrix} - \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix} = \underline{\underline{\begin{pmatrix} -3 & -5 \\ 4 & -1 \end{pmatrix}}}$$

L.2.4c) $A = \begin{pmatrix} 1 & -5 \\ 4 & 3 \end{pmatrix} \qquad B = \begin{pmatrix} 2 & -2 \\ 4 & 1 \end{pmatrix}$

$$(AB)^T = \left[\begin{pmatrix} 1 & -5 \\ 4 & 3 \end{pmatrix}\begin{pmatrix} 2 & -2 \\ 4 & 1 \end{pmatrix}\right]^T = \begin{bmatrix} 1\cdot 2+(-5)\cdot 4 & 1\cdot(-2)+(-5)\cdot 1 \\ 4\cdot 2+3\cdot 4 & 4\cdot(-2)+3\cdot 1 \end{bmatrix}^T$$

$$= \begin{bmatrix} -18 & -7 \\ 20 & -5 \end{bmatrix}^T = \underline{\underline{\begin{pmatrix} -18 & 20 \\ -7 & -5 \end{pmatrix}}}$$

$$A^TB^T = \begin{pmatrix} 1 & 4 \\ -5 & 3 \end{pmatrix}\begin{pmatrix} 2 & 4 \\ -2 & 1 \end{pmatrix} = \begin{pmatrix} 1\cdot 2+4\cdot(-2) & 1\cdot 4+4\cdot 1 \\ -5\cdot 2+3\cdot(-2) & -5\cdot 4+3\cdot 1 \end{pmatrix} = \underline{\underline{\begin{pmatrix} -6 & 8 \\ -16 & -17 \end{pmatrix}}}$$

$$B^TA^T = \begin{pmatrix} 2 & 4 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} 1 & 4 \\ -5 & 3 \end{pmatrix} = \begin{bmatrix} 2\cdot 1+4(-5) & 2\cdot 4+4\cdot 3 \\ -2\cdot 1+1\cdot(-5) & -2\cdot 4+1\cdot 3 \end{bmatrix} = \underline{\underline{\begin{pmatrix} -18 & 20 \\ -7 & -5 \end{pmatrix}}}$$

$$\therefore \boxed{(AB)^T = B^TA^T}$$

Matriser.

3.

L.2.7) $A = \begin{bmatrix} 1 & -5 \\ 4 & 3 \end{bmatrix}$ $\qquad A^{-1} = ?$

$$\boxed{A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \Rightarrow A^{-1} = \frac{1}{ad-cb} \cdot \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}}$$

$$A^{-1} = \frac{1}{1 \cdot 3 - 4(-5)} \cdot \begin{pmatrix} 3 & 5 \\ -4 & 1 \end{pmatrix} = \frac{1}{23} \begin{pmatrix} 3 & 5 \\ -4 & 1 \end{pmatrix}$$

$$= \begin{bmatrix} 3/23 & 5/23 \\ -4/23 & 1/23 \end{bmatrix}$$

Alt: $(A | I) \sim \sim (I | A^{-1})$

$$\left(\begin{array}{cc|cc} 1 & -5 & 1 & 0 \\ 4 & 3 & 0 & 1 \end{array}\right) (-4) \sim \left(\begin{array}{cc|cc} 1 & -5 & 1 & 0 \\ 0 & 23 & -4 & 1 \end{array}\right) 1/23 \sim \left(\begin{array}{cc|cc} 1 & -5 & 1 & 0 \\ 0 & 1 & -\frac{4}{23} & 1/23 \end{array}\right)$$

$$\left(\begin{array}{cc|cc} 1 & -5 & 1 & 0 \\ 0 & 1 & -4/23 & 1/23 \end{array}\right) (5) \sim \left(\begin{array}{cc|cc} 1 & 0 & 3/23 & 5/23 \\ 0 & 1 & -4/23 & 1/23 \end{array}\right)$$

$$\underbrace{\begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}}_{=I} \qquad \underbrace{\begin{matrix} 3/23 & 5/23 \\ -4/23 & 1/23 \end{matrix}}_{A^{-1}}$$

koll: $A \cdot A^{-1} = \begin{pmatrix} 1 & -5 \\ 4 & 3 \end{pmatrix} \frac{1}{23} \cdot \begin{pmatrix} 3 & 5 \\ -4 & 1 \end{pmatrix} = \frac{1}{23} \cdot \begin{pmatrix} 3+20 & 5-5 \\ 12-12 & 20+3 \end{pmatrix} =$

$$= \frac{1}{23} \cdot \begin{pmatrix} 23 & 0 \\ 0 & 23 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$

OK!

3)

P5, 21) $A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}$ invertera A

dvs bestäm A^{-1}.

$$(A|I) \sim (I|A^{-1})$$

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{array}\right) \overset{-1}{} \sim \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & -1 & 0 & 1 \end{array}\right) \overset{1}{} \sim$$

Gauss-Jordan

$$\sim \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 2 & -1 & 1 & 1 \end{array}\right) \cdot \frac{1}{2} \sim \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1/2 & 1/2 & 1/2 \end{array}\right) \overset{-1}{}$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1/2 & 1/2 & -1/2 \\ 0 & 0 & 1 & -1/2 & 1/2 & 1/2 \end{array}\right) \overset{-1}{} \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/2 & -1/2 & 1/2 \\ 0 & 1 & 0 & 1/2 & 1/2 & -1/2 \\ 0 & 0 & 1 & -1/2 & 1/2 & 1/2 \end{array}\right)$$

$$\underbrace{I}\qquad \underbrace{A^{-1}}$$

$$\underline{A^{-1} = \frac{1}{2} \cdot \begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ -1 & 1 & 1 \end{pmatrix}}$$

koll $A^{-1} \cdot A = \frac{1}{2}\begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ -1 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} =$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = I.$$

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4) $AXB + C = D$

$AXB = D - C$ mult med A^{-1} fr. vänster

$A^{-1}(AXB) = A^{-1}(D-C)$

$\underbrace{A^{-1}A}_{I}XB = A^{-1}(D-C)$ mult med B^{-1} fr. höger

$X\underbrace{B \cdot B^{-1}}_{I} = A^{-1}(D-C)B^{-1}$

$$\underline{X = A^{-1}(D-C)B^{-1}}$$

Egenvärden:

$$A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$$

$A\bar{x} = \lambda \bar{x}$ Bestäm λ

$A\bar{x} - \lambda\bar{x} = 0$

(*) $(A - \lambda \cdot I)\bar{x} = 0$ $\bar{x} = 0$ ointressant entydig trivial lösn.

andra lösningar om det=0

$\det(A - \lambda \cdot I) = 0$

$$A - \lambda I = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} - \lambda \cdot \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} = \begin{pmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{pmatrix}$$

$$\det(2 - \lambda I) = \begin{vmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{vmatrix} = (2-\lambda)^2 - 1 =$$

$$= 4 - 4\lambda + \lambda^2 - 1 =$$

$$= \lambda^2 - 4\lambda + 3$$

$\det = 0 \Leftrightarrow \lambda^2 - 4\lambda + 3 = 0$

$\lambda = 2 \pm \sqrt{4-3}$

$\lambda = 2 \pm 1 \begin{matrix} \nearrow 3 \\ \searrow 1 \end{matrix} \Big\}$ egenvärden!

$\underline{\lambda = 3}$: i (*) $\begin{pmatrix} 2-3 & 1 & | & 0 \\ 1 & 2-3 & | & 0 \end{pmatrix} \sim \begin{pmatrix} -1 & 1 & | & 0 \\ 1 & -1 & | & 0 \end{pmatrix} \sim \begin{pmatrix} -1 & 1 & | & 0 \\ 0 & 0 & | & 0 \end{pmatrix}$

sätt $y = t$ ger $x = t$ $\underline{\begin{pmatrix} x \\ y \end{pmatrix} = t \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix}}$ ← egenvektor till egenv. 3

$\underline{\lambda = 1}$: $\begin{pmatrix} 2-1 & 1 & | & 0 \\ 1 & 2-1 & | & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & | & 0 \\ 1 & 1 & | & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & | & 0 \\ 0 & 0 & | & 0 \end{pmatrix}$ sätt $y = t \Rightarrow x = -t$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -t \\ t \end{pmatrix} = t \cdot \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

egenvektor.

Svar: A har egenvektorn $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ med egenvärdet 3 och egenvektorn $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$ med egenvärdet 1