Diverse nyttigheter.

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1 Deriveringsregler.

$$\begin{aligned}
\frac{d}{dx}(f(x)+g(x)) &= f'(x)+g'(x) \\
\frac{d}{dx}(cf(x)) &= cf'(x) \\
\frac{d}{dx}(f(x)g(x)) &= f'(x)g(x)+f(x)g'(x) \\
\frac{d}{dx}(\frac{f(x)}{g(x)}) &= \frac{f'(x)g(x)-f(x)g'(x)}{(g(x))^2} \\
\frac{d}{dx}(f(g(x))) &= f'(g(x))g'(x)
\end{aligned}$$

2 Elementära derivator.

f(x)	f'(x)	
$\frac{1}{x}$	$-\frac{1}{x^2}$	
$\sqrt{x}$	$\frac{1}{2\sqrt{x}}$	
x^r	rx^{r-1}	
e^x	e^x	
a^x	$a^x \ln a$	$(a > 0)$
$\ln\lvert x\rvert$	$\frac{1}{x}$	
$\cos x$	$-\sin x$	
$\sin x$	$\cos x$	
$\tan x$	$\frac{1}{\cos^2 x}=1+\tan^2 x$	$(x \neq \frac{\pi}{2} + n\pi)$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$	$(-1 < x < 1)$
$\arctan x$	$\frac{1}{1+x^2}$	

3 Trigonometriska identiteteter.

$$\sin^2 x + \cos^2 x = 1$$

$$\frac{1}{\cos^2 x} = 1 + \tan^2 x$$

$$\sin(-x) = -\sin x$$

$$\cos(-x) = \cos x$$

$$\sin(x + \pi) = -\sin x$$

$$\cos(x + \pi) = -\cos x$$

$$\sin(\frac{\pi}{2} - x) = \cos x$$

$$\cos(\frac{\pi}{2} - x) = \sin x$$

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$$

$$\sin 2x = 2 \sin x \cos x$$

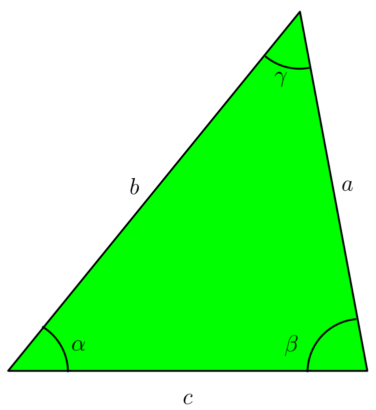
$$\cos 2x = 2\cos^2 x - 1 = 1 - 2\sin^2 x = \cos^2 x - \sin^2 x$$

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

$$
\begin{aligned}
\sin x + \sin y &= 2 \sin \frac{x+y}{2} \cos \frac{x-y}{2} \\
\sin x - \sin y &= 2 \cos \frac{x+y}{2} \sin \frac{x-y}{2} \\
\cos x + \cos y &= 2 \cos \frac{x+y}{2} \cos \frac{x-y}{2} \\
\cos x - \cos y &= -2 \sin \frac{x+y}{2} \sin \frac{x-y}{2}
\end{aligned}
$$

4 Triangeltrigonometri.



$$
\begin{aligned}
\frac{\sin \alpha}{a} &= \frac{\sin \beta}{b} = \frac{\sin \gamma}{c} \\
a^2 &= b^2 + c^2 - 2bc \cos \alpha \\
A &= \frac{bc \sin \alpha}{2}
\end{aligned}
$$

5 Vektoridentiteter.

$$\mathbf{u}\cdot\mathbf{v} = |\mathbf{u}|\,|\mathbf{v}|\cos\phi,\ \phi \text{ vinkel mellan } \mathbf{u} \text{ och } \mathbf{v}$$

Om

$$\mathbf{u} = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}, \mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} \text{ (\textit{ON-bas}), så}$$

$$\mathbf{u}\cdot\mathbf{v} = u_1v_1 + u_2v_2 + u_3v_3$$

$$\mathbf{u}\times\mathbf{v} = \begin{pmatrix} u_2v_3 - u_3v_2 \\ u_3v_1 - u_1v_3 \\ u_1v_2 - u_2v_1 \end{pmatrix}$$

$$|\mathbf{u}| = \sqrt{\mathbf{u}\cdot\mathbf{u}} = \sqrt{u_1^2 + u_2^2 + u_3^2}$$

6 Integrationsregler.

$$\int (Af(x) + Bg(x))\,dx = A\int f(x)\,dx + B\int g(x)\,dx$$

$$\int f(g(x))g'(x)\,dx = \int f(t)\,dt,\ [t = g(x)]$$

$$\int u(x)v(x)\,dx = U(x)v(x) - \int U(x)v'(x)\,dx \qquad \text{(U primitiv funktion till u)}$$

7 Elementära integraler.

$$\int x^a\,dx = \frac{x^{a+1}}{a+1} + C \text{ a} \neq -1$$

$$\int \frac{1}{x}\,dx = \ln|x| + C$$

$$\int e^x\,dx = e^x + C$$

$$\int a^x\,dx = \frac{a^x}{\ln\ a} + C$$

$$\int \sin\ x\,dx = -\cos\ x + C$$

$$\int \cos\ x\,dx = \sin\ x + C$$

$$\int \tan\ x\,dx = -\ln|\cos\ x| + C$$

$$\int \frac{1}{\cos^2 x}\,dx = \tan\ x + C$$

$$\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = arc\sin\ \frac{x}{a} + C$$

$$\int \frac{1}{a^2 + x^2}\,dx = \frac{1}{a}\ \text{arctan}\ \frac{x}{a} + C$$

$$\int \frac{u'(x)}{u(x)}\,dx = \ln|u(x)| + C$$