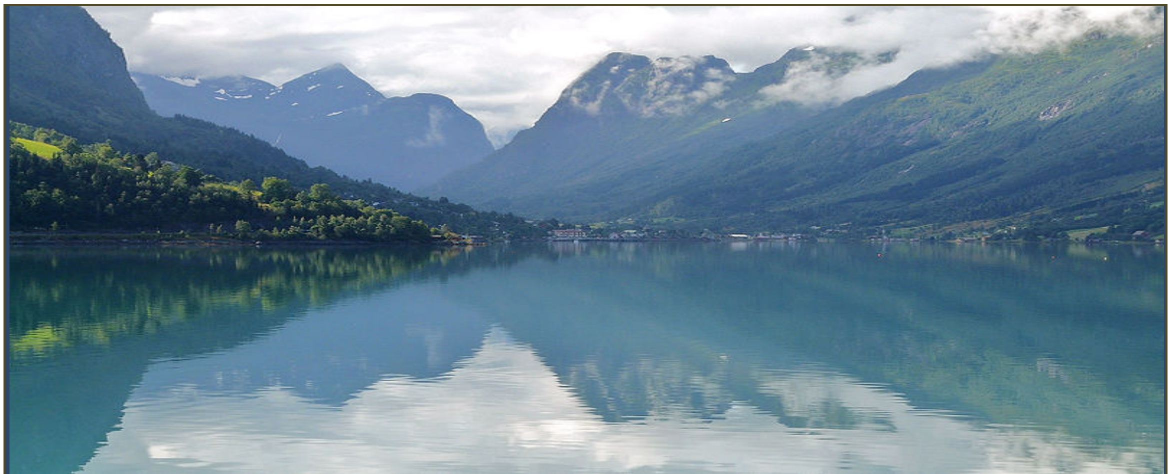

Conference on *Nonlinear Mathematical Physics: Twenty Years of JNMP*, June 4 - 14, 2013

Titles and Abstracts

of the

Talks and Posters

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Singular symmetric flat 3-webs and Frobenius 3-folds

Abstract: The theory of Frobenius manifolds, having its origin in theoretical physics, has deep interrelations with apparently very different areas of mathematics: Witten-Gromov invariants and quantum cohomology, deformation of flat connections, integrable systems, singularity theory etc. We discuss a new aspect of this fruitful and fast developing theory: its relations with the classical chapter of differential geometry, namely the web theory. Using the structure of a given semi-simple Frobenius 3-fold, we construct a 3-web in the plane. This web enjoys the following properties:
1) it is flat,
2) it admits at least one-dimensional symmetry algebra and
3) its Chern connection remains holomorphic in singular points, where at least 2 web directions coincide.
We present a classification of singularities of 3- webs with such properties and show that any such web is obtained by the presented construction. We give also a geometrical interpretation of the associativity equation, describing the corresponding Frobenius 3-fold.

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Symmetries and invariant solutions of mathematical models of plastic flow during linear friction welding

Abstract: In this research three models of plastic flow are suggested and justified. These models are investigated using methods of group analysis and a comparison of them is given.

Depending on the type of relation between stress deviator tensor's intensity and strain rate deviator tensor's intensity, three different mathematical models are suggested for the description of plastic flow of metal in a contact zone during the linear friction welding (LFW) process. General principles of theory of plasticity on par with assumptions based on properties of LFW like thinness of plastic layer are used for the construction of these models. The suggested models are solved by using methods of group analysis, in particular by finding invariant solutions corresponding to the admitted operators of the equations.

For models of perfectly plastic flow and viscoplastic flow asymptotics of solutions close to the edges of sliding are obtained. This allows us to estimate the heat release on the edges and the rate of thermal sludge. For the model of viscous flow, solution in the whole investigated area is obtained.

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Scalar and vector nonlinear Schrödinger systems with non-zero boundary conditions

Abstract: Despite having been intensely investigated over the last forty years, nonlinear Schroedinger (NLS) systems still offer a number of surprises. This talk with discuss recent results on both focusing and defocusing, both scalar and vector, NLS equations with non-zero boundary conditions at infinity. A number of explicit soliton solutions will be discussed, as well as spectral problems for special classes of initial conditions.

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Integrability criteria and wave-like solutions for BSR systems

(Joint work with N. Euler, M. Euler, Lulea University, Sweden, A. Prykarpatski, AGH University, Poland and H. Wu*, NJIT, USA)

Abstract: The integrability and existence and stability of wave-like solutions of higher-dimensional BSR systems is investigated in depth. These systems are of the form

$$\frac{\partial u_i}{\partial t} + u_1 \frac{\partial u_i}{\partial x_1} + \cdots + u_n \frac{\partial u_i}{\partial x_n} = F_i \left(t, x_1, \ldots, x_n, u_1, \ldots, u_n, \partial_{x_1} u_1, \ldots, \partial_{x_n} u_1, \ldots, \partial_{x_1} u_n, \ldots, \partial_{x_n} u_n \right),$$

$1 \leq i \leq n$, which can be represented far more concisely as

$$\boldsymbol{u}_t + < \boldsymbol{u}, \nabla > \boldsymbol{u} = \boldsymbol{F} \left(t, \boldsymbol{x}, \boldsymbol{u}, \boldsymbol{u}_{\boldsymbol{x}} \right),$$

where t represents the evolution parameter, time, $\boldsymbol{x} := (x_1, \ldots, x_n) \in \mathbb{R}^n$, $\boldsymbol{u} := (u_1, \ldots, u_n) \in \mathbb{R}^n$, $\nabla := \left(\partial / \partial x_j : j = \overline{1, n} \right)$, $\boldsymbol{u}_{\boldsymbol{x}} := \left(\partial u_i / \partial x_j : i, j = \overline{1, n} \right)$ and $\boldsymbol{F}$ is a C^∞ n-vector valued function of all its variables. BSR systems were first developed as models of particulate flows in which case the F_i were local integrals, so any results concerning integrability and wave-like solutions have interesting applications to a variety of physical phenomena. Recent results that have been obtained for the BSR model in one space dimension are extended to higher dimensions using various combinations of old and new symplectic and differential-algebraic techniques. For example, conditions on $\boldsymbol{F}$ guaranteeing complete integrability are obtained and corresponding infinite hierarchies of independent mutually commuting conservation laws (for appropriate Poisson brackets) are constructed, thereby identifying new classes of physically meaningful higher dimensional integrable systems. Moreover, criteria for the existence of traveling-wave and decaying traveling-wave solutions are derived, and the stability of these types of solutions is analyzed for a number of cases. Finally, some applications of the theoretical results to physical flow phenomena are briefly described, where, for example, physical interpretations of certain invariants are discussed.

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Canonical quantization in curvilinear coordinates

Abstract: We present a consistent way of quantizing a classical Hamiltonian system in arbitrary canonical coordinates. We pay a particular attention to the case when a phase space is the cotangent bundle to a configuration space being in turn an Euclidean space. The theory is developed within a context of phase space quantum mechanics, which seems to be the most natural approach to quantization, allowing a straightforward development of transformations of coordinates in a complete analogy with classical theory. Then the passage to Hilbert space approach of quantum mechanics is presented.

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A vacuum field theory approach to the classical Lorentz force problem

(Joint work with D. Blackmore*, NJIT, USA and A. Prykarpatski, AGH University, Krakow, Poland)

Abstract: We review modern classical electrodynamics charged particle problems and analyze the related fundamental least action principles characterizing their electrodynamical vacuum field theory interaction structure. In addition, we study electrodynamical models of the charged point particle dynamics based on a Maxwell type vacuum field medium description and some new field theory concepts that are developed. We also revisit and reanalyze the classical Lorentz force expression in arbitrary inertial reference frames and present new interpretations of some classical special relativity theory relations. The well-known Feynman approach to Maxwell's electromagnetic equations and the Lorentz force derivation are also analyzed in detail in the context of vacuum field theory. The old classical electromagnetic electron mass problem and related charged particle radiation force are discussed. Using the Gelfand-Vilenkin representation theory of infinite-dimensional groups and the Goldin-Menikov-Sharp theory for generating Bogolubov type functionals, the problem of constructing Fock type representations of current algebras and their application to studying classical dynamical systems in Hilbert spaces is reviewed. An application of a suitable current algebra representation for describing the non-relativistic Aharonov-Bohm paradox is also presented.

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On Bianchi permutability of Bäcklund transformations for asymmetric quad-equations

Abstract: In the last years the concept of multidimensional consistency has reached undoubtedly an outstanding importance in the theory of discrete integrable systems. Recently, we gave a classification of all 3D consistent systems of quad-equations with the tetrahedron property, where several novel asymmetric systems have been found. This classification comes along with a slightly generalized definition of multidimensional consistency which was necessary to enlarge the class of integrable 2D systems of quad-equations and allows for derivation of Bäcklund transformations and zero-curvature representations for all 2D systems of quad-equations participating in the above mentioned classification. This is a justification for using 3D consistency as a synonym for integrability. In the present talk, we will report on the proof of the Bianchi permutability (existence of superposition principle) of these (slightly generelized) Bäcklund transformations, i.e., we show that these are of the same value as the Bäcklund transformations which can be derived from the symmetric systems of quad-equations. We perform this proof by using 4D consistent systems of quad-equations, the structural insights through biquadratics patterns and the consideration of super-consistent eight-tuples of quad-equations on decorated cubes.

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Nonlinear self-adjointness and conservation laws for a Kuramoto-Sivashinsky equation with dispersive effects

Abstract: Blă and Niesen in [1] proposed a new procedure for finding nonclassical symmetries. The authors introduced a new algorithm for computing the determining equations of the nonclassical symmetries of a given PDEs system. In [3] we extend the procedure described in [1] to a different case. We offered some examples of how our method works for these equations. In [2] we gave a Lie group classification for a a Kuramoto-Sivashinsky equation with dispersive effects. By using this new algorithm [3] we derived nonclassical symmetries for this equation that yield to new solutions. Ibragimov [8, 9, 6] proved a general theorem on conservation laws. In order to apply this theorem is necessary to know the symmetries of the equation. We determine the subclasses of equations of the Kuramoto-Sivashinsky equation with dispersive effects which are nonlinear self-adjoint. From the general theorem on conservation laws proved by Ibragimov we obtain conservation laws for this equation.

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Geometrical Langlands ramifications and differential operators classification by coherent *D*-modules in field theory

Abstract: Spaces of equivalence modulo a relation of congruence are constructed on field solutions to establish a theory of the universe that includes the theory *QFT (Quantum Field theory)*, the *SUSY (Super-symmetry theory)* and *HST (heterotic string theory)* using the sheaves correspondence of differential operators of the field equations and sheaves of coherent D-Modules [1]. The above mentioned correspondence uses a Zuckerman functor that is a factor of the universal functor of derived sheaves of Harish-Chandra to the Langlands geometrical program in mirror symmetry [2]. The obtained development includes complexes of D-modules of infinite dimension, generalizing for this way, the $BRST$-cohomology in this context. With it, the class of the integrable systems is extended in mathematical physics and the possibility of obtaining a general theory of integral transforms for the space-time (*integral operator cohomology* [4]), and with it the measurement of many of their observables [3]. Also tends a bridge to complete a classification of the differential operators for the different field equations using on the base of *Verma modules* that are classification spaces of $SO(1,\ n+1)$, where elements of the Lie algebra $sl(1,\ n+1)$, are differential operators, of the equations in mathematical physics [1].

The cosmological problem that exists is to reduce the number of field equations that are resoluble under the same gauge field *(Verma modules)* and to extend the gauge solutions to other fields using the topological groups symmetries that define their interactions. This extension can be given by a global Langlands correspondence between the Hecke sheaves category on an adequate moduli stack and the holomorphic ${}^{L}G$-bundles category with a special connection *(Deligne connection)*. The corresponding D-modules may be viewed as sheaves of conformal blocks (or co-invariants) (images under a version of the Penrose transform [1]) naturally arising in the framework of conformal field theory.

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Magnetic fields on 2D and 3D spheres

Abstract: Let (M^m, g) be a Riemannian manifold and denote by ∇ the Levi-Civita connection. A *magnetic field* on (M^m, g) is a closed 2-form F on M^m. The *Lorentz force* of F is the skew-symmetric tensor field Φ defined by $g(\Phi(X), Y) = F(X, Y)$ for any vector fields X, Y on M^m. A smooth parametrized curve $\gamma(t)$ in M^m is a *magnetic curve* of F if it satisfies the Lorentz force equation $\nabla_{\gamma'}\gamma' = \Phi(\gamma')$, which is a nonlinear second order system of differential equations for the coordinates of $\gamma(t)$. For the trivial magnetic field $(F = 0)$ we see that $\Phi = 0$ and the magnetic curves are the geodesics of (M^m, g). As it is well-known, they represent charged particles in free fall travelling under the influence of only gravity.

In this talk we give a geometrical approach to this problem that allows us to have a good picture of how the solution curves behave. In the case $m = 2$ we see that the Lorentz force of the magnetic field completely determines the curvature κ of the magnetics curves. In particular, normal magnetic curves in the sphere $\S^2$ corresponding to an uniform magnetic field F are circles. In the three-dimensional case there exists a bijective correspondence between divergence-free vector fields and magnetic fields. As a consequence, we show that normal magnetic curves of a Killing field in the sphere S^3 are helices with the given Killing field as its axis.

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Isochronous systems are not rare

Abstract: A survey will be given of *isochronous* systems, i. e. systems that oscillate with a *fixed* period (for largely arbitrary initial data). It will be shown how to manufacture many such models—including "realistic" many-body problems whose time evolution is characterized by Newtonian equations of motion. In particular a fairly general technique will be described to modify fairly general models describing a time evolution so that the modified systems are *isochronous* (with period T) yet mimic closely (or even exactly) the behavior of the unmodified system for a time interval $Ttilde$ much smaller (or just smaller) than T.

As a particularly remarkable example (joint work with **F. Leyvraz**), it will be shown how—given the (autonomous) Hamiltonian H describing the most general (standard) nonrelativistic many-body problem (*arbitrary* number N of particles, *arbitrary* masses, *arbitrary* dimensions of ambient space, forces depending *arbitrarily* from all the particle coordinates–with the only restriction that the system be overall

translation-invariant, i. e. no external forces)—it is possible to construct another (also autonomous) Hamiltonian $Htilde$ (in fact, an infinity of such Hamiltonians) featuring the same dynamical variables and parameters as H and in addition two *arbitrary* positive parameters T and $Ttilde$ with $T > Ttilde$, and having the following two properties. (i) The new Hamiltonian $Htilde$ yields, over the (*arbitrarily* long!) time interval $Ttilde$, a dynamical evolution *identical* to that yielded by H. (ii) The Hamiltonian $Htilde$ is *isochronous*: all its solutions (for *arbitrary* initial data) are *completely periodic* with period T.

This finding raises (interesting?) questions about the difference among *nonintegrable* and *integrable* dynamics (*all isochronous systems are integrable*, indeed *more than superintegrable*), about the definition of *chaotic* behavior (including the apparent need to invent some such notion for a *finite* time interval), about the validity (say, for $N \approx 10^{23}$) of statistical mechanics and of the second principle of thermodynamics, about cosmology (say, for $N \approx 10^{85}$). It also demonstrates the impossibility to ascertain which dynamical theory is the correct one, out of an infinity of different theories predicting the same (*exactly the same*) evolution over an *arbitrarily long* time interval, but being *qualitatively different* (*isochronous* versus *chaotic*, *integrable* versus *nonintegrable*).

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Evolution problems in materials with memory: a singular kernel viscoelasticity problem

Abstract: The wider class of *smart* materials is more and more widely used in different applications; among them, here, materials with memory are considered. The characteristic peculiar physical property is that their behaviour depends on time not only through the present time but also through their past history. Under the analytical point of view, this corresponds to model problems represented by integro-differential equations exibiting a non local kernel. This is the case of linear viscoelasticity where the kernel represents the relaxation modulus. In dealing with classical viscoelastic materials the kernel is a regular function. Results obtained in [1,2] the case of magneto-viscoelastic materials are recalled. Then, aiming to study new materials which are characterized by a relaxation modulus which is unbounded at the origin, a singular integro-differential problem is considered [3]. Notably, the choice of suitable expressions of the free energy turns out to be crucial to prove existence and uniqueness results.

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Minimal rational surfaces and discrete Nahm equations

Abstract: Some birational dynamical systems may have "useless" singularities and, by blowing them up, the corresponding rational surface is not minimal. We minimize such rational surfaces by blowing down structures and apply this procedure to mappings with higher order invariants like discrete Nahm equations.

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On the blow-up problem for the Euler equations

Abstract: In this talk we present some recent progresses on the finite time blow-up problem for the 3D Euler equations. We first consider the scenarios of the self-similar blow-up and its generalizations. We also discuss recent developments on the exclusion of the asymptotically self-similar blow-up, and its localized version. We also present some results on the exclusion of the discretely self-similar blow-ups. For the axisymmetric Euler equations we show that the arbitrary long time preservation of the radial local minimum property of the pressure on the axis is not consistent with the global regularity. We also present Liouville type theorems for the steady equations of the Euler equations with suitable perturbations. Finally we discuss the refinements of the classical energy equality in terms of the level sets of the Bernoulli function.

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Zero curvature representation, bi-Hamiltonian structure, and an integrable hierarchy for the Zakharov-Ito system

Abstract: In the present paper, we present an integrable hierarchy for the Zakharov-Ito system. We first construct the Lenard recursion sequence and zero curvature representation for the Zakharov-Ito system, following Cao s method as generalized by other authors. We then construct the bi- Hamiltonian structures employing variational trace identities. From this, we are in a position to construct an integrable hierarchy of equations from the Zakharov-Ito system, and we obtain the hereditary operator for constructing this hierarchy. Finally, we demonstrate that the obtained hierarchy is indeed Liouville integrable.

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Building integrable NLPDE hierarchies with temporally and spatially varying coefficients (poster)

Abstract: In the present talk, we present two techniques, based on generalized Lax Pairs and similarity transformation methods, to derive generalizations of various integrable (in the Lax sense) NLPDE hierarchies. As illustrative examples, we consider a generalized NLS equation, as well as a very recent non-local and PT-symmetric NLS equation. We demonstrate that the techniques yield integrable equations with both time- AND space-dependent coefficients, and are thus more general than cases with only temporally varying coefficients treated earlier via the Painlevé Test and the use of Bell polynomials.

The methods have been applied to generalize some well-known integrable NLPDE hierarchies, and are currently being applied to others as well.

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Structure-preserving dynamic equations on time scales

Abstract: A time scale $\mathbb{T}$ is defined as a closed subset of $\mathbb{R}$. Time scales were introduced to unify discrete and continuous calculus, including difference and differential equations (*dynamic equations on time scales*) and, to some extent, q-calculus.

In the standard calculus of time scales two derivatives are commonly used: delta derivative and nabla derivative. In the discrete case they correspond to the forward and backward Euler numerical scheme, respectively. We point out that using other numerical schemes we can get dynamic equations on time scales with qualitative properties resembling the corresponding continuum system. For instance, using the trapezoidal rule, we get new exponential and trigonometric functions on time scales. The exponential function maps the imaginary axis onto the unit circle and Pythagorean trigonometric identity is satisfied. We present also structure-preserving time scale analogues of:
– Discrete gradient schemes for Hamiltonian systems,
– Dynamic equations on Lie groups,
– Sine-Gordon equation, solitons and pseudospherical surfaces.

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Intermediate Toda and Lotka-Volterra systems

Abstract: We define some new, evidently integrable Toda and Lotka-Volterra type systems. More specifically, we construct a large family of Hamiltonian systems which interpolate between the classical Kostant-Toda lattice and the full Kostant-Toda lattice and we discuss their integrability. There is one such system for every nilpotent ideal $\mathcal{I}$ in a Borel subalgebra $\mathfrak{b}_+$ of an arbitrary simple Lie algebra $\mathfrak{g}$. The classical Kostant-Toda lattice corresponds to the case of $\mathcal{I} = [\mathfrak{n}_+, \mathfrak{n}_+]$, where $\mathfrak{n}_+$ is the unipotent ideal of $\mathfrak{b}_+$, while the full Kostant-Toda lattice corresponds to $\mathcal{I} = \{0\}$.

In the second part of the talk, we construct in a similar fashion an infinite family of integrable Hamiltonian systems which are generalizations of the KM system (also known as the Voterra lattice). The Hamiltonian vector field is homogeneous cubic but in a number of cases a simple change of variables transforms such a system to a quadratic Lotka-Volterra system. We present in detail all such systems in dimensions 3, 4 and 5 and we also give some examples from higher dimensions. This construction generalizes easily to each complex simple Lie algebra. In particular we study a number of Lotka-Volterra systems associated with a simple Lie algebra of type D_4.

The first part of this talk (Toda sytems) is in collaboration with H. Sabourin and P. Vanhaecke. The second part (Lotka-Volterra sytems) is in collaboration with S. Charalambides and C. Evripidou.

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Numerical normal forms for codimension 2 bifurcations of limit cycles

Abstract: In this talk we concentrate on the computation of the normal form coefficients for codim 2 bifurcations of limit cycles. These formulas are important since they allow one to distinguish between the complicated bifurcation scenarios which can happen near these codimension 2 bifurcations of limit cycles.

Typically, the dynamics of a system show bifurcations upon variation of a parameter. Numerical continuation software such as AUTO or MatCont may be used to track bifurcations from a stable equilibrium to a periodic oscillation by a Hopf bifurcation and even the appearance of (un)stable invariant tori with multi-frequency oscillations by a secondary Hopf, or Neimark-Sacker bifurcation. Bifurcations of these invariant tori into other tori or chaos, however, are out of reach of the standard numerical analysis.

One possibility to study bifurcations of stable tori is to compute Lyapunov exponents. The dimension of the torus for a given parameter value then equals the number of exponents equal to zero. Varying one parameter one can observe that exponents become zero and this indicates a bifurcation. The exact nature of the bifurcation is however obscured from this analysis and should be elucidated with additional means. Yet, in many cases, bifurcations of tori first emerge from codim 2 bifurcations of limit cycles.

There is a straightforward approach to obtain the critical normal forms of the codim 2 bifurcations of the limit cycle. In the Poincaré map, the limit cycle corresponds to a fixed point and one can use techniques developed for maps to obtain the critical normal form. However, these techniques require accurate higher-order derivatives of the Poincaré map which are hardly available numerically. Methods which handle this last problem have two drawbacks that make them less (time) efficient. First, these are shooting methods that are slower when the system is very sensitive to perturbations and they are difficult to use in a continuation context. Second, the full Poincaré map is computed while only certain expressions are needed. Moreover, the number of derivatives of the Poincaré map to be computed is $O(n^k)$ if n is the dimension of the system and derivatives up to order k are needed (in several cases $k = 5$). Even for moderate values of n this involves a great deal of work which is in fact unnecessary.

Indeed, our method completely avoids the numerical computation of Poincaré maps and their derivatives. Instead, the computation of the normal form coeffcients is reduced to solving certain linear boundary value problems and evaluating certain integrals. In the implementation of our method in MatCont, we make use of robust numerical boundary value algorithms which are based on orthogonal collocation with piecewise polynomial functions.

We derive explicit formulas to compute the critical normal form coefficients. Note that the formulas are independent of the dimension of the phase space. The implementation in MatCont is freely available. We apply our techniques to the study of a laser model, a model from population biology, and one for

mechanical vibrations to predict the bifurcation diagram near a codim 2 point in these examples. We are in particular interested in the existence of stable invariant tori and chaos. We corroborate the predictions using Lyapunov exponents. In fact, we argue that the classification from the critical normal form guides the correct interpretation of the Lyapunov exponents.

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Canonical quantization in Riemann spaces

Abstract: We propose a quantization scheme of a classical Hamiltonian system which configuration space is a general Riemannian manifold. The theory is based on our previously developed approach of quantization in Euclidean spaces in arbitrary canonical coordinates. Examples of quantum systems on Riemann spaces are presented and discussed.

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Three approaches to conservation laws for difference ODEs and PDEs

Abstract: The talk is a short review of different approaches to conservation properties of finite difference equations possessing Lie point symmetries. Firstly, within Lagrangian formalism the invariant variational problems for difference equations are considered. The Noether's type constructions for difference fuctionals, meshes and difference equations are reviewed.

In the second approach we consider the relation between symmetries and first integrals for canonical Hamiltonian equations. Based on a newly established identity, this approach provides a simple and clear way to construct first integrals with the help of symmetries of a Hamiltonian.

The third way is based on the wellknown identity which links together given linear equation (the determining equation for a symmetry in our case), adjoint equation and a divergence of some function. This approach seems to be useful in all cases when the first two failed. The peculiarity of difference case is that we have to rewrite the determining equation in the non-evolutionary form and then to generalize the identity for a difference equations. The reviewing approaches are illustrated by a number of examples.

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Flag structures on smooth manifolds: equivalence and applications

Abstract: Flag structure on a smooth manifold M defined as a one-dimensional subbundle of the projectivized tangent space $P(TM)$, or, more generally, of the Grassmann bundle $\mathrm{Gr}_r(TM)$. Such structures are naturally defined on the solution space for any non-linear system of ODEs (possibly of mixed order). They also naturally appear in the context of integrability of PDEs.

We prove the existence of a natural frame bundle associated with any non-degenerate flag structure, show how to distinguish flag structures that come from systems of ODEs, and find the necessary and sufficient criteria when a flag structure defines a conformal, symplectic or G_2-structure on the base manifold.

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The Ricci flow on 4-dimensional space with metric of Walker and its properties

Abstract: The system of equations which describe properties of the Ricci flow

$$\partial_s g_{ij} = -2R_{ij}, \tag{1}$$

where $R_{ij} = R_{ij}(\vec{x}, s)$, $(\vec{x} = x, y, z, \tau)$ -is the tensor Ricci, on the metrics of a Walker type

$$ds^2 = 2\,\rho_{\tau\tau} dx^2 - 4\,\rho_{z\tau} dx\,dy + 2\,\rho_{zz} dy^2 + 2\,dy\,d\tau + 2\,dx\,dz \tag{2}$$

is considered. In this case the system of equations (1) is reduced to a single equation for the function $\rho(\vec{x}, s)$

$$\rho_s - 2\rho_{y\tau} - 2\,\rho_{xz} + 2\,\rho_{\tau\tau}\rho_{zz} - 2\,\rho_{z\tau}^2 = 0. \tag{3}$$

Remark that the equation (3) is generalization of corresponding equation which was considered in [1-2]. Simplest solution of the equation (3) is obtained under substitution of the form $\rho(\vec{x}, s) = H(x+y+z+\tau, s)$ when the equation (3) takes form the heat equation $\phi_s - 4\,\phi_{\xi\xi} = 0$, where $\xi = x + y + z + \tau$. It has solutions that depend on solutions of the Chazy equation $\dddot{h}(s) - 24\ddot{h}(s) + 36(\dot{h}(s))^2 = 0$ [2] (and its generalizations) which can be useful in theory of the Ricci-flow of the metric (2). As example of more complicated solution of equation (3) we can bring the solution of the form

$$\rho(x, y, z, \tau, s) = s^k H\left(\frac{y - A(x)}{s^2}, \frac{\tau}{s^m}, zs\right).$$

In general case equation (3) is reduced to a more simple p.d.e., which can be integrated by help of generalized method of Monge-Ampere transformations proposed in works of author (details see in [1]) or to the linear p.d.e. integrated by cascade method of Laplace. The solutions of the equation (3) which are expressed trough the solutions of ordinary differential equations the first order$F(x, y, y') = 0$ have special interest. As a simplest example we consider properties the Ricci-flow of the metric (2) which depend on solutions of the equation $y' = \dfrac{8 + xy - 2\,y^2}{y\,(2\,y - x)}$.

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Discrete Schlesinger transformations and difference Painlevé equations

Abstract: We known that differential Painlevé equations can be obtained as reductions of Schlesinger equations describing isomonodromic deformations of Fuchsian systems. Similarly, discrete Schlesinger transformations of Fuchsian systems give rise to discrete Painlevé equations. Sometimes both can be obtained for the same Fuchsian system, e.g., in the rank-two system with three finite poles, whose isomonodromic transformations are described by Painlevé-VI equation P_{VI}, and discrete Schlesinger transformations are described by the difference Painlevé-V equation d-P_V, which also corresponds to the Bäcklund transformations of P_{VI}. However, some discrete equations do not have a continuous counterpart. In [3] Sakai posed a problem of representing such equations using Schlesinger transformations, we consider here one such equation of type d-$P(A_2^{(1)*})$. Corresponding Fuchsian system was described by Boalch in [1]. Using a recently obtained discrete Hamiltonian of elementary Schlesinger transformations [2] we explicitly compute this example, verify that we indeed obtain discrete Painlevé equation type d-$P(A_2^{(1)*})$, and compare it to the usual form of d-$P(A_2^{(1)*})$ by comparing the blow-up structures of the corresponding rational surfaces.

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Interaction of lumps in a Bogoyavlenskii-Kadomtsev-Petviashvili equation in 2+1 dimensions

Abstract: The Singular manifold method is applied to an equation in 2+1 dimensions [4]

$$(4u_{xt} + u_{xxxy} + 8u_x u_{xy} + 4u_{xx}u_y)_x - u_{yyy} = 0,$$

which can be considered as a generalization of the Calogero- Bogoyavlenski- Schift equation [2]. The Lax pair, Darboux transformation and τ-functions are obtained. Lump solutions can be derived through an iterative method that yields a classification of lumps in terms of two integer numbers. We prove that these integer numbers are related with the numbers and order of the poles of the eigenfunctions of the Lax pair. This result allows us to connect this method with the method described by Ablowitz and Villarroel [1], [10] for the Kadomset-Petviashvili equation.

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Dispersionless integrable systems in 3D and Einstein-Weyl geometry

Abstract: We demonstrate that integrability of multi-dimensional PDEs can be seen from the geometry of their formal linearizations. Thus, for several classes of second order dispersionless PDEs that are integrable by the method of hydrodynamic reductions, the symbols of their formal linearizations define conformal structures which must be Einstein-Weyl in 3D, and self-dual in 4D. The talk will be based on the recent paper [1].

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About the group classification of the third-order ODE $y''' = F(x, y, y')$

Abstract: The problem of group classification of differential equations was first posed by Norwegian mathematician Sophus Lie, the founder of the theory of continuous groups. He also began to solve the problem of group classification of the second-order ordinary equation

$$y'' = f(x, y, y') \tag{4}$$

and proved that this equation admits no more than an eight-parameter transformation group of the space $R^2(x, y)$.

S. Lie solved in 1884 the problem of group classification of the particular type of Eqs. (4), where the right side does not depend on the first derivative, namely of the equations

$$y'' = F(x, y). \tag{5}$$

This condition appears very stringent and leads to a relatively small list of the possible forms of the equations.

The group classification of the equations (5) has been repeated later by L.V. Ovsiannikov. In our work we extend Lie's classification to the third-order equations

$$y''' = F(x, y, y'). \tag{6}$$

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Maple software package for computation of conservation laws of nonlinear self-adjoint systems of differential equations

Abstract: A recently developed Maple-based software package is presented for automatized conservation law analysis of systems of differential equations in light of the theory of nonlinear self-adjoint equations presented by N.H. Ibragimov. The package contains a collection of routines for construction of adjoint system, calculation of substitution that provides nonlinear self-adjointness for system, construction of conserved vectors corresponding to a certain symmetry operator, and verification if a vector satisfies the conservation equation for a given system. As a result of application of the package to various differential systems some exact, approximate and nonlocal conservation laws are investigated.

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New classes of nonlinearly self-adjoint porous medium equation

Abstract: In [3] and [10], the concepts of self-adjoint and quasi self-adjoint equations were generalized and the definition of weak self-adjoint equations and nonlinear self-adjoint equations were introduced. In this paper a nonlinear self-adjoint classification of a general class of porous medium equations is presented. We show, that the theorem in conservation laws proved in [8] can be applied to construct conservation laws associated with nonclassical generators. By using the property of nonlinear self-adjointness we construct some conservation laws associated with some classical and nonclassical generators of a porous medium equation.

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Nonlinear conformal-invariant electrodynamics in four and six dimensions

Abstract: It is well known that the conformal compactification of $(3+1)$-dimensional Minkowski spacetime can be identified with the projective light cone in a $(4+2)$-dimensional spacetime. In this space the conformal group acts linearly via rotations, and fields satisfying conformally-invariant linear Maxwell equations are defined. Here we consider nonlinear, conformal-invariant equations for the higher-dimensional Maxwell fields. Nonlinear constitutive equations are expressed in terms of invariant functionals, which we write in some different coordinate systems. We explore their properties in relation to the class of nonlinear constitutive equations for classical electromagnetism in $3+1$ dimensions respecting conformal symmetry, including Lagrangian and non-Lagrangian systems. The talk is based on joint work by the presenter with Steven Duplij and Vladimir Shtelen.

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Discretising systematically integrable systems

Abstract: We present two different systematic methods for the construction of discrete analogues to integrable differential systems. The first method is tailored to first order differential equations and relies on a formal linearisation, followed by a Padé-like rational approximation of the exponential evolution operator. We apply our method to a host of systems for which there exist discretisation results obtained

by what we call the "intuitive" method and compare the discretisations obtained. An application of our method to discrete Painlevé equations is also presented. The second method aims solely at discretising the Painlevé equations. It is inspired by the method of Hirota (while extending it) and by that of Mickens (by specifying it to the case at hand). We derive various discrete analogues of Painlevé I and II. We obtain forms that have been previously derived as well as new ones, in particular equations with a geometry described by the affine Weyl group $E_8^{(1)}$. As a by-product we obtain also linearisable equations, some of which are new.

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A noncommutative version of the bispectral problem

Abstract: The so called "*bispectral problem*" was raised and solved in a joint paper with Hans Duistermaat in CMP back in 1986. Many people have made very important contributions to this problem afterwards. I want to consider a few noncommutative versions of this problem. In the scalar case this leads to all sorts of unexpected connections, including the appearance of the KdV, KP, Toda and other nonlinear integrable evolution equations.

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Noether symmetries and the quantization of a Liénard-type nonlinear oscillator

Abstract: The classical quantization of a Liénard-type nonlinear oscillator is achieved by using Noether symmetries. This method straightforwardly yields the correct Schrödinger equation in the momentum space, and sheds light into the apparently remarkable connection with the linear harmonic oscillator.

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Representations of simple algebraic groups with vector stabilizers having positive dimensions

Abstract: Let G be a simply connected simple algebraic group over an algebraically closed field K of characteristic $p > 0$ with root system R, and let $\mathfrak{g} = \mathcal{L}(G)$ be its restricted Lie algebra. Let V be a finite dimensional $\mathfrak{g}$-module over K. For any point $v \in V$, the *isotropy subalgebra* of v in $\mathfrak{g}$ is $\mathfrak{g}_v = \{ x \in \mathfrak{g} / x \cdot v = 0 \}$. A restricted $\mathfrak{g}$-module V is called **exceptional** if for each $v \in V$ the isotropy subalgebra $\mathfrak{g}_v$ contains a non-central element (that is, $\mathfrak{g}_v \not\subseteq \mathfrak{z}(\mathfrak{g})$). This talk is about results obtained on the classification of irreducible exceptional $\mathfrak{g}$-modules. After finding a necessary condition for a $\mathfrak{g}$-module to be exceptional, the modules over groups of exceptional type are completely classified, while for modules over groups of classical type the general problem is reduced to a short list of unclassified modules. This work was inspired by the characteristic zero case done by Andreev, Elashvili and Vinberg on the late 60's. The classification of exceptional modules in prime characteristic is expected to have applications in modular invariant theory and in classifying modular simple Lie superalgebras.

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Formal diagonalization of the discrete Lax operators and construction of conserved densities for dynamical systems

Abstract: A method for constructing conservation laws for discrete models based on the formal diagonalization of the Lax pairs is discussed (see [1]). An algorithm of finding formal asymptotic expansion of eigenfunctions of discrete Lax operators is suggested. Discrete potential KdV equation, dressing chain, Toda lattice, Discrete Toda lattice corresponding to the Lie algebra $A_1^{(1)}$ etc. are considered as illustrative examples.

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Discrete Boussinesq equations

Abstract: We discuss lattice versions of the Boussinesq equation. Since the continuous form is not evolutionary (i.e., first order in time derivatives) but second order in time, the Boussinesq equation cannot be discretized as a one-component equation on an elementary quadrilateral of the Cartesian lattice. Instead there are one-component discretizations on a 3×3 stencil [1] and three-component versions on the quadrilateral [2, 3]. Furthermore discrete bilinear versions are also known [4, 5, 6]. We compare these different approaches and the related soliton solutions.

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Solitons and large-time behavior of solutions for the Novikov-Veselov equation

Abstract: The Novikov-Veselov equation is mathematically the most natural $(2+1)$-dimensional analog of the classic Korteweg-de Vries equation, integrable via the inverse scattering transform for the 2-dimensional stationary Schrödinger equation at a fixed energy. Another well-known $(2+1)$-dimensional analog of KdV, Kadomtsev-Petviashvili equation, can be viewed as a limit case of the Novikov-Veselov equation, when energy tends to infinity. In the present talk we consider the question of existence/absence of solitons and the question of asymptotic behavior of inverse scattering solutions for the Novikov-Veselov equation. We start by studying nonsingular algebraically localized rational solutions of the Novikov-Veselov equation at positive energy constructed by P.G. Grinevich and V.E. Zakharov. We show that, asymptotically at large times, these solutions are sums of solitons that interact completely elastically. Grinevich-Zakharov solutions are localized as $O\left(|x|^{-2}\right)$, $|x| \to \infty$, and we prove that this localization is almost the strongest possible: we show that the Novikov-Veselov equation at nonzero energy does not possess solitons localized stronger than $O\left(|x|^{-3}\right)$, $|x| \to \infty$. For the case of zero energy we show that the Novikov-Veselov equation does not possess sufficiently localized solitons of conductivity type. We also study the question of the asymptotic behavior of inverse scattering solutions of the Novikov-Veselov equation at nonzero energy (at positive energy we restrict ourselves to transparent solutions). Under assumption that the scattering data for the solutions are nonsingular, we obtain that these solutions decrease uniformly with time and we find an estimate on the velocity of decrease. Thus we show, in particular, that the solitons of the Novikov- Veselov equation are generated by the singularities of the associated scattering data.

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Observer's mathematics approach to the quantum mechanics

Abstract: When we consider and analyze physical events with the purpose of creating corresponding models we often assume that the mathematical apparatus used in modeling is infallible. In particular, this relates to the use of infinity in various aspects and the use of Newton's definition of a limit in analysis. We believe that is where the main problem lies in contemporary study of nature. This work considers Physical aspects in a setting of arithmetic, algebra, geometry, analysis, topology provided by Observer's Mathematics (see www.mathrelativity.com). As the Geometry application, we prove that Euclidean Geometry works in sufficiently small neighborhood of the given line, but when we enlarge the neighborhood, non-Euclidean Geometry takes over. As the Analysis application, we prove that the physical speed is a random variable, cannot exceed some constant, and this constant does not depend

on an inertial coordinate system. As the Physical application, we consider the Schrodinger equation, the Airy equation, the Korteweg-de Vries equation, Lorentz transform, geodesic equation and the Schwarzian derivative. The probability appears automatically, without apriori assumptions. Certain results and communications pertaining to solution of these problems are provided.

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Non-Abelian Lie algebroids over jet spaces

Abstract: We associate a class of non-Abelian Lie algebroids with zero-curvature representations (ZCR) for partial differential equations (PDE). Let $\mathfrak{g}$ be the finite-dimensional Lie $\Bbbk$-algebra of a gauge group G (here $\Bbbk = \mathbb{R}$ or $\mathbb{C}$). Take a basis $e_1, \ldots, e_d$ in $\mathfrak{g}$; locally, every $\mathfrak{g}$-valued ZCR $\alpha = \alpha^k_\mu e_k \, \mathrm{d}x^\mu$ for a given PDE system $\mathcal{E}$ upon sections of a vector bundle π is a solution of the Maurer–Cartan equation $\overline{\mathrm{d}}_h \alpha = \frac{1}{2}[\alpha, \alpha]$ (here $\alpha^k_\mu \in C^\infty(\mathcal{E}^\infty)$ for $\mathcal{E}^\infty \subseteq J^\infty(\pi)$; we denote by x^μ local coordinates on the base M^n; the horizontal differential d_h on $J^\infty(\pi)$ is inherited from $\mathrm{d}_{\mathrm{dR}(M)}$ under $\pi_\infty \colon J^\infty(\pi) \to M$, and $\overline{\mathrm{d}}_h = \mathrm{d}\big|_{\mathcal{E}^\infty}$). Construct two auxiliary vector bundles over M, namely $\chi \colon \Lambda^1(M) \otimes \mathfrak{g} \to M$ and $\xi \colon M \times \mathfrak{g} \to M$; denote by $\Pi\xi$ the odd neighbour of ξ, i.e., the $\mathbb{Z}_2$-graded bundle with reverse parity of fibre coordinates. Take Whitney's sum $J^\infty(\chi) \times_M J^\infty(\Pi\xi)$ of the bundles of infinite jets of sections for χ and $\Pi\xi$. Each $\mathfrak{g}$-valued zero-curvature representation yields a cohomology theory the differential Q in which encodes a non-Abelian Lie algebroid [c.f. Vaintrob (1997), Kiselev and van de Leur (2011)].
Proposition
This homological evolutionary vector field Q is

$$Q = \partial^{(\alpha)}_{[b,\alpha] + \mathrm{d}_h b} + \tfrac{1}{2}\partial^{(b)}_{[b,b]}, \qquad [Q,Q] = 0 \quad \Longleftrightarrow \quad Q^2 = 0,$$

where $\alpha = (\alpha^k_\mu,\ 1 \leqslant k \leqslant d,\ 1 \leqslant \mu \leqslant n)$ are fibre variables in χ encoding $\mathfrak{g}$-valued one-forms on M^n, $b = (b^i)$ is the d-tuple of odd coordinates along fibres in $\Pi\xi$, $[b,b]^k = b^i c^k_{ij} b^j$ and by c^k_{ij} we denote the structure constants w.r.t. e_ℓ in $\mathfrak{g}$, $[b,\alpha]^k = b^i c^k_{ij} \alpha^j_\mu \, \mathrm{d}x^\mu$, d_h is the horizontal differential in the Whitney sum, the differential operator $\partial_\alpha = \mathrm{d}_h + [\cdot, \alpha]$ is the anchor, and $\partial^{(\alpha)}$, $\partial^{(b)}$ are evolutionary fields in the superbundle.

In the non-graded phrasing of the non-Abelian geometry of ZCR, the anchor $\overline{\partial}_\alpha = \overline{\mathrm{d}}_h + [\cdot, \alpha]$ is the differential which was discovered by Marvan (2002) in the context of (non)removability of parameters λ in families $\alpha(\lambda)$ under gauge transformations. We conclude that this object is natural because it exemplifies –in the non-Abelian case– a classical construction of Lie algebroid in the framework of jet spaces [c.f. Alexandrov, Kontsevich, Schwarz, and Zaboronsky (1997)].

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Differential invariants of Riemannian and Einstein manifolds

Abstract: We describe the differential algebra of invariants of Riemannian and Einstein metrics which gives a local classification with respect to the group of diffeomorphisms in the regular case.

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Approximate RG-symmetries in application to nonlinear problems of mathematical physics

Abstract: Recent results in application of the renormgroup algorithm and the related approximate renormgroup symmetries to solutions of boundary-value problems in nonlinear mathematical physics are reviewed. Different methods of calculating these symmetries, based on modern group analysis are described. Presented illustrations from nonlinear optics and acoustics, and kinetic plasma theory demonstrate the potentialities of the algorithm in construction of new analytic solutions to nonlinear problems of mathematical physics. These solutions enable to describe the singularity structure for self-focusing of a laser beam in a nonlinear medium, to study nonlinear acoustic waves propagation in channels with variable cross-sections, and to investigate the energy spectra of accelerated ions in expanding plasma.
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SU(2s+1) symmetry and nonlinear dynamic equations of spin magnets

Abstract: Landau-Lifshitz equation describes the evolution of non-equilibrium states of magnets in terms of the spin vector. Such an equation is well established and widely used for magnetic insulators, which are composed of particles with spin $s = 1/2$. Mathematics paid great attention to this equation for finding the nonlinear solutions.

Currently, there is an increasing interest in research of high spin magnets with $s > 1/2$. This is caused by the discovery of the quadrupole magnetic states, the synthesis of stable high-spin molecules, creating crystals in optical lattices at low temperatures. These many-particle states and the Hamiltonian may have unusual properties SU($2s + 1$) symmetry. There is therefore a necessity of building dynamic theory for high spin condensed matter.

Based on the variation principle, the authors obtained the Poisson brackets for the macroscopic quantities that characterize the physical state of high spin magnets and the dynamical equations for normal and degenerate states are derived. The symmetry of normal equilibrium states coincides with symmetry of Hamiltonian. In the degenerate case, the symmetry of the equilibrium state below the symmetry of the Hamiltonian. It was examined the role of Casimir invariants in constructing exchange magnetic Hamiltonians.

As an application, it was studied magnets with spin $s = 1$[1, 2] in a detail. We calculate the spectra of spin and quadrupole waves. At last, the nonlinear dynamical equations for the magnets with spin $s = 3/2$ are obtained for cases when the Hamiltonian has the properties of SU(4), SU(2) x SU(2), SU(3), SO(4) and SO(5) symmetries

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On symmetries of stochastic differential equations

Abstract: Recently there appeared applications of Lie group theory to stochastic differential equations (SDEs). Different approaches were suggested. Here we deal with infinitesimal Lie group transformations which leave the form of SDEs and the framework of Itô calculus invariant. There will be reviewed results concerning with Lie point symmetries of

1. scalar stochastic differential equations with one-dimensional Brownian motion: the admitted symmetry group is at most three-dimensional;
2. systems of stochastic differential equations with diffusion matrices of full rank: the admitted symmetry group of a system of n SDEs is at most $(n+2)$-dimensional;
3. scalar stochastic differential equation of order $n \geq 3$ with one-dimensional Brownian motion: the admitted symmetry group is at most $(n+2)$-dimensional.

Finally, we consider the relation between symmetries of SDEs and symmetries of the associated Fokker-Planck equation. The relation between first integrals of SDEs and symmetries of the associated Fokker-Planck equation is also mentioned.

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Gaps, symmetry, integrability

Abstract: It is often the case in geometry and differential equations that maximally symmetric structures are distinguished (in particular, they allow complete description). The smaller the size of the symmetry is, the bigger the moduli space grows. Another issue is the phenomenon of gaps in dimension study of the symmetry groups (for infinite pseudogroups this appeals to functional dimensions). I will illustrate this with many examples, like G2 structures, conformal algebras, almost complex bundles and heavenly equations, and I will draw a speculative bridge towards integrability problem.

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PT(reversible) symmetric nonlinear Lienard Type oscillators: nonstandard Lagrangian/Hamiltonian description and exact quantization

Abstract: We identify a class of nonlinear Lienard type oscillators of N-degrees of freedom, exhibiting nonstandard type Lagrangian and Hamiltonian descriptions and possessing reversible/PT symmetry in real dynamical variables. We prove the exact quantization of these systems and solve the underlying time-independent Schrdinger equations, not in coordinate space, but in momentum space corresponding to nonsymmetric non-Hermitian Hamiltonians. We deduce the eigenvalue spectrum corresponding to both positive and negative eigenvalues and obtain the corresponding eigenfunctions exactly. Generalization to quadratic Lienard type equations will be indicated.

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Differential invariants and their applications

Abstract: Differential invariant algebras of Lie (pseudo) group actions as an alternative way of description of infinite dimensional manifolds will be discussed. Various methods of finding differential invariants as well as a structure of the differential invariant algebras together with applications to classical classification problems and integrable systems will be shown.

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Invariant and partially invariant solutions of gas dynamic equations on three-dimensional subalgebra

Abstract: The gas dynamic equations

$$\rho D\vec{u} + \nabla p = 0, D\rho + \rho\nabla \cdot \vec{u} = 0, Dp + \rho c^2 \nabla \cdot \vec{u} = 0, \tag{7}$$

where $D = \partial_t + \vec{u} \cdot \nabla$ – total derivative with respect to time, $\vec{u}$ – velocity vector, p – pressure, $c^2 = \dfrac{\partial p}{\partial \rho}$ – squared sound velocity, with the state equation of separate density

$$\rho = h(p)S \tag{8}$$

(S - entropy) admit 12-dimensional Lie algebra L_{12}. Certain results were obtained earlier: the optimal system of subalgebras for a 12-dimensional Lie algebra admitted by equations (7) was constructed, hierarchy of submodels for 5 dimensional self-normalized subalgebra from the optimal system of subalgebras was considered, the graph of nested subalgebras for 5 dimensional self-normalized subalgebra was constructed, the partially invariant solution of rank 2, defect 0 on 4-dimensional subalgebra from graph was constructed and explored. In this work the 3-dimensional subalgebra nested in 4-dimensional subalgebra is considered. The aim is to find the set of solutions in order to join them with the solutions on subalgebras of higher dimension. The aim is not reached yet, but invariant solutions of rank 1 and partially invariant solutions of the rank 1 defect 1 are constructed. We got two submodels - invariant and partially invariant, 5 solutions that depend on arbitrary function and 19 exact solutions.

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Invertible coupled KdV and coupled Harry Dym hierarchies

Abstract: The energy-dependent Schrödinger spectral problem has been introduced by Jaulent and Miodek in [1] in the two-field case. It has been generalized to an arbitrary number of components by Martínez Alonso [2] who also presented its multi-Hamiltonian structure. Antonowicz and Fordy have further investigated this problem in a series of papers (see for example [3],[4],[5],[6]). They demonstrated that this spectral problem leads to two families of coupled (multicomponent) soliton hierarchies: the coupled KdV (cKdV) and the coupled Harry-Dym (cHD) hierarchies.

In this talk I complete their work by discussing the conditions under which both types of hierarchies have also negative parts (i.e., when the recursion operator is explicitly invertible) and present a first

few flows of these negative hierarchies. Further, using the results from [3], I investigate the structure of nonlocal parts of all Hamiltonian structures associated with both types of hierarchies: I show that all Hamiltonian and symplectic structures of the cKdV-type hierarchies are at most weakly nonlocal while the cHD-type hierarchies have Hamiltonian and symplectic structures that are nonlocal up to third order, i.e., they have terms up to ∂^{-3} in their nonlocal parts. I also present compact formulas for the highest nonlocalities of all these quantities.

The presented results were obtained in cooperation with M. Blaszak, Poznan, Poland; they are accepted for publication in Studies in Applied Matthematics.

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Self-adaptive mesh discrete integrable systems

Abstract: In a series of our papers, we have investigated integrable discretizations of nonlinear evolution equations in which loop-type and cusp-type soliton solutions exist. In those studies, discrete hodograph transformations, tau-functions and bilinear equations have played important roles. Recently, we found that a geometric formulation of integrable PDEs plays an important role in discretizations. Discrete integrable systems obtained from the above class of integrable PDEs have a property of self-adaptive mesh. In this talk, I will show the geometric construction of self-adaptive mesh discrete integrable systems. These new discrete integrable systems include the discrete WKI elastic beam equation, the discrete Dym equation, the discrete short pulse equation, the discrete Hunter-Saxton equation and the discrete Camassa-Holm equation. This type of discrete integrable systems can be obtained easily through a geometric approach. This is a part of joint work with Bao-Feng Feng, Junichi Inoguchi, Kenji Kajiwara and Yasuhiro Ohta.

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On commuting ordinary differential operators of arbitrary rank and genus

Abstract: The talk is devoted to the theory of commuting ordinary scalar differential operators. Recently Andrey Mironov has constructed remarkable examples of commuting ordinary scalar differential operators of ranks 2 and 3 with polynomial coefficients for any genus $g > 1$ that generalize naturally the famous Dixmier examples of ranks 2 and 3, genus 1. We construct examples of commuting ordinary scalar differential operators with polynomial coefficients that are related to a spectral curve of an arbitrary genus g and to an arbitrary even rank $r = 2k$, and also to an arbitrary rank of the form $r = 3k$, of the vector bundle of common eigenfunctions of the commuting operators over the spectral curve. At present we know no explicit examples of commuting ordinary scalar differential operators that are related to a spectral curve of genus $g > 1$ for rank of the form $r = 6s \pm 1$, $s \geq 1$. For all other values of genus g and rank r, explicit examples of commuting operators even with polynomial coefficients are constructed. We conjecture that there exist commuting ordinary scalar differential operators with polynomial coefficients that are related to a spectral curve of an arbitrary genus $g > 1$ also for an arbitrary rank of the form $r = 6s \pm 1$, $s \geq 1$, but such examples are not known for now, – this is a very interesting problem.

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Reduction of ODEs: a general approach

Abstract: We discuss reduction of ODEs in a geometric framework that generalizes reduction by a symmetry group. Our approach can be applied either to systems of first order equations or to higher order ODEs and elucidates the analogies in the reduction by using symmetries, lambda-symmetries and solvable structures. Given a vector field Z on a manifold M, we consider a distribution $\mathcal{D}_1$ such that the distribution $\mathcal{D} = Z \oplus \mathcal{D}_1$ is completely integrable. If $\mathcal{A}$ is the Pfaffian System generated by one-forms that are annihilated by $\mathcal{D}$, the research of integral manifolds of $\mathcal{A}$ gives us the reduced equations. In this way the problem of finding the integral lines of Z is split into two parts. In particular, if we look for a complete set of invariants of Z, we can find a part of them by looking for the joint invariants of the distribution $\mathcal{D}$. The possibility of integrating the second set of equations, or equivalently of finding the residual invariants, depends on the nature of distribution $\mathcal{D}$. In the particular case of a symmetry group G for Z the generators for the Lie algebra $\mathcal{G}$ can be taken as generators for the distribution $\mathcal{D}_1$ and our reduction procedure completely agrees with the standard reduction by a symmetry group. However our setting allows us to deal with more general situations. In particular if $\mathcal{D}_1$ is a solvable structure for the vector field Z, we have only one reduced equation and we can integrate the initial problem by quadratures. Moreover, given a k-order ODE represented as a vector field Z on a suitable jet bundle, if we have a λ-symmetry Y for Z our result still holds since the distribution $\mathcal{D} =< Z, Y >$ is completely integrable and we can find reduced equations just looking for invariant of Z that are also invariant of Y. In the general case, our reduction procedure achieves the result to split the initial equations into two systems, one of which depends on less variables and should be in principle easier to integrate.

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Variational problems with free boundary values: a geometric perspective

Abstract: Let Y be a smooth (real) manifold of dimension $n + 1$, with nonempty boundary ∂Y. An n–dimensional submanifold $L \subseteq Y$ such that

1. L is connected, compact and oriented;
2. $\partial L = L \cap \partial Y$;

3. L is nowhere tangent to ∂Y,

is called *admissible*; their totality is denoted by $\mathcal{A}_Y$. Introduce a local coordinate system $(\boldsymbol{x}, u)$ on Y, where $\boldsymbol{x} := (x^1, \ldots, x^n)$. Let $\lambda = \mathcal{L}\mathrm{d}^n\boldsymbol{x}$ be an r^{th} order Lagrangian, i.e., let $\mathcal{L} = \mathcal{L}(\boldsymbol{x}, u, u_I)$ and I denote a multi–index of length $\leq r$. Suppose that, in such coordinates, an element $L \in \mathcal{A}_Y$ is the graph of a function $u = u(\boldsymbol{x})$, defined on a connected and bounded domain $\Omega \subseteq \mathbb{R}^n$: then the integral

$$\boldsymbol{S}_\lambda[L] := \int_\Omega \mathcal{L}\left(\boldsymbol{x}, u(\boldsymbol{x}), \frac{\partial^{|I|}u}{\partial \boldsymbol{x}^I}\right) \mathrm{d}^n\boldsymbol{x}$$

makes sense. The *variational problem with free boundary values* determined by the Lagrangian λ on Y consists of finding the elements of $\mathcal{A}_Y$ which are critical for $\boldsymbol{S}_\lambda$.

In this talk I will show that, by extending the well–known geometric framework for Variational Calculus based on jet fibrations through Grassmann fribrations, one can encompass also the above–defined variational problem with free boundary values. Conceptually, such a construction is rather obvious; nevertheless there are few delicate steps which need to be enlightened (full details can be found in my recent publication [1]). Once the geometric setting has been established, I will exploit it to derive a complementary equation to the Euler–Lagrange equation, which generalizes a notion known as *transversality condition* in Variational Calculus (a result obtained in collaboration with M.E. Stypa [2]). I stress that, in principle, such an equation may be obtained by direct analytical techniques; on the contrary, the geometric approach I proposed here makes it almost self–evident.

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Jacobi last multipliers and some classes of nonlocal symmetries for ODEs

Abstract: We study the role of the Jacobi last multipliers in the order reductions of ordinary differential equations derived by the existence of some nonlocal symmetries obtained by embedding the equation in an auxiliary system. Some connections of such nonlocal symmetries with exponential vector fields, $\lambda-$symmetries and telescopic vector fields are also established. We include examples to illustrate that the combined use of Jacobi last multipliers and these symmetries leads to the complete solution of nonlinear ODEs, even if they lack Lie point symmetries.

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New variational principle for integrable systems

Abstract: A new point of view on what constitutes in our view the appropriate variational approach for integrable systems, was introduced by S. Lobb and the author in 2009. This approach in terms of

what was coined Lagrangian multiforms, is understood as the Lagrangian description of multidimensional consistency. The talk reviews the results obtained so far and points to directions of travel for the future.

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Qualitative analysis of weakly coupled Landau-Lifshits equations with spin-transfer torque terms

Abstract:
A recent modification of a classic Landau-Lifshitz (LL) equation that includes so-called spin-transfer torque is widely recognized in physics community as a model of magnetization dynamics in certain nanodevices [1]. We perform rigorous qualitative analysis of this model and its generalization, weakly coupled LL equations with spin-transfer torque terms. There is some experimental evidence that the coupled LL system models magnetization dynamics of bi-layered nanodevices when both layers are active [2]. We analyze stability of invariant sets of the models using tools of dynamical systems theory. A surprising analytic criterion of stability of equilibrium points of the coupled system that involves all physical and geometric parameters of the model is found [3]. Our analysis also reveals certain analytic structure of the coupled system that is an interesting mathematical fact in its own right. We also explore a problem of existence, number, and stability of periodic solutions to the systems under consideration.

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Superintegrability? Hidden linearity? Classical quantization? Symmetries and more symmetries!

Abstract: I will present and exemplify new venues provided by Lie and Noether symmetries when dealing with superintegrable systems, their possible hidden linearity, and classical quantization. If time allows I will also present and exemplify the relationship between $\lambda-$symmetries and Jacobi last multiplier, and that between nonclassical symmetries and heir-equations.

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New conservation laws for helically symmetric flows

Abstract: Helically invariant reductions due to a reduced set of independent variables (t, r, ξ) with $\xi = az + b\varphi$ emerging from a cylindrical coordinate system of viscous and inviscid time-dependent fluid flow equations, with all three velocity components generally nonzero, are considered in primitive variables and in the vorticity formulation. Full sets of equations are derived.

Special cases of rotationally symmetric and plane flows are obtained as limiting cases of helical invariance.

Local conservation laws of helically invariant systems are systematically sought through the direct construction method. Various new sets of conservation laws for both inviscid and viscous flows, including families that involve arbitrary functions, are derived. For both Euler and Navier-Stokes flows, infinite sets of vorticity-related conservation laws are derived. In particular, for Euler flows, we obtain a family of conserved quantities that generalize helicity.

The special case of two-component flows, with zero velocity component in the invariant direction, is additionally considered, and special conserved quantities that hold for such flows are computed. In particular, it is shown that the well-known infinite set of generalized enstrophy conservation laws that holds for plane flows also holds for the general two-component helically invariant flows and for axisymmetric two-component flows.

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Classical algebras understood as graded-commutative algebras

Abstract: The classical algebras (quaternions, Clifford algebras, octonions,...) are viewed as graded-commutative algebras over the abelian group $\mathbb{Z}_2^n$. I will explain this point of view, and show a number of application in algebra, geometry and combinatorics.

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On finite W-algebras for Lie superalgebras in the regular case

Abstract: The finite W-algebras are University of Texas - Pan American, USAcertain associative algebras associated to a complex semisimple Lie algebra g and a nilpotent element e of g. Due to recent results of I. Losev, A. Premet and others, W-algebras play a very important role in description of primitive ideals. It is a result of B. Kostant that for a regular nilpotent element e, the finite W-algebra coincides with the center of $U(g)$.

In the full generality, the finite W-algebras were introduced by A. Premet. His definition makes sense for a simple Lie superalgebra $g = g_{\bar{0}} \oplus g_{\bar{1}}$ in the case when $g_{\bar{0}}$ is reductive, g has an invariant symmetric bilinear form, and e is an even nilpotent element. We show that certain results of A. Premet can be generalized for classical Lie superalgebras. We study the case when e is an even regular nilpotent element. Kostant's result does not hold in this case.

We obtain the precise description of finite W-algebras for regular e for classical Lie superalgebras of Type I and defect one and for the exceptional Lie superalgebra $D(2,1;\alpha)$. We show that the finite W-algebra for the queer Lie superalgebra $Q(n)$ is isomorphic to a truncation of the super-Yangian of $Q(1)$.
This is joint work with V. Serganova.

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Lump Solutions in a 2+1 dimensional equation

Abstract: In this work we consider the 2+1 dimensional equation

$$(4u_{xt} + u_{xxxy} + 8u_x u_{xy} + 4u_{xx}u_y)_x - u_{yyy} = 0$$

or the system

$$\begin{aligned} 4u_{xt} + u_{xxxy} + 8u_x u_{xy} + 4u_{xx}u_y &= \omega_{yy} \\ u_y &= \omega_x \end{aligned}$$

which represents a modification to the Calogero-Bogoyavlenskii-Schiff (CBS) equation [1, 2, 3]

$$4u_{xt} + u_{xxxy} + 8u_x u_{xy} + 4u_{xx}u_y = 0$$

The system arises as the compatibility of the following pair of operators,

$$\mu_{xx} + i\mu_y + 2k\mu_x + 2u_x\mu = 0 \qquad -2i\mu_{yy} + \mu_t + 4k^2\mu_y + 4u_y\mu_x + (4ku_y - 2u_{xy} - 4iw_y)\mu = 0$$

Using this convenient form of the Lax pair, depending on a complex spectral parameter k, we will determine eigenfunctions μ that are meromorphic functions of k, having single poles.

The potential is recovered, up to an arbitrary function of time $\varphi(t)$, by the following formula:

$$u = -\mu^{1)}$$

where $\mu^{1)}$ is the first term of the expansion of u in powers of $\frac{1}{k}$ as $|k| \to \infty$.

Finally certain classes of potentials are determined. We follow the ideas put forward in [4, 5]. First we suppose that $\mu(k)$ is a meromorphic function of the form with poles $k_0, -k_0^*$ and residues ϕ_1, ϕ_2:

$$\mu(k) = 1 + \frac{\phi_1}{k - k_0} + \frac{\phi_2}{k + k_0^*}$$

We find the potential

$$u = \frac{\Delta_x}{\Delta} = \partial_x \log \Delta$$

where

$$\Delta = f(k_0)f(-k_0^*) + \frac{1}{(k_0 + k_0^*)^2}$$

and $f(x, y, t, k) = -x - 2iky + 8ik^3t + C$.

We next extend these ideas to the case of a meromorphic function μ with a finite number $2N$ of simple poles coming in pairs $k_j, -k_j^*, j = 1, \ldots, N$.

We give conditions which guarantee that the potential obtained is regular on the entire plane and express it as the determinant of a certain matrix. The potential is a rational, infinitely differentiable function with no singularities.

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Solitons in quantum systems with spectral self-similarity and complexity of symbolic dynamical systems

Abstract: We study a class of quantum dynamical systems with the property of multiplicative self-similarity $\mathscr{L} = q\mathscr{L}$, where q is a real parameter close to 1. In this class the existence of soliton solutions is established associated with special values of the parameter q providing simultaneous Diophantine approximation of the sequence $\log n$. This construction is connected to spectral analysis of ergodic dynamical systems with invariant measure. A system with finite entropy is encoded by a random process generating infinite words in a finite alphabet $\mathbb{A}$. We know due to contribution of Shannon, Kolmogorov and Sinai that there exists an invariant of combinatorial origin, *entropy* distinguishing Bernoulli dynamical systems having same spectral. Though it is still an open question how we can classify all spectral types observed for ergodic dynamical systems? We consider two kinds of ergodic flows T^t on a Borel probability space $(X, \mathscr{A}, \mu)$ with zero entropy, simple spectrum and correlation decay $\langle T^t f, g\rangle = O(t^{-1/2+\epsilon})$ having, respectively, singular and Lebesgue spectral type.

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On the inverse scattering transform for the defocusing nonlinear Schrödinger equation with non-vanishing boundary conditions

Abstract: This talk will report on a recent work aimed at developing a rigorous theory of the inverse scattering transform for the defocusing nonlinear Schrödinger equation: $iq_t = q_{xx} - 2|q|^2, q$ with nonvanishing boundary conditions $q(x,t) \sim q_\pm$ as $x \to \pm\infty$ (same amplitude $|q_\pm| \equiv q_0 > 0$ is assumed at both space infinities).

The direct scattering problem is shown to be well-posed for potentials q such that $q - q_\pm \in L^{1,2}(R^\pm)$. As to the inverse problem, we formulated and solved it both via Marchenko integral equations, and as a Riemann-Hilbert problem in terms of a suitable uniform variable, determined the asymptotic behavior of the scattering data and showed that the linear system solving the inverse problem is well-defined.

An important open issue is whether an "area" theorem can be established, to relate the existence and location of discrete eigenvalues of the scattering problem to the area of the initial profile of the solution, suitably defined to take into account the boundary conditions. In this regard, we proved that discrete eigenvalues of the scattering problem, if they exist, are confined to two real semi-intervals of $\pm q_0$ whose sizes decrease to zero with the area: $\int_{-\infty}^{0} dx|q(x) - q_-| + \int_0^{+\infty} dx|q(x) - q_+|$.

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Lax integrability of new spatially one-dimensional Riemann type hydrodynamical systems

(Joint work with O. Artemowych, Krakow Polytechnical University, Poland, D. Blackmore*, NJIT, USA and Y. Prykapatsky, University of Agriculture, Krakow, Poland)

Abstract: Some innovative approaches involving both symplectic and differential-algebraic tools are used to study the Lax integrability of the following hydrodynamic hierarchies of the generalized Riemann type

$$D_t^{N-1} u = \bar{z}_x^2, \quad D_t \bar{z} = 0,$$

and the Ostrovsky-Vakhnenko type

$$D_x[D_t^{N-1} u + D_x^{-1}(u + \beta)] = 0, \qquad D_t[D_t^{N-1} u + D_x^{-1}(u + \beta)] = 0,$$

for $\beta \in \mathbb{R}$. They are defined on a 2π-periodic manifold $\bar{M}^N \subset C^\infty\left(\mathbb{R}/2\pi\mathbb{Z}; \mathbb{R}^N\right)$, where $N \in \mathbb{N}$ is a natural number, the manifold $\bar{M}^N$ is comprised of vectors $\left(u, D_t u, D_t^2 u, \ldots, D_t^{N-2}, \bar{z}\right)$ for $N \geq 2$, and the relevant differential operators are defined as $D_x := \partial/\partial x$ and $D_t := \partial/\partial t + u\partial/\partial x$. These operators satisfy the Lie-algebraic commutator equation

$$[D_x, D_t] = u_x D_x,$$

with $t \in \mathbb{R}$ as the evolution parameter. New Lax representations for the generalized Riemann type hierarchy at $N = 2$ and 3 are constructed in exact form, and related bi-Hamiltonian integrability and compatible Poissonian structures are also obtained by means of a synthesis of gradient-holonomic and geometric methods. Thus, the complete integrability of a new generalized Riemann type hydrodynamic system is studied by means of a novel combination of symplectic and differential-algebraic tools. In particular, a compatible pair of polynomial Poissonian structures, a Lax representation and a related infinite hierarchy of conservation laws are constructed. Concerning the Ostrovsky-Vakhnenko type hierarchy at $N = 2$ the complete Lax integrability is also studied by means of analogous symplectic gradient-holonomic and differential-algebraic tools, an associated compatible pair of polynomial Poissonian structures, Lax representation and related infinite hierarchies of conservation laws are obtained. For the case $N = 3$ the existence of an infinite hierarchy of polynomial conservation laws is stated.

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Integrability, peakons and wave breaking for a generalized μ-Camassa-Holm equation

Abstract: In this talk, a μ-type integrable equation with cubic and quadratic nonlinearities is introduced and studied, which can be regarded as a generalization to both μ-Camassa-Hlom and modified μ-Camassa-Holm equation. It is shown that the proposed equation is integrable in the sense that it admits Lax-pair and bi-Hamiltonian structure. It's scaling limit is an integrable model which describe short capillary gravity waves. Local well-posedness of the initial value problem to the equation in the Sobolev space is established by the viscosity method. Existence of peaked traveling-wave solutions and formation of singularities of solutions for the equation are investigated. It is verified that the equation admits a single peaked soliton and multi-peakon solutions. Blow-up criterion and wave-breaking for solutions with certain initial profiles are also given. It turns out that singularities of the solutions occur only in the form of wave-breaking. This is a joint work with Ying Fu and Yue Liu.

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Dynamics of the many particle Jaynes-Cummings Model

Abstract: We consider the dynamics of a system consisting of N two-level atoms interacting with a multi-mode cavity field, as an example of the generalized Jaynes-Cummings model. Based on formulation of the collective atom variables the Jaynes-Cummings model is generalized to a system of N two-level atoms. For the given system, the generalized kinetic equation is obtained and conditions are given under which its solution is reduced to solution of the linear equation, and of the nonlinear Schrödinger equation, respectively.

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How many integrals are needed for integrability?

Abstract: For a general autonomous dynamical system of n equations each integral of motion reduces the order by one. Then all together $n-1$ integrals are needed for integrability by quadratures. According to the Liouville-Arnold theorem a hamiltonian system of $2n$ equations needs n independent, commuting integrals for complete integrability.

It may seem surprising that there are non trivial integrable dynamical systems having any given prescribed number of integrals with m between 1 and $n-1$. For separable potential systems and for

cofactor-pair Newton systems the knowledge of 2 quadratic w.r.t. velocities, functionally independent integrals of motion and depending on all dynamical variables, implies existence of n integrals and the hamiltonian integrability. For triangular systems of Newtons equations $\ddot{q}_r = M_r(q_1, \ldots, q_r)$, $r = 1, \ldots, n$ it appears that knowledge of only 1 essential, quadratic w.r.t. velocities integral of motion implies existence of $n-1$ further integrals and makes possible sequential separability of equations of motion.

I shall discuss the relationship between the structure of equations of motion, the number of integrals and integrability for certain known classical systems of dynamical equations and for new classes of dynamical equations that we have found.

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Nonlocal symmetries, integrability of equations of pseudo-spherical type and the Virasoro algebra

Abstract: I will introduce a simplified version of the classical Krasil'shchik-Vinogradov geometric theory of nonlocal symmetries, [5], and I will present several applications, [1–8]. For example, [2, 4, 5], the theory can be used to find highly non-trivial explicit solutions and Darboux-like transforms to nonlinear equations such as the Kaup-Kupershmidt equation

$$q_t = q_{xxxxx} + 5\,q\,q_{xxx} + \frac{25}{2}\,q_x q_{xx} + 5\,q^2 q_x \ .$$

Moreover, I will argue that nonlocal symmetries can be used to *uncover* integrable equations, at least within the class of equations of pseudo-spherical type, [1, 4, 4]. I will also present classifications of nonlocal symmetries of integrable equations such as the Camassa-Holm equation

$$m + u_{xx} - u = 0 \qquad m_t + m_x\,u + 2\,m\,u_x = 0 \ ,$$

and I will show that the (centerless) Virasoro algebra appears as a Lie algebra of *nonlocal* symmetries, [4, 5, 2, 3, 4]. Finally, I will present a recent generalization of the Krasil'shchik-Vinogradov theory, [5], and I will show how it applies to the Degasperis-Procesi equation

$$m + u_{xx} - u = 0 \qquad m_t + m_x\,u + 3\,m\,u_x = 0 \ .$$

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Nonlocal equations, fluids and strings
(Poster)

Abstract: I summarize some recent work on *nonlocal* differential equations appearing in the theory of fluids and in some modern approaches to cosmology and string theory. Specifically, I present results on the existence and uniqueness of solutions to the so-called "modified" Camassa-Holm and Hunter-Saxton equations (which depend on inverses of second order differential operators), and also to the generalized euclidean bosonic string equation (which depends on an exponential of a second order differential operator). The first two equations appear formally in the theory of integrable equations admitting traveling wave solutions. The last equation is an euclidean version of an equation of interest for bosonic and supersymmetric string theory. Moreover, I also discuss the delicate issue of the initial value problem for nonlocal equations.

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Some approaches to partial differential equations with state-dependent delays

Abstract: Partial differential equations with state-dependent delays (SDD) are considered. Different types of bounded delays (discrete, distributed and mixed) are studied. We mention that the nonlinearities with discrete SDD (in contrast to constant or time-dependent cases) are not Lipschitz continuous on the

space of continuous in time functions - the classical space of initial data. We discuss different formulations of the problem based on different choices of the phase spaces and types of solutions. For the constructed dynamical systems we study the long-time asymptotic behavior and find conditions for the existence of compact global attractors [1], [2].

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Regular N-gon as a model of discrete gravitational system

Abstract: A system of N points, each having mass m, forming a planar regular polygon (N-gon), and a central mass M, are considered. The motion equations for a testing particle are given in different coordinates. To provide stability, N-gon must always rotate, including case zero central mass. The velocity of rotation is calculated. In a large distance from N-gon, potential is close to gravitational potential of homogenous disk or ring, as well as in case N tends to infinity. But, discrete character of modeling system leads to some new features relative continues case. Numerical modeling show that at fixed total mass of N-gon perturbation nonlinearly depends on N. 3N-1 stationary points (libration points) appear in a system. Studying of libration point behavior is a main result of this paper. It is shown, that the stationary solution (libration points) in considered system may be determined from algebraic equation of 5-th degree. Considered equation for determination libration points coordinates always have only one real root. In case small m / M coordinates of these points may be calculated as a generalization of classical gravitation 3-body problem. In present research, the dependence of libration points coordinates on mass and number of particles is studied. At large N, libration point coordinates close to a fixed value, slowly depends on mass ratio. On the other hand, at large m coordinates have a maximal value, which respect a case infinitesimal central mass. Positions of non-collinear libration points are slightly differ from n-gon described circle. Obtained results are discussed and compared with another publications. Area, where potential allows a linearization is described in small vicinity of N-gon. However, oscillations even in this small area become strongly nonlinear due to differential rotation of system.

In fact, approximation ring potential by N-gon is an example of a point-mass modeling. But specific of potential expansion gives an ability to study more complex (non-homogenous) systems on base of considered model.

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Infinite symmetries and infinite conservation laws

Abstract: We will consider variational partial differential equations possessing infinite symmetry algebras parametrized by arbitrary functions of independent, as well as dependent variables and their derivatives. We will give classification of infinite symmetries with arbitrary functions and corresponding local conservation laws. Specifically we will examine essential (integral) conservation laws which lead to non-vanishing conserved quantities, and study the role of boundary conditions for generation of essential conservation laws. We discuss classes and examples of variational PDE possessing infinite sets of symmetries and essential conservation laws with arbitrary functions of dependent variables and their derivatives. We also analyze boundary conditions for infinite conserved densities of known soliton equations, and compare them with boundary conditions giving rise to essential conservation laws related to infinite symmetries with arbitrary functions of independent and dependent variables.

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Lie point symmetries for an infinite set of PDEs appearing in the theory of turbulence

Abstract: We consider the infinite set of the so-called multi-point correlation equations (MPCE), which naturally arise in turbulence theory. It is our main aim to derive the admitted Lie point symmetries for the latter system and in turn derive invariant solutions thereof.

The starting point are the Navier-Stokes equations, which represent a system of PDEs and describe the flow of an incompressible viscous fluid. Employing the so-called Reynolds decomposition for the velocity U it may be re-written as a sum of the averaged ($\bar{U}$) and a fluctuation velocity (u), i.e. $U = \bar{U} + u$. Implementing this into the Navier-Stokes equations and taking the average, a transport equation for the mean velocity is derived, leading to a system of four equations for ten unknown functions. Hence, additional equations for the unknown moments are needed, which can also be derived in a successive manner and an infinite set of MPCE is obtained.

In the present context the advantage of this infinite set of equations is the possibility to separately investigate each of the n-th equation. As the coupling between the differential equations is simple enough, the Lie algorithm can directly be applied to each equation. Although it cannot be carried out for the infinite set of equations to the very end, at least some necessary conditions for the infinitesimals can be deduced.

Finally, we give several symmetries of the MPCE, which form a Lie (sub)algebra [1]. We give relevant applications such as homogeneous isotropic turbulence or the turbulent channel flow and determine the behaviour of the averaged velocity including higher order statistical moments [2].

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Dynamics of the tippe top

Abstract: The tippe top (TT) has the shape of a truncated sphere with a peg attached to the flat surface so that the centre of mass is shifted below the geometrical centre. When spun sufficiently fast on its spherical bottom the tippe top behaves in a counterintuitive manner, it turns upside down and continues spinning on the peg with raised centre of mass. We call this behaviour inversion.

The inversion takes place for the values of parameters $1 - \alpha < \gamma = I_1/I_3 < 1 + \alpha$, where $0 < \alpha < 1$ measures the eccentricity of the centre of mass, and for the vertical angular momentum $L_{\hat{z}}$ above the

threshold value

$$L_{\hat{z}} > \frac{\sqrt{mgR\alpha I_3}(1+\alpha)^2}{(1-\alpha)\sqrt{1+\alpha-\gamma}}.$$

The only rigorous mathematical argument proving the inversion is based on the LaSalle invariance priciple stating that if on a compact positively invariant set there exist a continuously differentiable function $V(x)$ such that $\dot{V}(x) < 0$ then all solutions go to the invariant subset of the set $\{x : \dot{V}(x) = 0\}$. This existential type statement says however nothing about the dynamics of the symmetry axis $\hat{\mathbf{3}}(t)$ of TT and how it approaches the inverted spinning state. The study of dynamics is a relatively virgin area of research.

In this talk I shall present an approach to study the dynamics based on deformation of integrals of motion of an integrable sub-case when the TT is rolling without sliding. This approach leads to one (nonintegrable) Main Equation for the inclination angle θ between vertical axis $\hat{z}$ and the symmetry axis $\hat{\mathbf{3}}$. In particular we show that we can take values for physical parameters such that the potential function $V(\cos\theta, D, \lambda)$ in the Main Equation becomes a rational function of $\cos\theta$, which is easier to analyse. We estimate quantities characterizing an inverting Tippe Top, such as the period of oscillation for $\theta(t)$ as it moves from a neighborhood of $\theta = 0$ to a neighborhood of $\theta = \pi$ during inversion.

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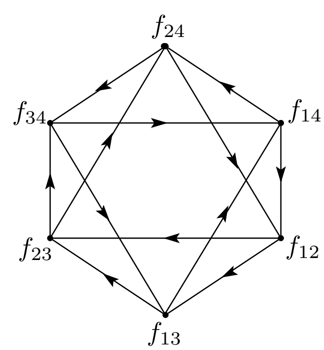
Exact triangles and integrability of rational maps

Abstract: The theory of category is often said a general nonsense in mathematics. Nevertheless we would like to say it is useful in physics, in the sense that it might characterize integrability of physical phenomena.

A large class of known integrable equations are in the KP hierarchy, which is derived from a single discrete equation,

$$a_{12}a_{34}f_{12}f_{34} + a_{13}a_{24}f_{13}f_{24} - a_{14}a_{23}f_{14}f_{23} = 0,$$

$$f_{ij} \in \mathbb{C}, \quad a_{ij} = -a_{ji} \in \mathbb{C},$$



called the Hirota-Miwa (HM) equation. We show that the solutions of HM eq. can be described by means of the theory of triangulated category (TC). In particular the singularity confinement (SC) of a map is associated to the localization of the TC. After the SC, iteration of the map is shown to generate algebraic varieties of periodic points of all periods, which are determined by the invariants of the map alone. A preliminary result of this work has been partially reported in [1].

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Degeneration of the Julia set of rational maps in integrable limit
(poster)

Abstract: The Julia set is the closure of repulsive periodic points of a map, which is fractal and characterizes non-integrability of the map. On the other hand periodic points of an integrable map form a variety for each period, which is determined by invariants of the map, hence is called invariant varieties of periodic points, or IVPP. We would like to explore the mechanism which provides such different nature of periodic points and discriminates integrable maps from non-integrable ones.

To study this problem analytically, we consider a map with a parameter a, which is integrable at $a = 0$, but non-integrable otherwise. The Julia set of the map in the complex space of initial points, moves around as a varies. In the small a limit, the Julia set must totally disappear from the complex space. We will show that some points of the set approach to IVPPs as a becomes small, but a large part degenerate and eventually form a variety consisting of indeterminate points in the integrable limit. A preliminary result of this work has been partially reported in [1].

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Schlesinger transformations for the second and third members of a third Painlevé hierarchy

Abstract: In this article, we derive the Schlesinger transformations for second and third members of a third Painlevé hierarchy. The procedure involves formulating a Riemann-Hilbert problem for a transformation matrix which transforms the solution of the linear problem but leaves the associated monodromy data the same. Using the Schlesinger transformations, we obtained the corresponding Bäcklund transformations for each of the considered equations. Furthermore we discussed some special solutions of the second and third members of the third Painlevé hierarchy.

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Multiple-pole solutions of the nonlinear Schrödinger equation

Abstract: We will start by a short resume on an operator theoretic approach to the Nonlinear Schrödinger equation, with the aim to motivate a solution formula which gives a unified access to the multiple-pole solutions. The main result is a complete asymptotic description of these solutions, which was so far only achieved for cases of low complexity by Olmedilla. After an overview on the geometric an algebraic ingredients of the proof, we will conclude by a discussion of cases of higher degeneracy and a comparison with the situation for the KdV equation.

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Peculiarities of nonlinear dynamics of radiation damage evolution: phase portraits and bifurcations

Abstract: It is considered the system of nonlinear differential equations that describes evolution of radiation damage of materials under irradiation. The variables of the system which describe material under irradiation are densities of different radiation defects, defect characteristics and temperature of the material. Nonlinear feedbacks between different elements of the model take into account. The qualitative analysis of this non-linear dynamical system is carried out. All critical points and limit cycles of the system, their stability and type are found. A phase portrait of the system is constructed. The change of the phase portrait depending on the system parameters is investigated. All phase portraits that have qualitative differences are obtained. The bifurcation surfaces are obtained in the space of the parameters of the system (environment temperature, defect generation rate and so on). It is found the peculiarities of the appearance and disappearance of the limit cycle and the peculiarities of the development of the self-oscillation in the system. It is shown there are two ways of the self-oscillation development. In the first casethe limit cycle appears when stable spiral transform into the unstable one through the center. In the second case the limit cycle is formed by separatrices of the saddle. The separatrices coincide at bifurcation. It is given physical interpretation of obtained results. Frequency of the self-oscillation and conditions of their development is rated. Results of the experiments and simulation are compared.

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Multidimensional dispersionless integrable hierarchies: recursion operators and all that

Abstract: It is well known that the recursion operators provide a standard tool for the construction of the integrable hierarchy associated with a given integrable system. We present here a new explicit construction of recursion operators for integrable multidimensional dispersionless systems that admit a Lax representation in terms of commuting vector fields of certain special form. The efficiency of our approach is demonstrated on several examples.

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Asymptotic expansion for the Korteweg-de Vries-Burgers equation

Abstract: It is common knowledge that many physical problems (such as non-linear shallow-water waves and wave motion in plasmas) can be described by the Korteweg-de Vries (KdV) equation, which possesses certain special solutions, known as solitary waves or solitons. As a marriage of the KdV equation and the classical Burgers (KdVB) equation, the Korteweg-de Vries-Burgers (KdVB) equation is a mathematical model of waves on shallow water surfaces in the presence of viscous dissipation.

Asymptotic analysis is a method of describing limiting behavior and is a key tool for exploring the differential equations which arise in the mathematical modeling of real-world phenomena. By using variable transformations, the asymptotic expansion of the KdVB equation is presented in this paper. The asymptotic expansion may provide a good gauge on the validation of the corresponding numerical scheme.

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Discretizations of two-dimensional Toda lattice: symmetries and integrals

Abstract: In the two-dimensional case symmetries and integrals of the infinite Toda lattice are expressed in terms of non-local variables. For the finite reductions of the infinite lattice, corresponding to simple Lie algebras, symmetries and integrals appear to be local and therefore provide Darboux integrability of these finite reductions. I'll introduce non-local variables for discrete Toda lattices in order to obtain symmetries and integrals in (semi)discrete case.

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Commuting phase flows in the tropical periodic Toda lattice

Abstract: The Toda lattice is one of the most important examples of classical integrable systems that appears in many topics in modern mathematics and physics. Among the growing interest in *tropical mathematics* which combines piecewise linear geometry and min-plus algebra, a tropical counterpart of periodic Toda lattice is attracting attentions [1]. It is a dynamical system derived from a time-discretized version of the Toda lattice with a periodic boundary condition through a limiting procedure called *tropicalization*. Its time evolution is described by a piecewise linear equation known as the ultradiscrete periodic Toda equation. The initial value problem of this equation was solved in terms of tropical Riemann theta functions, but no phase flows of the system was known except that for the original time evolution.

In this work we present a new combinatorial description of this dynamical system with a representation by two-colored strips which enables us to construct a family of commuting phase flows. The commutativity is based on Yang-Baxter relations satisfied by the tropicalization of a birational intertwining map [2]. The construction of such flows will be significant for the future studies of various tropical/ultradiscrete dynamical systems, because they can determine the global structure of their iso-level sets as in the case of Liouville-Arnold's theorem for integrable Hamiltonian systems.

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The extended Skyrme-Faddeev model and its topological solitons

Abstract: In this talk, we present static soliton solutions with non-zero Hopf topological charges, for a field theory that has found interesting applications in many areas of Physics. It is a $(3+1)$-dimensional Lorentz invariant field theory for a triplet of scalar fields $\vec{n}$, living on the two-sphere S^2, $\vec{n}^2 = 1$, and defined by the Lagrangian density[1]:

$$\mathcal{L} = M^2 \partial_\mu \vec{n} \cdot \partial^\mu \vec{n} - \frac{1}{e^2} \left(\partial_\mu \vec{n} \wedge \partial_\nu \vec{n}\right)^2 + \frac{\beta}{2} \left(\partial_\mu \vec{n} \cdot \partial^\mu \vec{n}\right)^2 , \tag{9}$$

where the coupling constants e^2 and β are dimensionless, and M has dimension of mass. The first two terms correspond to the so-called Skyrme-Faddeev model, as the generalization to $3+1$ dimensions of the CP^1 model in $2+1$ dimensions. This model is called the extended Skyrme-Faddeev model. In a Minkowski space-time the static Hamiltonian associated to (9) is

$$\mathcal{H}_{\text{static}} = M^2 \partial_i \vec{n} \cdot \partial_i \vec{n} + \frac{1}{e^2} \left(\partial_i \vec{n} \wedge \partial_j \vec{n}\right)^2 - \frac{\beta}{2} \left(\partial_i \vec{n} \cdot \partial_i \vec{n}\right)^2 , \tag{10}$$

with $i, j = 1, 2, 3$. Therefore, it is positive definite for $M^2 > 0$, $e^2 > 0$ and $\beta < 0$. By an axially symmetric ansatz based on toroidal coordinates, we construct numerical solutions with Hopf charge up to four, and calculate their analytical behavior in some limiting cases. The solutions present an interesting behavior under the changes of a special combination of the coupling constants of the quartic terms. Their energies and sizes tend to zero as that combination approaches a particular special value. In addition, the model presents an integrable sector with an infinite number of local conserved currents which apparently are not related to symmetries of the action. In the intersection of those two special sectors the theory possesses exact vortex solutions (static and time dependent), which were constructed in our previous researches. It is believed that such model describes some aspects of the low energy limit of the pure SU(2) Yang-Mills theory, and our results may be important in identifying important structures in that strong coupling regime.

This contribution will be based upon [2].

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Discrete integrable equations over finite fields

Abstract: Discrete integrable equations over finite fields are investigated. Firstly we treate discrete Painelevé equations over finite fields. As concrete examples, we consider the discrete Painlevé II equations ($\mathrm{dP_{II}}$ and $\mathrm{qP_{II}}$ equations). They are well defined by extending the domain according to the theory of the spaces of initial conditions. Then we treat them over local fields $\mathbb{Q}_p$ and observe that they have *almost good reduction* to the finite fields $\mathbb{F}_p$. We can use this property, which seems an arithmetic analogue of the singularity confinement, to avoid the indeterminacy of the equations over finite fields and to obtain the special solutions from those defined originally over the fields of characteristic zero. Then we consider soliton equations. The indeterminacy of the equation is resolved by using a rational function field instead of the finite field itself. The main discussion concerns a generalized discrete KdV equation related to a Yang-Baxter map. Explicit forms of soliton solutions and their periods are obtained.

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Universal formulae in Lie algebras and Chern-Simons theory

Abstract: In 1990s Vogel introduced an interesting parametrisation of simple Lie algebras by 3 parameters defined up to common multiple and permutations. Numerical characteristic of Lie algebra is universal if it can be expressed rationally in terms of Vogel's parameters (example - dimension of Lie algebra). I will present new universal formulae for certain Casimir eigenvalues as well as for some quantities in Chern-Simons theory on a 3D sphere, found in a joint work with Mkrtchyan and Sergeev.

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Dual pairs of momentum maps in fluid dynamics

Abstract: A pair of Poisson mappings defined on a symplectic manifold and taking values in two Poisson manifolds build a dual pair if their kernels are symplectic orthogonal complements of one another. Nice examples are provided by pairs of equivariant momentum maps for two commuting Hamiltonian actions on a symplectic manifold. We will discuss such dual pairs related to fluid dynamical settings: Euler's ideal fluid motion, multi-dimensional Camassa-Holm equation, non-abelian fluid, and free boundary incompressible fluid. In all these cases the acting groups are (extensions of) groups of diffeomorphisms. Dual pairs can explain geometrically some fluid dynamical features like nature of Clebsch variables, singular point vortex configurations in the plane, singular solution ansatz (filaments, sheets), and conservation laws arising from particle relabeling along the singular solutions.

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The ultradiscrete sine-Gordon equation and its exotic soliton interactions

Abstract: Soliton solutions to the ultradiscrete sine-Gordon equation that exhibit fission and fusion-type interactions were discovered more than 13 years ago, but an explanation of their exotic properties has been lacking ever since. In this talk I will describe a general framework for treating these solitons and their interactions, in terms of so-called "oistons" which are in fact part of the solutions to an alternate (i.e. non-Box & Ball) ultradiscrete KdV equation, related to the ultradiscrete sine-Gordon equation by a remarkable Miura transformation. This is joint work with B. Grammaticos and A. Ramani.

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Homogeneous almost complex- and related structures in dim 6

(joint work with B.Kruglikov and D.Alekseevsky)

Abstract: We classify homogeneous spaces $M = G/H$ of real dimension 6 with semi-simple H and G-invariant almost complex structure J by representation theory. These turn out to consist of a short discrete list of known spaces and proper homogeneous spaces, as well as several families of (solvable) Lie groups equipped with left invariant J. We investigate existence of almost-symplectic forms and hermitian triplets on these spaces, and their integrable versions such as symplectic and Kähler structures. We also remark on the significance of this work to the question of sub-maximally symmetric non-degenerate almost complex structures.

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Reduction groups and related integrable difference systems of NLS type

Abstract: Elementary Darboux transformations along with the corresponding Bäcklund transformations are constructed for some of the Lax operators classified using the reduction group method. Subsequently, the matrices defining the Darboux transformations are employed in the construction of a discrete Lax pair and the derivation of some novel integrable systems of difference equations. For these systems, generalized symmetries are presented and the initial value problem is discussed.

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On the first integrals of two nonlinear oscillator equations

Abstract: The aim of this work is to construct first integrals of the following nonlinear oscillator equations:

$$\ddot{x} - \frac{kx\dot{x}^2}{(1+kx^2)} + \frac{\alpha^2 x}{(1+kx^2)} = 0 \ ,$$

$$\ddot{x} + \frac{kx\dot{x}^2}{(1+kx^2)} + \frac{\alpha^2 x}{(1+kx^2)^3} = 0.$$

The $\lambda-$symmetry and Prelle-Singer method is applied to these equations. The results were combined with those gained by other methods.

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Random attractor of the 3D stochastic convective Brinkman-Forchheimer equations

Abstract: For the three-dimensional stochastic convective Brinkman-Forchheimer equations served as a model of the incompressible fluid flow in a saturated porous media on a bounded domain with multiplicative white noise,

$$\frac{\partial u}{\partial t} - \nu \Delta u + (u \cdot \nabla) u + \alpha u + \beta |u|^2 u + \nabla p = g(x) + \sigma u \circ \frac{dW}{dt},$$
$$\operatorname{div} u = 0, \quad x \in \Omega,\ t > 0,$$

and with the homogeneous Dirichlet boundary condition, where the Darcy coefficient α and the Forchheimer coefficient β are positive constants, the existence of a random attractor for the semiflow of weak solutions is proved through pullback absorbing property and pullback asymptotic compactness with sharp uniform estimates. Then we prsesent the upper semicontinuity (robustness) of the random attractors when the Darcy coefficient converges to zero.

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A new statistical scaling group and its impact on decaying turbulence

Abstract: Classical decay laws of isotropic turbulence usually derived from the von Kármán Howarth equation are essentially based on two paradigms: First, scaling symmetries of space and time, both tracing back to the Navier-Stokes equations in the limit of large Reynolds numbers (or $r \gg \eta$), give rise to a temporal power law decay for the turbulent kinetic energy and at the same time an algebraic growth of the integral length scale at an exponent that is uniquely coupled to the latter energy decay. Second, global invariants such as Birkhoff's or Loitsyansky's integrals determine the exponent of both power laws. We presently show, that this class of decay laws may be considerably extended considering the entire set of multi-point correlation equations which admit a much wider class of symmetries. It was recently shown, that this new symmetries are of paramount importance e.g. in deriving the logarithmic law of the wall being an analytic solution of the multi-point equations. For the present case it is particularly an additional scaling group, which we call statistical scaling group, that gives rise to two additional families of "canonical" decay laws including those with an exponential characteristics for both the kinetic energy and the integral length scale. Finally, a second rather generic group admitted by all linear differential equations corresponding to the superposition principle induces an infinite set of scaling laws of rather complex form that may match rather generic initial conditions. All scaling laws are analyzed in the light of the above mentioned integral invariants which have been further extended in the present contribution to an exponential type invariant.
