

Shape Inspection by Vision in Production

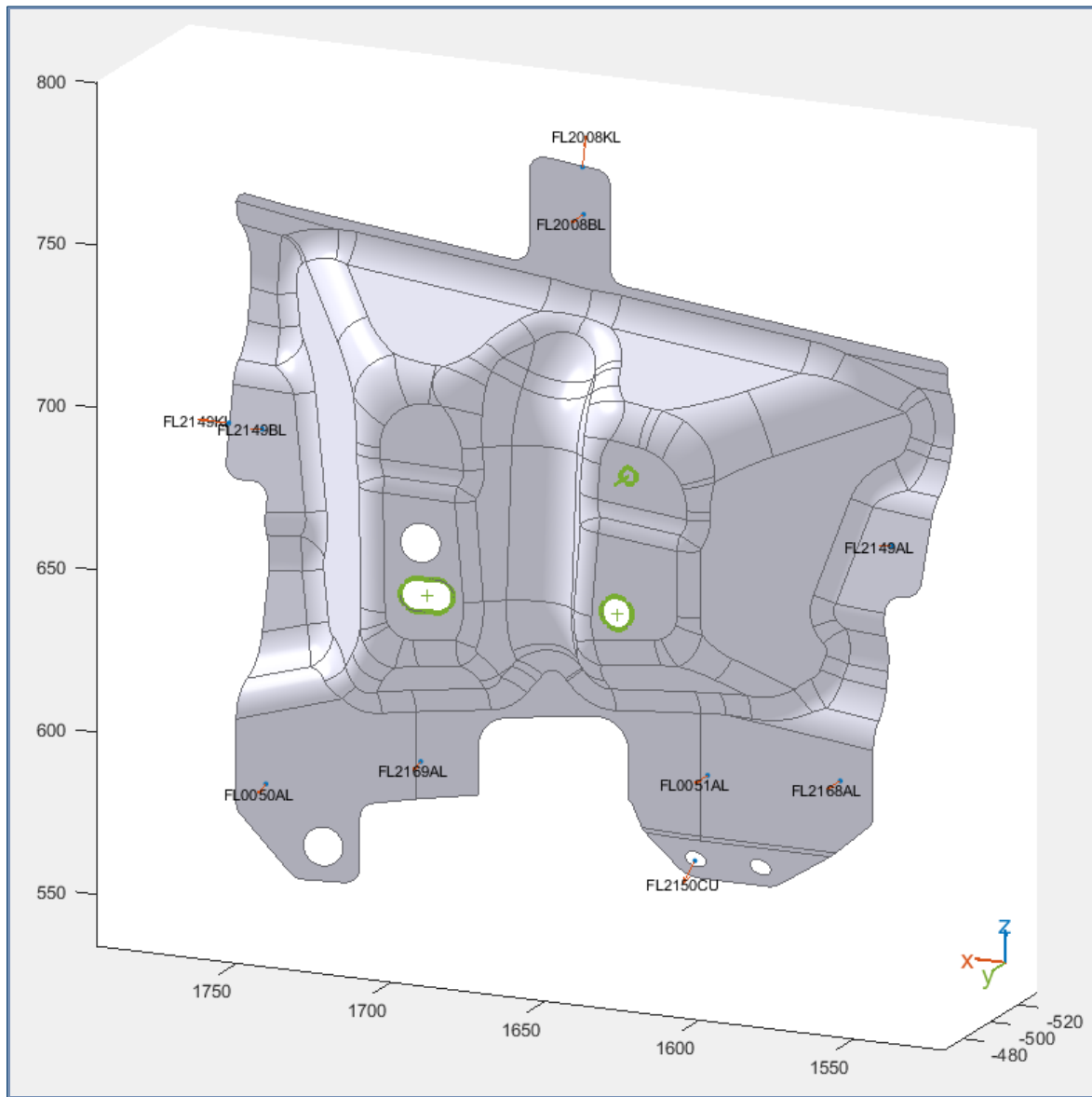
Per Bergström

Luleå University of Technology, 23 November 2016

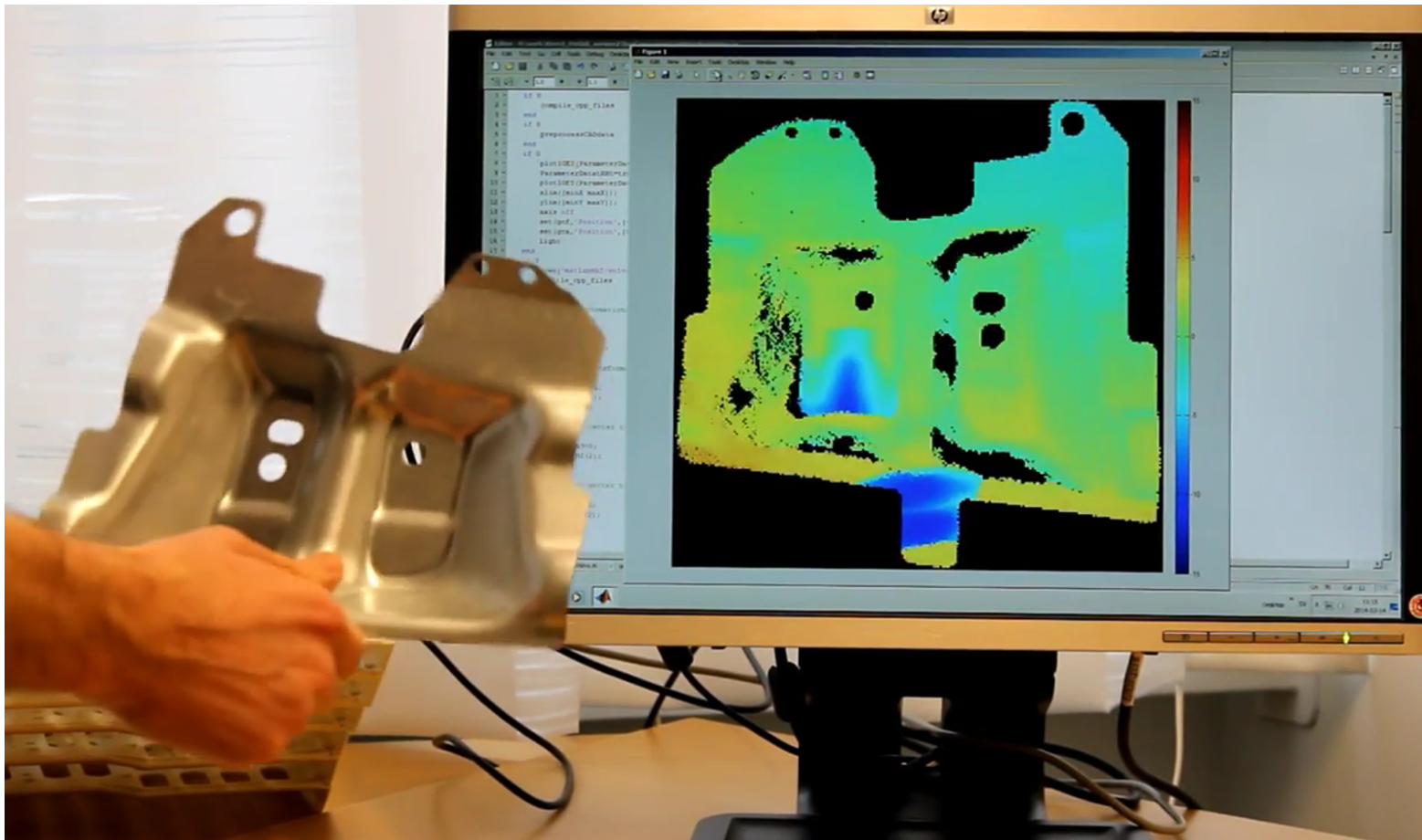






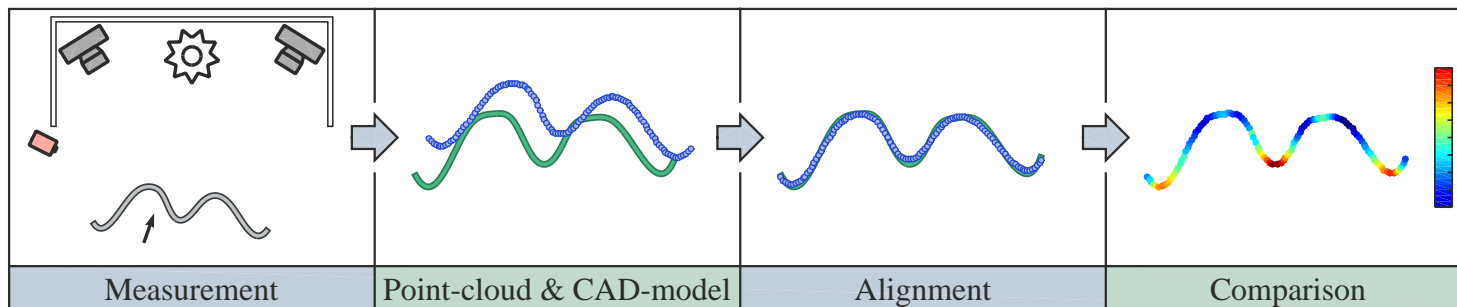
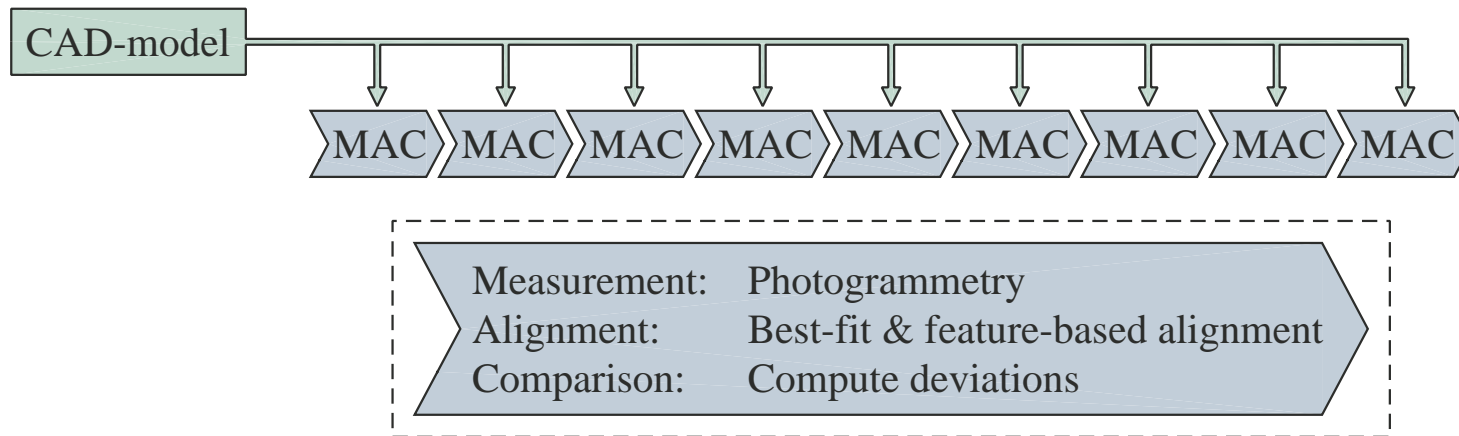


Video



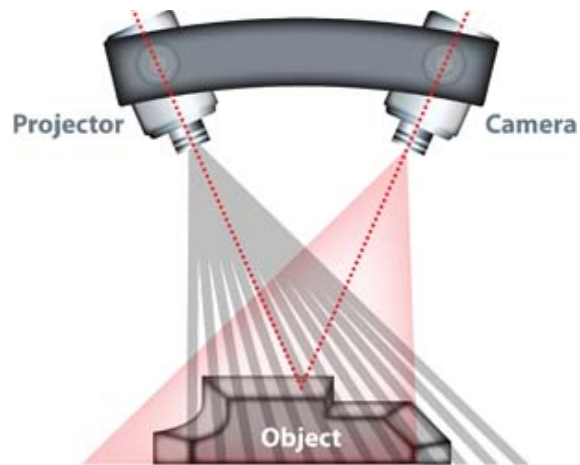
https://youtu.be/lm7_mwpOk0E

Repeated shape inspection

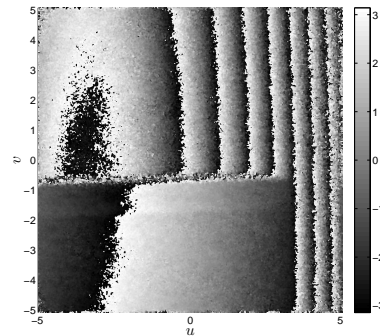
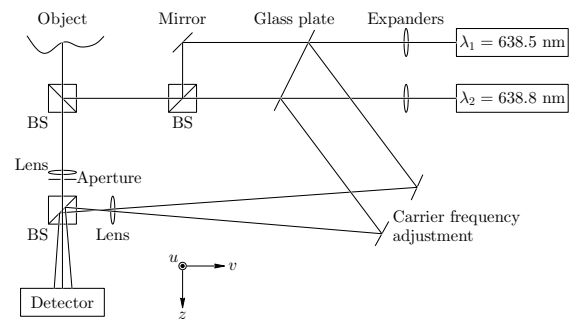


Shape measurement methods

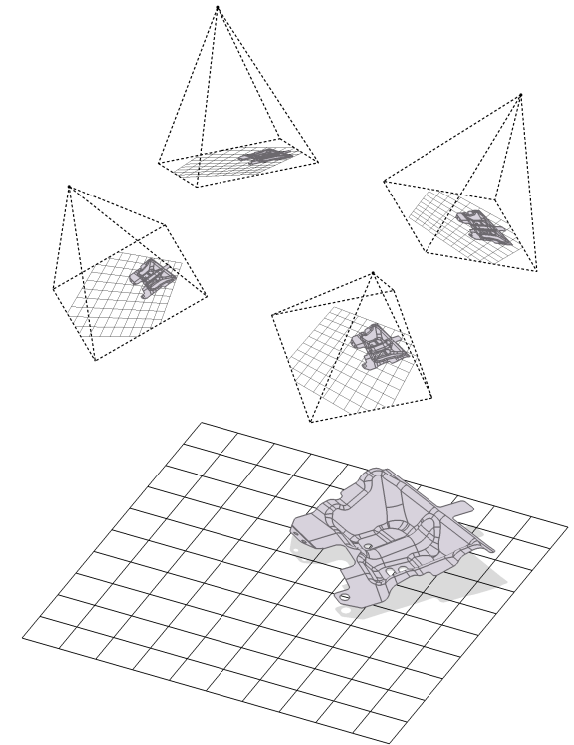
Projection of pattern



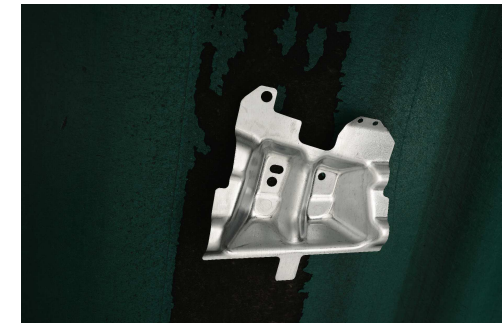
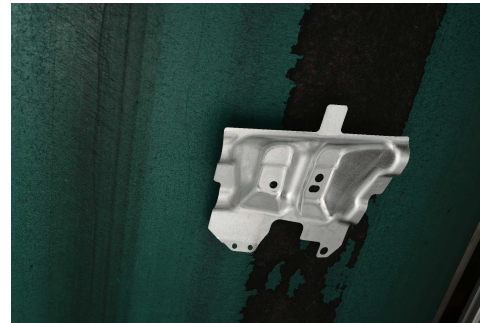
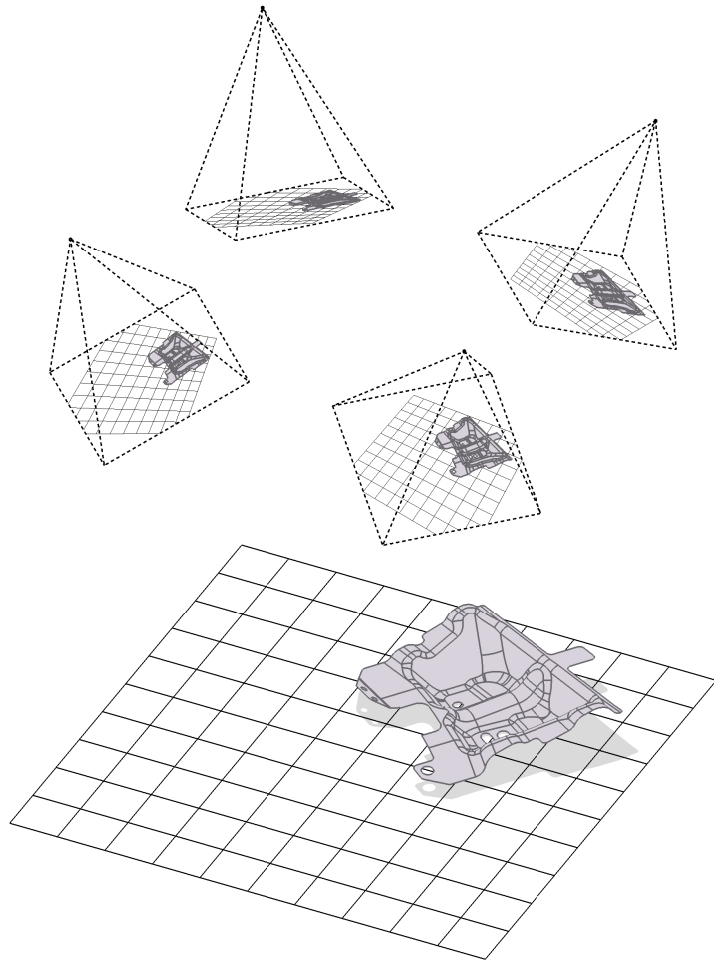
Holography



Photogrammetry



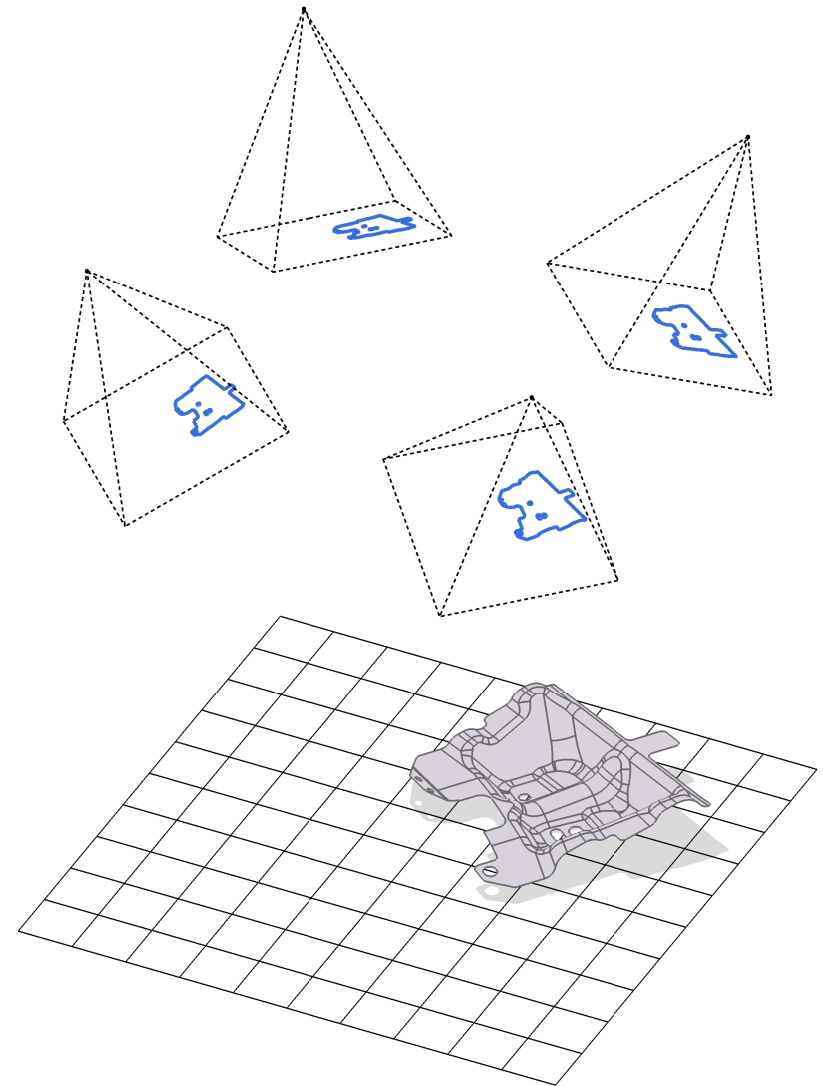
Photogrammetry



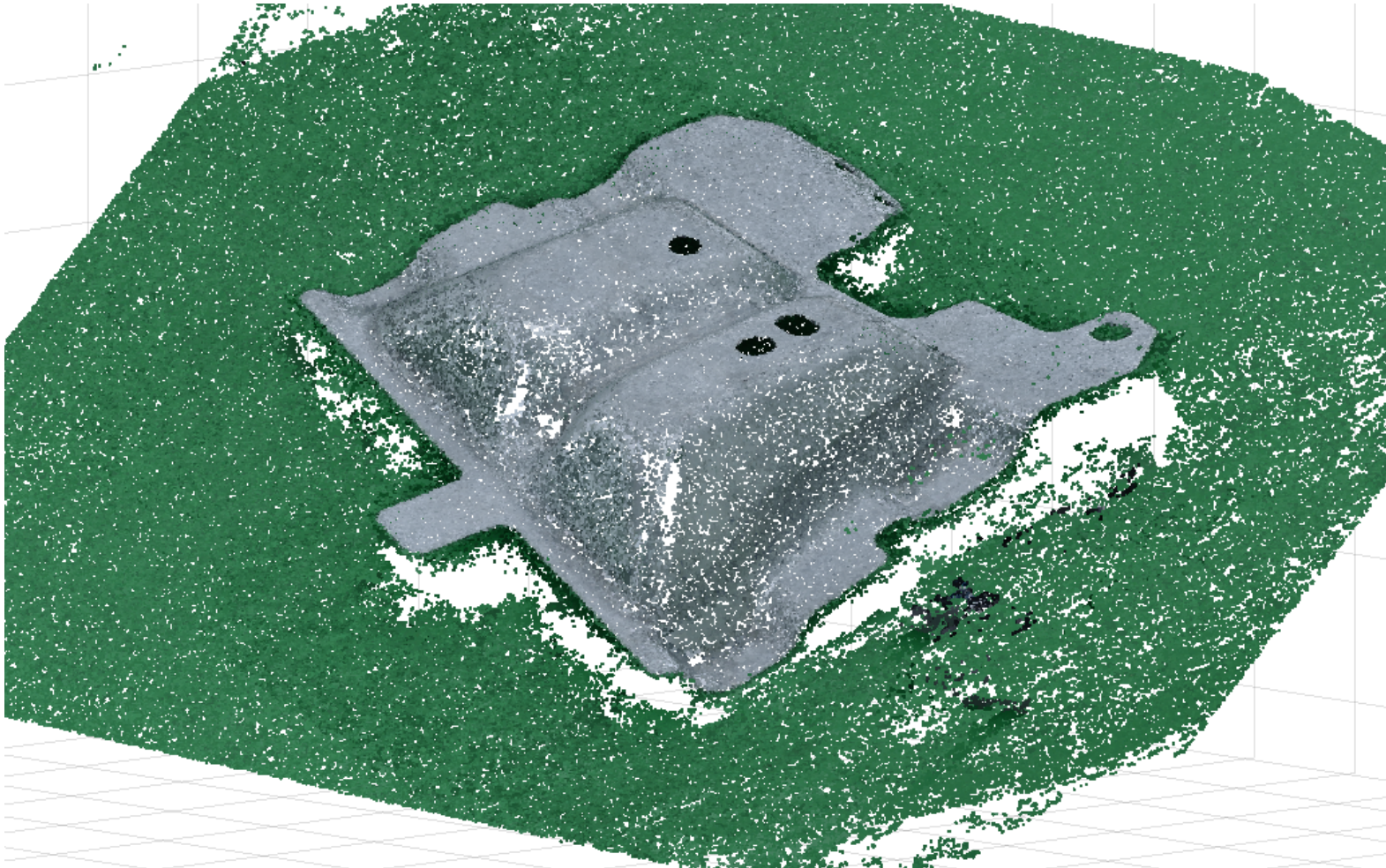
- Identification of homologous points

Photogrammetry




- Identification of homologous points
 - Edges
 - Surface texture
- No projected pattern is required
 - Diffuse illumination
 - Avoiding specular reflections



Point cloud

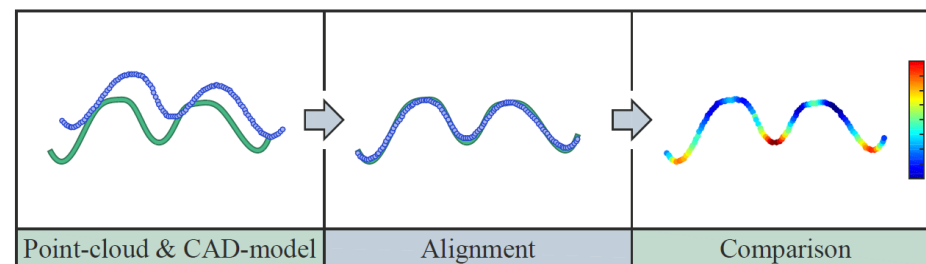
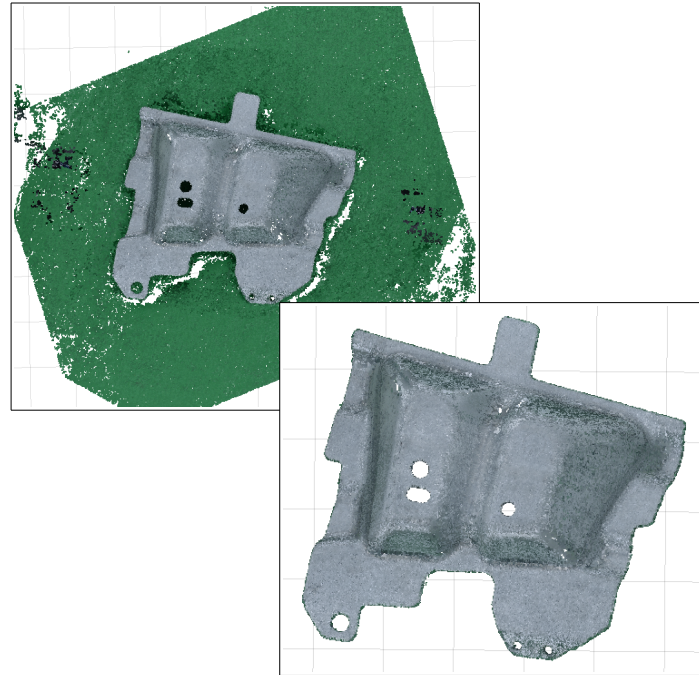


Photogrammetry

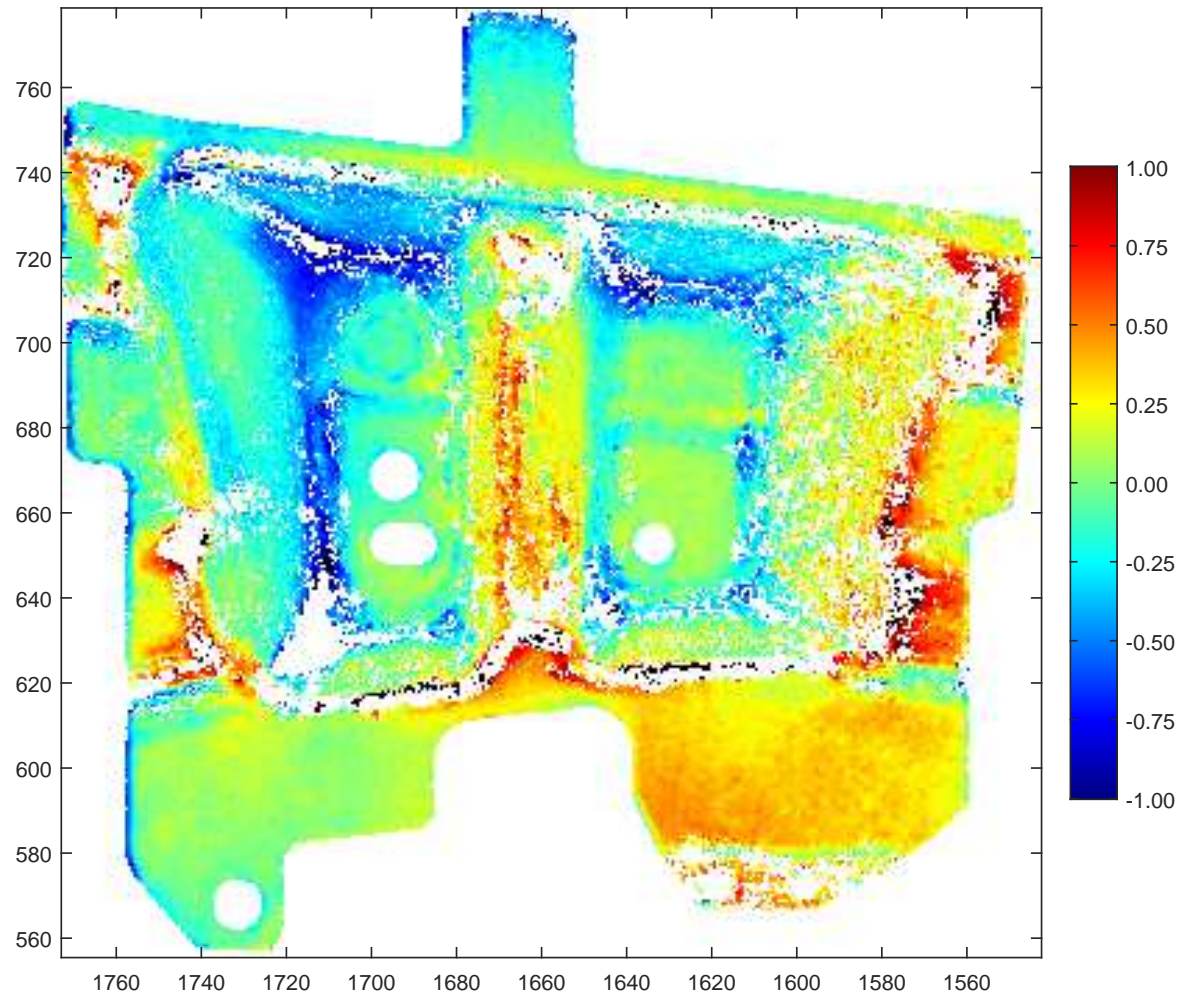
-  Enables shape measurements on moving objects (~ 1 m/s)
-  High spatial resolution
-  Long computational time

Outline of analysis

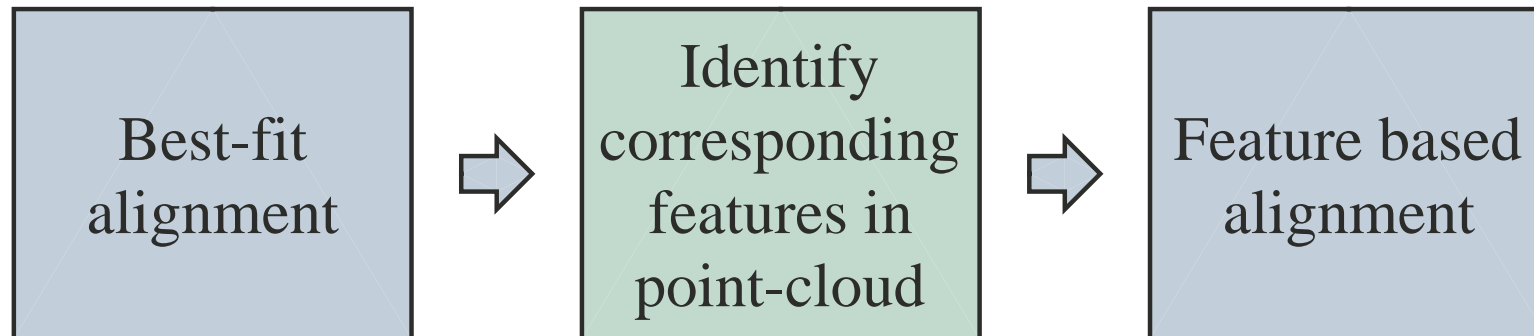
1. Background removal
2. Alignment
3. Comparison



Result: deviations in mm



Alignment process



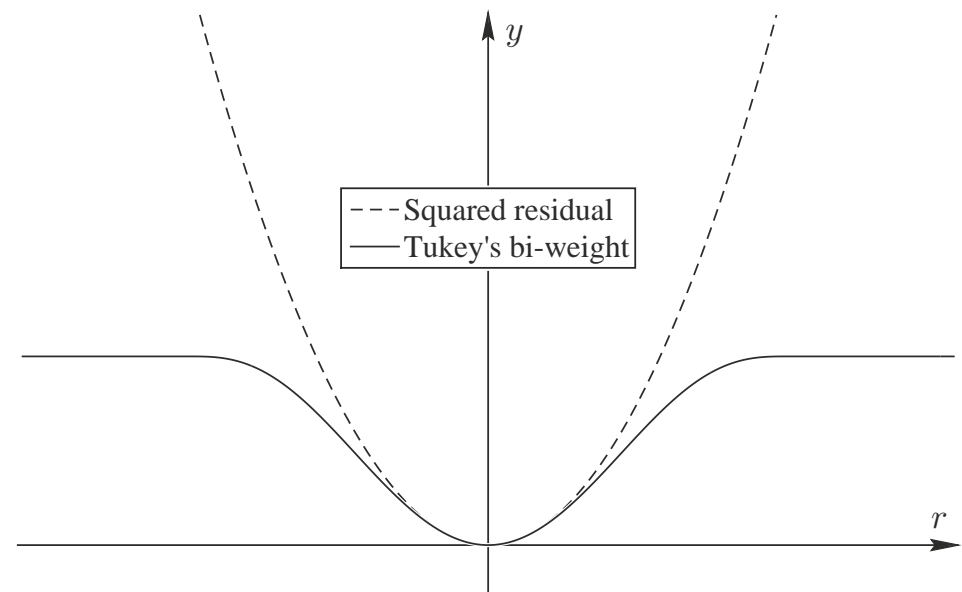
Robust estimation

Estimation of rigid body transformation

$$\min_{\mathbf{R}, \mathbf{t}} \sum_{i=1}^N \varrho(d(\mathbf{R}\mathbf{p}_i + \mathbf{t}, S)),$$

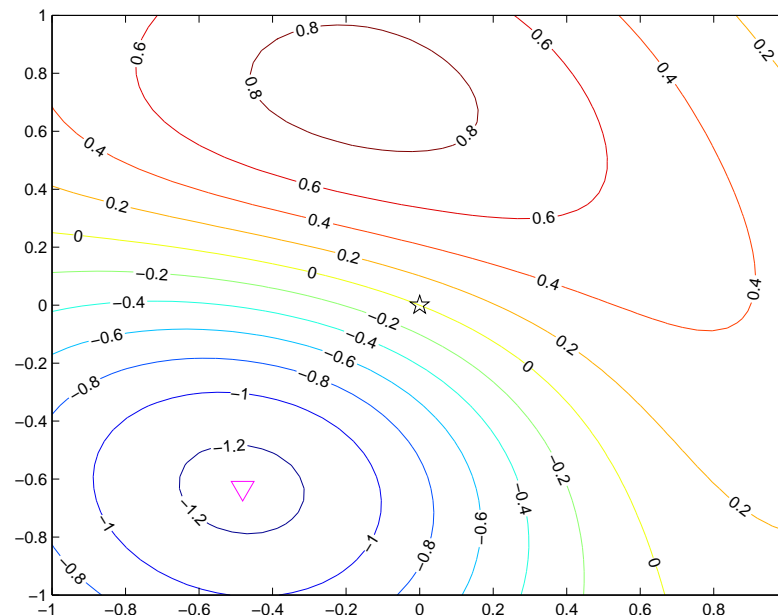
$d(\mathbf{p}, S)$ distance operator, ϱ robust criterion function

$$\varrho_{\text{Tu}}(r) = \begin{cases} \frac{\kappa_{\text{Tu}}^2}{6} \left\{ 1 - \left(1 - \frac{r^2}{\kappa_{\text{Tu}}^2} \right)^3 \right\} & \text{if } |r| \leq \kappa_{\text{Tu}}, \\ \frac{\kappa_{\text{Tu}}^2}{6} & \text{if } |r| > \kappa_{\text{Tu}} \end{cases}$$



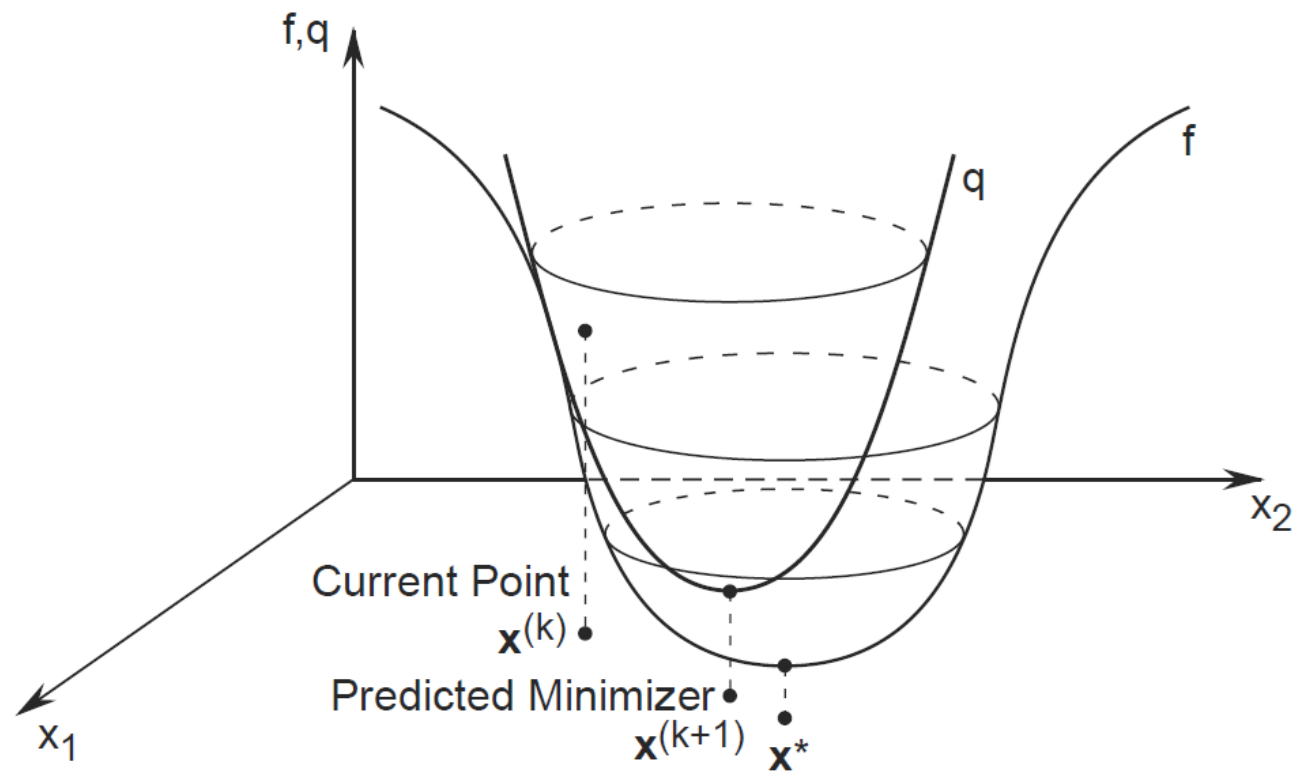
Optimization

- Finding the maximum/minimum of an objective function
 - Constrained
 - Unconstrained



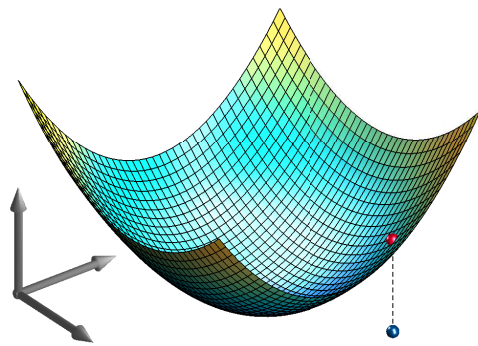
Newton's method

- Quadratic approximation of objective function at current point

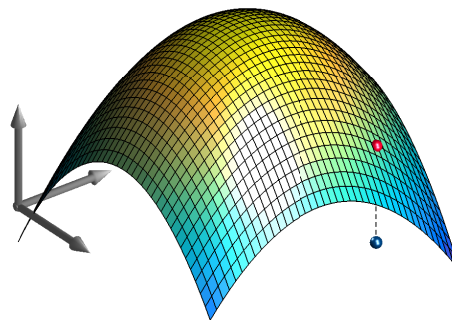


Newton's method

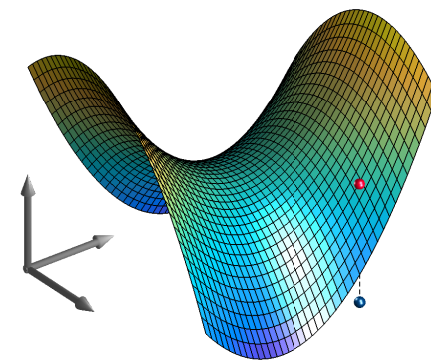
- Advantage if Hessian is positive definite
- Linesearch procedure
- Fast asymptotic convergence



Positive definite



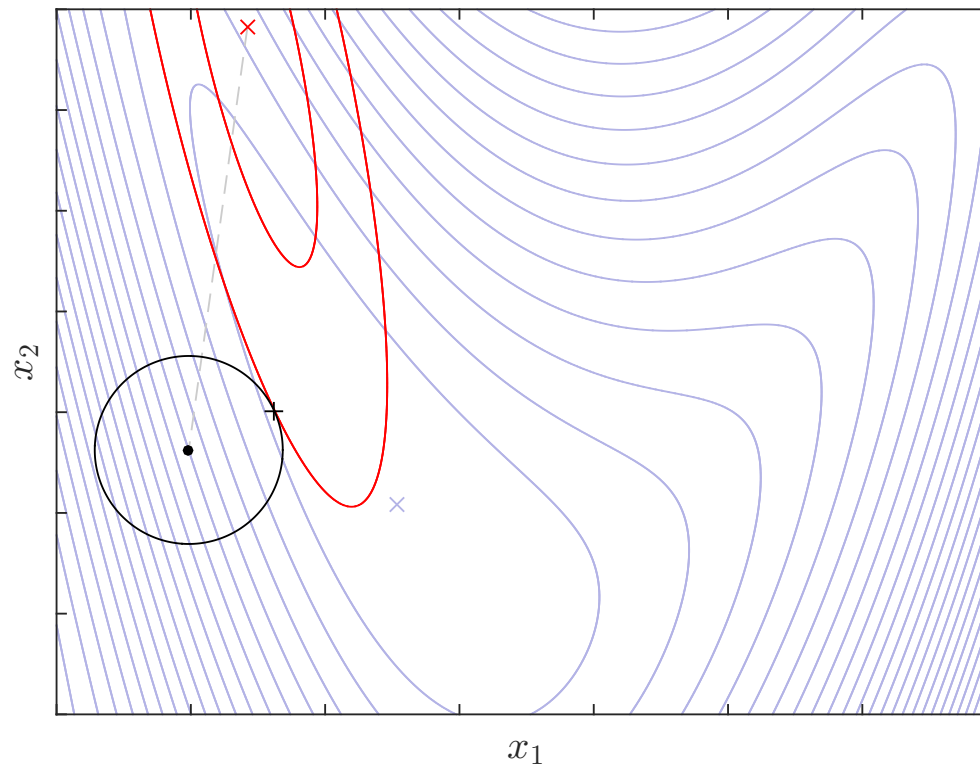
Negative definite



Indefinite

Trust-region method

- Model function (quadratic)
- Restricted to a region in the domain



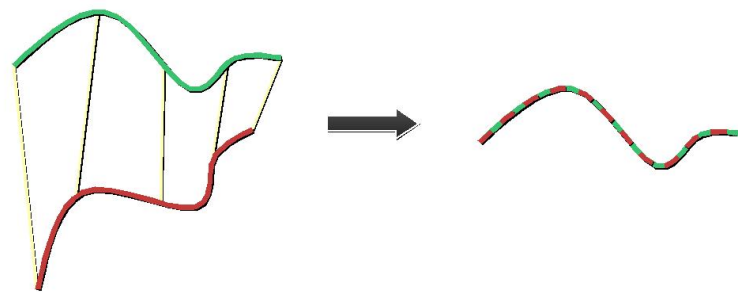
ICP-algorithm, estimation of \mathbf{R} , \mathbf{t}

repeat

Find closest points

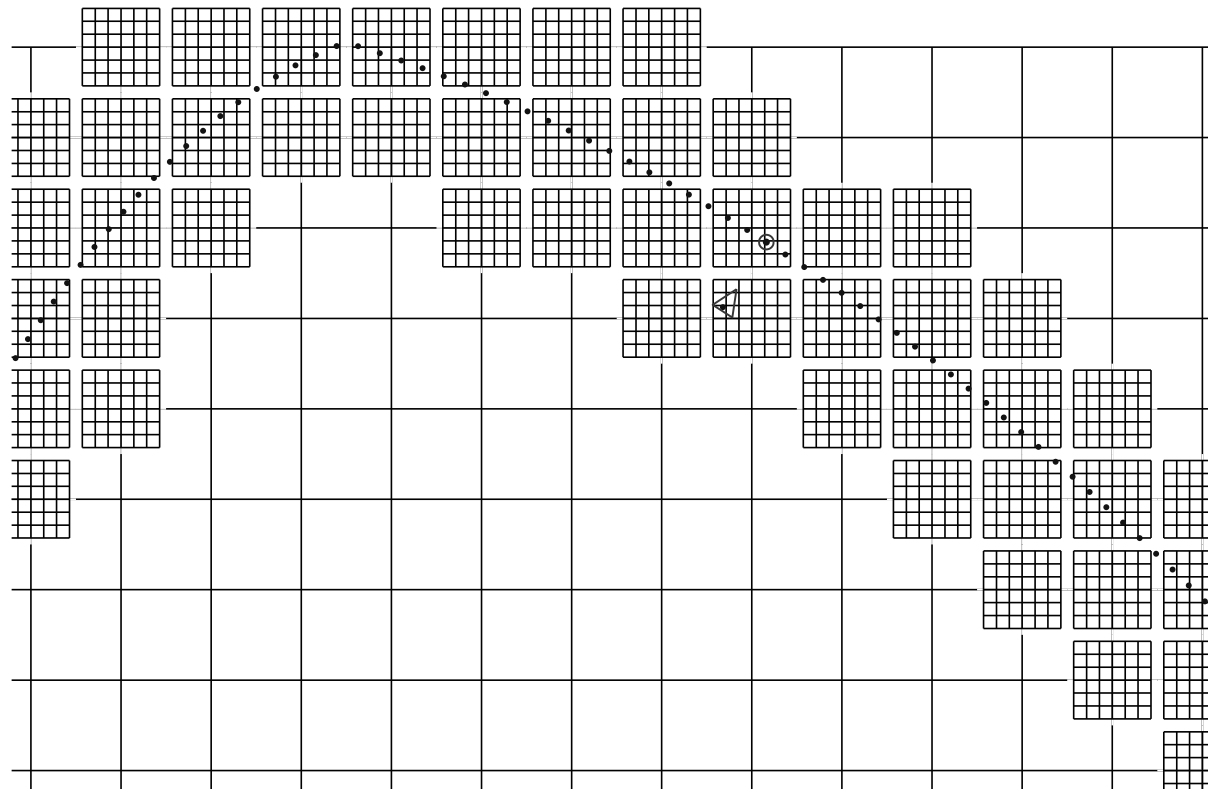
Find an appropriate update of the rigid body transformation

until convergence



- Point-to-point distance minimization
- Point-to-plane distance minimization
- Pottmann, H. et al., “Geometry and Convergence Analysis of Algorithms for Registration of 3D Shapes”, International Journal of Computer Vision (2006) 67: 277–296

Fast closest point search



Pre-processed CAD-model

- Bergström, P., Edlund, O. & Söderkvist, I., “Repeated surface registration for on-line use”, The International Journal of Advanced Manufacturing Technology (2011) 54: 677–689

Robust ICP-algorithm

Point-to-point distance minimization

repeat

Find closest points

Find an appropriate update of the rigid body transformation

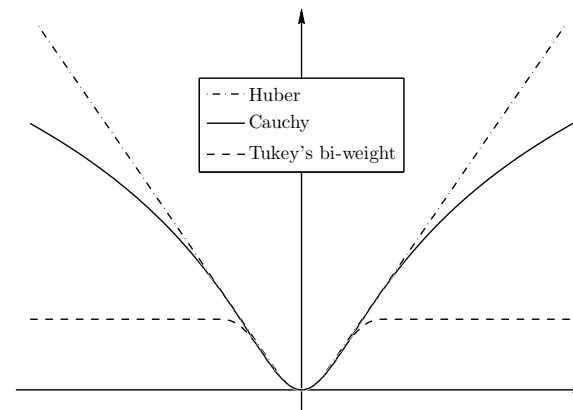
until convergence

- Bergström, P. & Edlund, O., “Robust registration of point sets using iteratively reweighted least squares”, Computational Optimization and Applications (2014) 58: 543–561

Criterion function ϱ

Let $\psi(r) = \varrho'(r)$. A criterion function ϱ belongs to a set \mathcal{Q} if the following conditions are fulfilled:

1. $\varrho(r)$ is an even function and \mathbf{C}^1 -continuous on \mathbb{R} where $\varrho(0) = 0$,
2. $\varrho(r)$ is monotonically increasing on $[0, \infty)$,
3. $\psi(r)/r$ is monotonically decreasing and bounded above on $(0, \infty)$.



A weight function w is defined as

$$w(r) = \begin{cases} \frac{\psi(r)}{r} & \text{if } r \neq 0 \\ \lim_{r \rightarrow 0} \frac{\psi(r)}{r} = \psi'(0) & \text{if } r = 0 \end{cases}$$

A quadratic approximation to $\varrho(s)$ about $s = r$ is

$$\eta(s, r) = \begin{cases} \varrho(r) - \frac{\psi(r)}{2}r + \frac{\psi(r)}{2r}s^2 & \text{if } r \neq 0 \\ \frac{\psi'(0)}{2}s^2 & \text{if } r = 0 \end{cases}$$

$$\eta(s, r) = \varrho(r), \quad \frac{\partial}{\partial s}\eta(s, r) = \psi(r) \text{ at } s = r,$$

$$\frac{\partial}{\partial s}\eta(s, r) = w(r)s \quad \forall s, r \in \mathbb{R}$$

Lemma

For all criterion functions $\varrho \in \mathcal{Q}$ the corresponding quadratic approximation $\eta(s, r)$ fulfills $\eta(s, r) \geq \varrho(s) \quad \forall s, r \in \mathbb{R}$.

A proof is given in

- Bergström, P. & Edlund, O., “Robust registration of point sets using iteratively reweighted least squares”, Computational Optimization and Applications (2014) 58: 543–561

Quadratic approximation

Finding a rigid body transformation

$$[\mathbf{R}^*, \mathbf{t}^*] = \arg \min_{\mathbf{R}, \mathbf{t}} g(\mathbf{R}, \mathbf{t})$$

$$g(\mathbf{R}, \mathbf{t}) = \sum_{i=1}^N w_i \|\mathbf{R}\mathbf{p}_i + \mathbf{t} - \mathbf{y}_i\|_2^2$$

$$g(\mathbf{R}, \mathbf{t}) =$$

$$\sum_{i=1}^N w_i \|\mathbf{p}_i - \bar{\mathbf{p}}\|_2^2 + \sum_{i=1}^N w_i \|\mathbf{y}_i - \bar{\mathbf{y}}\|_2^2 + \hat{w} \|\mathbf{R}\bar{\mathbf{p}} + \mathbf{t} - \bar{\mathbf{y}}\|_2^2 - 2\hat{w} \text{trace}(\mathbf{R}\mathbf{C})$$

where

$$\hat{w} = \sum_{i=1}^N w_i, \quad \bar{\mathbf{p}} = \frac{1}{\hat{w}} \sum_{i=1}^N w_i \mathbf{p}_i, \quad \bar{\mathbf{y}} = \frac{1}{\hat{w}} \sum_{i=1}^N w_i \mathbf{y}_i$$

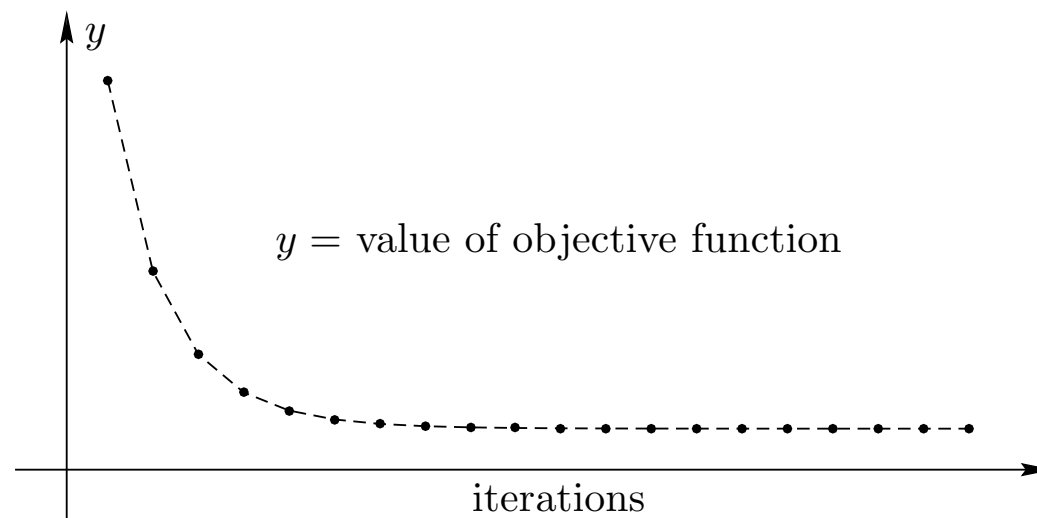
$$\mathbf{C} = \frac{1}{\hat{w}} \sum_{i=1}^N [w_i \mathbf{p}_i \mathbf{y}_i^T] - \bar{\mathbf{p}} \bar{\mathbf{y}}^T \in \mathbb{R}^{3 \times 3}$$

Properties of the un-weighted rigid body transformation problem

- Söderkvist, I., “Perturbation Analysis of the Orthogonal Procrustes Problem”, BIT (1993) 33: 687–694

Convergence

- Convergence of the objective function
- Optimality condition eventually satisfied



Robust ICP-algorithm

Point-to-plane distance minimization, trust region approach

Finding a rigid body transformation using

$$v(\mathbf{z}) = \sum_{i=1}^N \varrho\left(\mathbf{n}_i^T (\mathbf{R}(\mathbf{z})\mathbf{p}_i + \mathbf{t}(\mathbf{z}) - \mathbf{y}_i)\right)$$

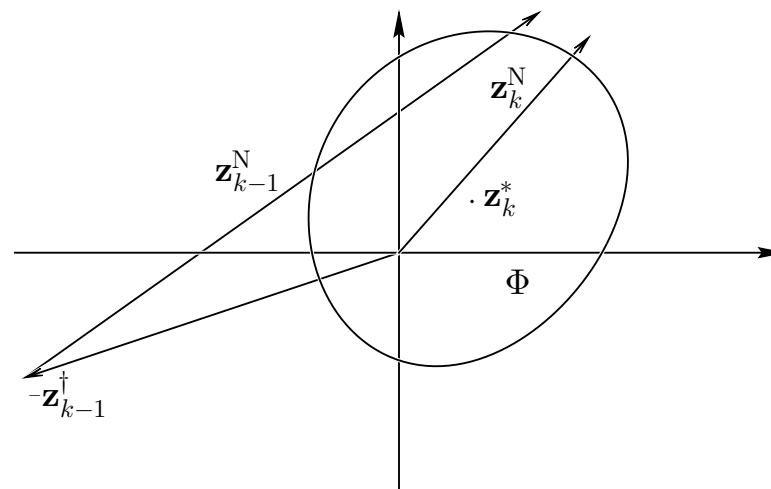
A second-order approximation to the function v about $\mathbf{z} = \mathbf{0}$ with the constant term excluded

$$q(\mathbf{z}) = \mathbf{a}^T \mathbf{z} + \frac{1}{2} \mathbf{z}^T \mathbf{H} \mathbf{z}$$

Trust region subproblem

$$\min_{\mathbf{z} \in \Phi} \mathbf{a}^T \mathbf{z} + \frac{1}{2} \mathbf{z}^T \mathbf{H} \mathbf{z}$$

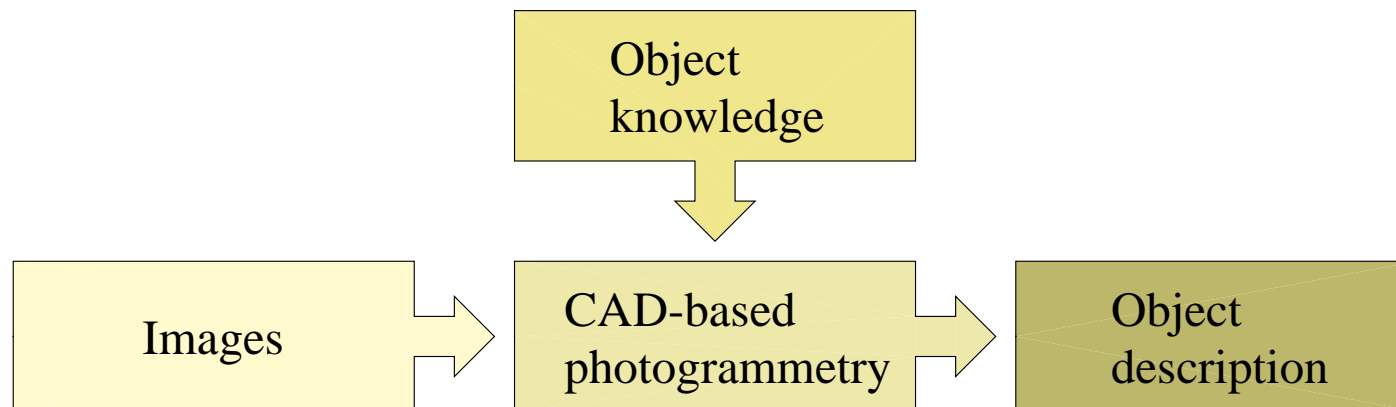
Region Φ derived from weighted point-to-point distances

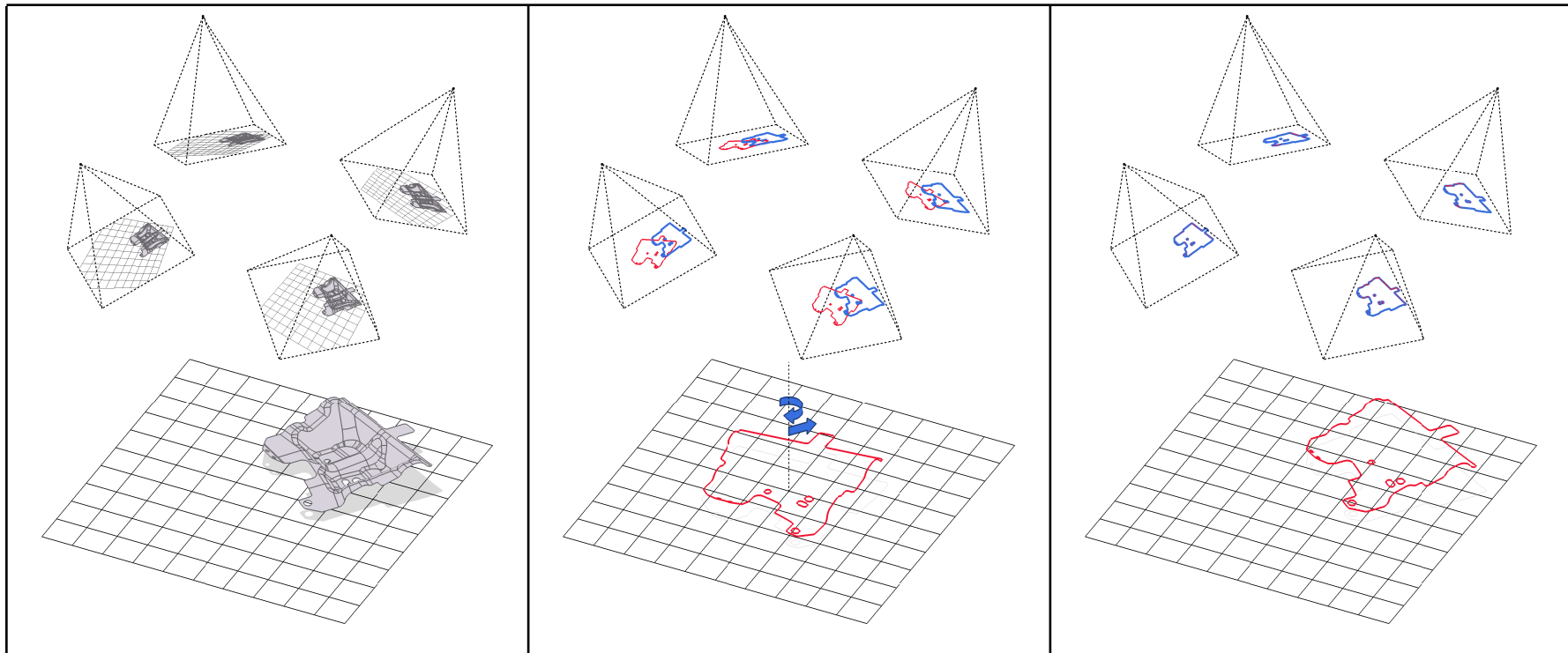


- Bergström, P. & Edlund, O., “Robust registration of surfaces using a refined iterative closest point algorithm with a trust region approach”, Numerical Algorithms (2016), doi:10.1007/s11075-016-0170-3

Future work

- CAD-based photogrammetry
 - Nominal shape is known
 - Faster identification of homologous points
 - High precision edge detection





Questions?

