Pre-Test 2: M0030M - Integral Calculus.

Test your knowledge on Integral Calculus for the course M0030M by solving the problems in this test.

It should not take you longer than 90 minutes.

Problem 1:

Evaluate the following definite integral, if possible,

$$\int_1^e \frac{1}{x} \cos\left(\frac{\pi}{2} \ln x\right) \, dx.$$

Svar 1/*Answer 1*:

A: $1/\pi$, B: $2/\pi$, C: 1, D: 1/2,

E: Inget av de givna svaren. / *None of the given answers.*

Problem 2:

Evaluate the average value of the function

$$f(x) = \frac{2x+4}{x^3+2x^2+2x}$$

on $[1,\ 2]$.

Svar 2/*Answer 2*:

A: $\ln(2)$, B: 3/2, C: $\ln(5)$, D: 1/2,

E: Inget av de givna svaren. / *None of the given answers.*

Problem 3:

Evaluate the area under the curve

$$y = \frac{x^2}{\sqrt{1-x^2}}$$

and above the x-axis between $x = 0$ and $x = 1$.

Svar 3/*Answer 3*:

A: 1 B: 1/2, C: $\pi/4$, D: The area is not finite.

E: Inget av de givna svaren. / *None of the given answers.*

Problem 4:

Let

$$I_n = \int \frac{1}{(x^2+a^2)^n}\,dx.$$

Then the following reduction formula is valid:

$$I_n = \frac{1}{2a^2(n-1)}\left[\frac{x}{(x^2+a^2)^{n-1}} + (2n-3)I_{n-1}\right], \qquad n = 2,\ 3,\ 4,\ \ldots.$$

Evaluate the definite integral

$$\int_0^1 \frac{1}{(x^2+1)^2}\,dx$$

by using the above reduction formula.

Svar 4/*Answer 4*:

A: $1/2$, B: $\ln(2)$, C: $\pi/4$, D: $1/4+\pi/8$

E: Inget av de givna svaren. / *None of the given answers.*

Problem 5:

Evaluate the following definite integral, if possible:

$$\int_0^1 x\ln(x)\,dx.$$

Svar 5/*Answer 5*:

A: $-1/2$, B: $-1/4$, C: -1, D: The integral diverges.

E: Inget av de givna svaren. / *None of the given answers.*

Problem 6:

Find the volume of the solid of revolution formed by revolving the plane region bounded by

$$y = x^2 - x^5, \qquad y = 0$$
$$0 \le x \le 1$$

about the y-axis.

Svar 6/*Answer 6*:

A: $3\pi/14$, B: $3\pi/28$, C: $\pi/7$, D: 1

E: Inget av de givna svaren. / *None of the given answers.*

Problem 7: Vilka av följande påståenden är sanna? *Which of the following statements are true?*

I: The arc length s of the curve $y = e^x \cos x$ in the interval $0 \le x \le \pi/2$ can be calculated by the following definite integral:

$$s = \int_0^{\pi/2} \left[1 + e^{2x}\left(1 - 2\cos x \sin x\right)\right]^{1/2} dx.$$

II: Consider the curve $x = e^y + y$ on the interval $0 \le y \le 2$ and rotate this curve about the x-axis to create a surface area S of revolution. The area S can be calculated by the following definite integral:

$$S = 2\pi \int_0^2 y\left[2 + 2e^y + e^{2y}\right]^{1/2} dy.$$

III: $\dfrac{d}{dx}\left(x \displaystyle\int_{x^2}^3 e^{-t^2}\, dt\right) = \displaystyle\int_{x^2}^3 e^{-t^2}\, dt - 2x^2 e^{-x^4}.$

IV: $\displaystyle\int_{-1}^1 \frac{1}{x}\, dx = 0.$

Svar 7/*Answer 7*:

A: Alla påståenden är sanna. *All statements are true.*

B: Bara **I**, **II** och **III** är sanna. *Only* **I**, **II** *and* **III** *are true.*

C: Bara **I**, **III** och **IV** är sanna. *Only* **I**, **III** *and* **IV** *are true.*

D: Bara **II** och **IV** är sanna. *Only* **II** *and* **IV** *are true.*

E: Bara **III** är sanna. *Only* **III** *is true.*

Problem 8:

Assume that $u(x)$ is a function that is twice differentiable for which the second derivative $u''(x)$ is continuous for all $x \in [0, 1]$. Assume further that

$$u(1) = -1, \quad u(0) = 1, \quad u'(1) = -2, \quad u'(0) = 3.$$

Use the above information to evaluate

$$\int_0^1 \left[u(x)u''(x) + (u'(x))^2\right] \, dx.$$

Svar 8/*Answer 8*:

A: 0, B: 1, C: 2, D: -1

E: Inget av de givna svaren. / *None of the given answers.*