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Sp(2) Quantization of Solitonic Theories

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Abstract

The gauge-field theoretical formulation of solitonic theories is quantized by using an extended version of the BRST Sp(2) symmetric formalism. The proposed method is based on a modified triplectic geometry which allows us to incorporate the linear and/or nonlinear global symmetries of the model and to perform the regularization and renormalization stages in a systematic way.

1 Introduction

In this paper, the quantization of solitonic theories is addressed in the BRST framework, starting from their gauge field theoretical formulation obtained by associating spectral parameters with the global degrees of freedom. It was shown [19, 20] (and references therein) that the $(1+1)$ dimensional projection of Chern-Simons gauge interacting theory contains the hidden hierarchy of integrable systems and that the nonlinear Schrödinger (NLS) model integrability follows from a U(1) gauge invariance of the gauged NLS.

On the other hand, we should underline that even the non-solvability of the constraints is treatable only by BRST methods ([2]–[7], [10]) if one aims to quantize the original integrable systems. In this respect, a challenging problem might be solved as a consequence, i.e. establishing the relation between the quantized integrable models and the corresponding quantized gauge theories.

A typical model which may be studied in this framework is described by the following Lagrangian [18] obtained by dimensional reduction from the $(2+1)$ dimensional nonlinear Schrödinger model interacting with a Chern–Simons gauge field:

$$\begin{aligned} L_{1+1} = {} & \frac{1}{2k} B\varepsilon^{\mu\nu} F_{\mu\nu} + i\hbar\psi^* \left(\frac{\partial}{\partial t} + iA_0\right)\psi \\ & - \frac{\hbar^2}{2m}\left|\left(\frac{\partial}{\partial x} + iA_x\right)\psi\right|^2 - \frac{mc^2}{2\hbar^2} B^2\rho \pm \frac{1}{\hbar}\frac{\partial B}{\partial t}\frac{\partial B}{\partial x}. \end{aligned} \tag{1}$$

The following crucial problems may be identified whenever one tries to apply the BRST Sp(2) formalism ([1, 9, 15]), which incorporates both the BRST and the anti-BRST symmetries, in order to perform the quantization, regularization and renormalization stages, by taking into account higher perturbative orders:

i) treating theories having both local and global linear and/or non-linear invariances;

ii) choosing the regularization imposed by the "delta"-operators problems ([8, 11, 14]) such that to keep the advantages of the Sp(2) symmetric formalism;

iii) the trivial and genuine anomalies calculation and the cohomological problems associated to the higher order expansions.

The method which eliminates these difficulties is based on the extended version of the Sp(2) quantization proposed in [16, 17] which will be given a geometrical interpretation in what follows, enhancing the role of the global symmetries and enlightening the advantages of the triplectic geometry.

2 Extended triplectic quantization

The starting point of our ψ^*-extended technique is the path integral of the gauge field theory:

$$Z = \int \left[\exp \left\{ \frac{i}{\hbar} \left(\widetilde{S} + \widetilde{X} \right) \right\} \right]_0 \rho\left(z\right) [dz] [d\psi] [d\psi^*] \tag{2}$$

after the complete set of fields and antifields is defined, according to the local and global symmetries of the model.

In the previous expression, we assume that:

i) $[F]_0$ signifies the function $F\left(\psi\right)$ in the expression $\widetilde{F} = F\left(\psi\right) + F^{Aa}\psi^*_{Aa} + \overline{F}^A \overline{\psi}_A + \cdots$ which contains multi-vectors;

ii) the particular hipergauge in [1, 9, 15] is avoided, the parametric variables imposed by the gauge-fixing process $\psi^*_{\alpha a}$, $\overline{\psi}_\alpha$ being kept manifest through the regularization process;

iii) $\widetilde{S} = S + \psi^*_{\alpha a} f^{\alpha a} + \overline{\psi}^\alpha \overline{f}_\alpha$ (extended master action) and $\widetilde{X} = X - \psi^*_{\alpha a} R^{\alpha a} - \overline{\psi}^\alpha \overline{R}_\alpha$ (gauge-fixing master action), where both $f^{\alpha a}$, $\overline{f}_\alpha$ and $R^{\alpha a}$, $\overline{R}_\alpha$ depend on z, ψ, $\hbar$ while S, X are also functions of z, ψ, $\hbar$;

iv) the generalized "coordinates" z are local coordinates (their number being $3M$ if the total number of fields, ghosts, ghosts of ghosts, ... is M) on the antisymplectic manifold $\mathcal{M}$;

v) we introduced the set of constant "ghosts" $\left(\psi^A\right)$ and the corresponding "anti-fields" $(\psi^*_{Aa}$ and $\overline{\psi}^A)$ associated to each representative of a basis of $H^*\left(s_{\text{total}}, M\right)$ over the ring of functions in the coupling constants, while M^* is the space of local functionals in the fields and anti-fields;

vi) the extended anti-brackets $(\cdot,\cdot)^a_e$ and V^a_e -operators are defined in order to include the ψ^A, ψ^*_{Aa} and $\overline{\psi}^A$ by:

$$(A, B)^a_e = (A, B)^a + (A, B)_{\psi, \psi^*_a, \psi}\,, \tag{3}$$

$$V^a_e = V^a + V^a|_{\psi, \psi^*_a, \overline{\psi}}\,. \tag{4}$$

The extended BRST anti BRST transformations acting on functionals $\widetilde{F}$ are given by:

$$\delta^a_{\widetilde{S}} \widetilde{F} = \left(\widetilde{S}, \widetilde{F} \right)^a_e + V^a_e \widetilde{F} \tag{5}$$

allowing us to derive:

$$\left(\delta_{\widetilde{S}} \widetilde{F} \right)^a_{\psi^*=0} = \overline{s}^a_S F + \frac{\partial S}{\partial \psi^\alpha} F^{\alpha a} \tag{6}$$

and:

$$\left(\delta_{\widetilde{S}}\widetilde{F}\right)_{(\psi^*)^2=0} = \widehat{s}_S\left(\psi^*_{\alpha a}F^{\alpha a} + \overline{\psi}^{\alpha}\overline{F}_{\alpha}\right), \tag{7}$$

where we denoted:

$$\overline{s}^a_S F = (S,F)^a + V^a F + (-)^{\alpha+1} f^{\alpha}\frac{\partial F}{\partial \psi^{\alpha}} \tag{8}$$

and respectively:

$$\widehat{s}^a_S = \left(\psi^*_{\alpha b}f^{\alpha b} + \overline{\psi}^{\alpha}\overline{f}_{\alpha}, \cdot\right)^a + V^a_{\psi}. \tag{9}$$

We must underline that if the theory is invariant with respect to the hiper-gauge fixing (non-anomalous theory) then $\left\langle \delta_{\widetilde{S}}\widetilde{F}\right\rangle_{\widetilde{X}}$ does not depend on the specific X, while $\left\langle \delta_{\widetilde{X}}\widetilde{F}\right\rangle = 0$ for any $\widetilde{F}$. The BRST invariant operators are thus the ones that satisfy the condition $\delta_{\widetilde{S}}\widetilde{F} = 0$.

The definition of $\delta_{\widetilde{X}}$, $\overline{s}^a_S$, $\widehat{s}^a_S$ are perfectly similar to the ones given in (6)–(9) and may be obtained by replacing the action S by the gauge-fixing action X, the corresponding terms $f^{\alpha b}$, $\overline{f}_{\alpha}$ by $R^{\alpha a}$, $\overline{R}^{\alpha}$ respectively and taking care to the minus sign of the V^a operator in the case of the X action.

The structure of the terms that does not depend on ψ^* is explicitly given by:

$$\begin{aligned} S &= S_0 + \sum_{m=1} S_{\alpha_1\ldots\alpha_m}\lambda^{\alpha_1}\ldots\lambda^{\alpha_m}, \\ X &= X_0 + \sum_{m=1} X_{\alpha_1\ldots\alpha_m}\lambda^{\alpha_1}\ldots\lambda^{\alpha_m}. \end{aligned} \tag{10}$$

The master equations being:

$$\begin{aligned} &\frac{1}{2}\left(\widetilde{S},\widetilde{S}\right)^a + V^a\widetilde{S} = 0, \\ &\frac{1}{2}\left(\widetilde{X},\widetilde{X}\right)^a - V^a\widetilde{X} = 0 \end{aligned} \tag{11}$$

we derive as a consequence the following equations fulfilled by the ψ^* independent and ψ^* linear terms respectively:

$$\begin{aligned} &\frac{1}{2}(S,S)^a + V^a S + \Lambda^a S = 0, \\ &\frac{1}{2}(X,X)^a - V^a X + \Lambda^a X = 0, \end{aligned} \tag{12}$$

$$\left(\psi^*_{Cc}f^{Cc} + \overline{\psi}^C\overline{f}_C, \psi^*_{Bb}f^{Bb} + \overline{\psi}^D\overline{f}_D\right) + V\left(\overline{\psi}^D\overline{f}_D\right) = 0, \tag{13}$$

$$\left(\psi^*_{Cc}R^{Cc} + \overline{\psi}^C\overline{R}_C, \psi^*_{Bb}R^{Bb} + \overline{\psi}^D\overline{R}_D\right) - V\left(\overline{\psi}^D\overline{R}_D\right) = 0 \tag{14}$$

encoding an entire set of equations for the coefficients $X_{\alpha_1\ldots\alpha_m}, S_{\alpha_1\ldots\alpha_m}$(in (10)) and the generalized Jacobi identities.

One has to remark that the involution problems encountered in the first version of the triplectic quantization [1, 9] are now replaced by sets of equations of the type:

$$(X_0, X_{\alpha_1}) = 0, \tag{15}$$

$$(X_0, X_{\alpha_1\alpha_2}) + \frac{1}{2}(X_{\alpha_1}, X_{\alpha_2})(-)^{\alpha_1(\alpha_2+1)} + X_\beta R^\beta_{\alpha_1\alpha_2} = 0. \tag{16}$$

The correspondence between the solution S of the master equation and the solution of the extended one is now recovered, being now able to establish the connection between the cohomology (using "ngh"number as degree) of the operator $\overline{s} = \overline{s}^1 + \overline{s}^2$, if:

$$\overline{s}^a = (S, \cdot)^a + V^a + \Lambda^a \tag{17}$$

and the quantum $\overline{\sigma}$ operator cohomology. One may straightforwardly check that all the standard properties (see [16]) of the $\overline{\sigma}^a$ operators and V^a are fulfilled by (5) and $\widetilde{V}^a = \pm V^a + \Lambda^a$ respectively.

3 Regularization and renormalization steps

In this section the dimensional regularization is proposed in order to solve the problem of ill-defined quantities and the renormalization stage for the general, anomalous case. The consequences of regularization non-invariances and anomalies are treated in a compact form, keeping the genuine anomalies manifest and the algorithm very systematic.

On this purpose, we start with the anomalous, extended master equations (treated in [16, 17] for the Sp(2) formalism), written for a complete $(\hbar, \tau)$ double-expansion of the master action:

$$\frac{1}{2}(S, S)^a + \widetilde{V}^a S = -i\hbar\overline{\mathcal{A}}^a, \tag{18}$$

where $\overline{\mathcal{A}}^a$ (for $a = 1, 2$) incorporate the effects of: local contributions to the anomaly, the quantum dressings of the non-trivial anomalies in the previous stages and the breakings of the master equation due to the regularization non-invariances (if it is the case), at every order in perturbation theory. The action S and the complex terms $\overline{\mathcal{A}}^a$ are regarded, at each perturbative step, as the ones generated in the previous one, after the divergences substraction.

The following definitions will be used:

$$S = \sum_{p=0} h^p S^{(p)}_{R_{p-1}}, \tag{19}$$

where each term in the expansion may be explicitly given as:

$$S^{(p)}_{R_{p-1}} = \sum_{n=n_p} \tau^n S^{(p)n}_{R_{p-1}} \tag{20}$$

after the $(p-1)$ step $(p \geq 1)$ has been completed by eliminating the divergencies. This necessary step is the one that determines the value of the lower limit n_p of the power series in τ. We denoted by: $S^{(1)}_{R_0} \equiv S^{(1)}$; $S^{(0)n} \equiv S_n$; $n_1 = -1$.

The tower of equations obtained for higher order terms encodes all the contributions we had mentioned:

$$-i\left(\overline{\mathcal{A}}^{a}\right)^{(p)}_{R_{p-1}} = \left(S^{(p)}_{R_{p-1}}, S^{(0)}_{R_{p-1}}\right)^{a} + \widetilde{V}^{a} S^{(p)}_{R_{p-1}} + \sum_{q=1}^{p-1}\left(S^{(p)}_{R_{p-1}}, S^{(p-q)}_{R_{p-1}}\right)^{a}. \tag{21}$$

On the other hand, an important consequence of the "consistency conditions" in [17] is that:

$$\left(\left(\overline{\mathcal{A}}^{\{a}\right)^{(p)}_{R_{p-1}}, S^{(0)}_{R_{p-1}}\right)^{b\}} + \widetilde{V}^{\{a}\left(\overline{\mathcal{A}}^{b\}}\right)^{(p)}_{R_{p-1}} = -\sum_{q=1}^{p-1}\left(\left(\overline{\mathcal{A}}^{\{a}\right)^{(p)}_{R_{p-1}}, S^{(p-q)}_{R_{p-1}}\right)^{b\}} \tag{22}$$

for $p \geq 2$ and plays an important role in the substraction procedure.

All of these equations (12)–(14) have to be written in τ^n, for $n = n_p, \ldots, -1, 0, 1, \ldots$ at each value of p, while the limit $\tau \to 0$ may be taken (removing the regularization) only when this process does not generate any divergencies, i.e. when the terms with poles in τ have been substracted.

The order $p = 0$ in (21), shows us that the starting order of the $\left(\overline{\mathcal{A}}^{a}\right)^{(p)}_{R_{p-1}}$-terms in τ has to be chosen such that to allow us to eliminate the regularization:

$$\left(S^{(0)}, S^{(0)}\right)^{a} + \widetilde{V}^{a} S^{(0)} \equiv \tau\theta^{(0)a}, \tag{23}$$

where we use a notation similar to the one in [14], in order to suggest the direct application of our procedure to the anti-field and antisymplectic regularization formalism. One can easily check that if the regularization is removed in (16), the master equation at classical level is recovered.

The general algorithm may now be detailed in order to derive the well defined expressions of the anomalies and renormalized action. At each stage, $S^{(n)-m}$ and the trivial anomalies may be eliminated by an appropriate $\hbar/\tau$-dependent BRST- anti- BRST change of variables as the ones defined in [6, 9].

The μ_a functions in the corresponding transformations leave a total change in the action equal to $-\frac{\hbar}{\tau}S^{(n)-m} - \hbar a_n$ if $\mu_a = \gamma_a$, $\widetilde{\mu}_a = \psi^*_{Aa}\left(\gamma^A + a^{Aa}\right)$ where γ, γ^A, a_n, a^{Aa} are given by $S^{(n)-m} = \overline{s}^{tot}\gamma + \frac{\partial S}{\partial\psi^A}\gamma^A$ and $A^d_n = \overline{s}^c a_n + \frac{\partial S}{\partial\psi^A}a^{Ad}$ but with $\hbar^2/\tau^2$ contributions that have to be taken into account.

The corresponding action S_{R_1} has for example the following form:

$$S_{R_1} = S^0_{R_1} + \hbar\sum_{n=0}\tau^n S^{(1)n}_{R_1} + \hbar^2\sum_{n=-2}\tau^n S^{(2)n}_{R_1} + O\left(\hbar^3\right) \tag{24}$$

while the new $-i\left(\overline{\mathcal{A}}^{a}\right)^{(2)}_{R1}$ contain two type of terms: $\tau\theta^{(2)a}_{R_1}$ and the renormalization dressing of the non-trivial one loop anomaly $\tau\overline{\theta}^{(1)a}_{R_1}$ due to an auxiliary $\hbar$-multiplication inherited from S_{R_1}.

One may obviously continue the procedure and obtain a completely regularized and substracted S_{R_∞} which fulfills the anomalous equations:

$$\frac{1}{2}\left(S_{R_\infty}, S_{R_\infty}\right)^{a} + \widetilde{V}^{a} S_{R_\infty} = -i\hbar\left(\overline{\mathcal{A}}^{a}\right)_{R_\infty} \tag{25}$$

working only with well defined quantities and were the genuine anomalies are written as:

$$-i\hbar\left(\overline{\mathcal{A}}^{a}\right)_{R_\infty} = \sum_{n=1}\hbar^n\overline{\theta}^{(n)a}_{R_n}. \tag{26}$$

4 Conclusions

The method we proposed here may be applied for higher loop expansions as previously exposed, once one keeps in mind the "extended" picture which works with $S(\psi)$ and defines correctly the changes of variables involved in the substraction steps after the conditions for $S^{(p)-n}_{Rp-1}$ $(n \leq p)$ to be $\overline{s}$-closed and the "consistency conditions" for A^a_p are made explicit.

The algorithm also relates the non-trivial anomalous breakings of the master equation to the cohomology of the total extended BRST differential in $ngh = 1$ and gives us the opportunity to control the anti-bracket algebra of the observables, as in the standard case.

We must emphasize that the triplectic formalism proves to be an appropriate method in treating the gauge-field theories corresponding to solitonic models, such that the involved global linear and/or nonlinear symmetries play a major role in both quantization and renormalization stages.

One can state once more that, in the modern sense of renormalizability given in [11], all theories are renormalizable. However, the cohomological analysis and the specific higher order maps in the triplectic formalism should also be carefully handled even the main conclusions anticipated by us are in direct correspondence with the Sp(2) and standard results.

References

[1] Batalin I A and Marnelius R, hep-th/9809208.

[2] Batalin I A and Vilkovisky G A, *Phys. Lett. B*, 1981, V.102, 27.

[3] Batalin I A and Vilkovisky G A, *Phys. Rev. D*, 1983, V.28, 2567.

[4] Batalin I A, Lavrov P and Tyutin I, *J. Math. Phys.*, 1990, V.31, 1487.

[5] Batalin I A, Lavrov P and Tyutin I, *J. Math. Phys.*, 1990, V.32, 532.

[6] Batalin I A, Lavrov P and Tyutin I, *J. Math. Phys.*, 1990, V.32, 2513.

[7] Henneaux M, *Nucl. Phys. B (Proc. Suppl.)*, 1990, V.18, 47.

[8] Brandt F, Henneaux M and Wilch A, hep-th/9705007.

[9] Batalin I A and Marnelius R, hep-th/9502030, hep-th/9502031.

[10] Damgaard P H, De Jonghe F and Sollacher R, hep-th/9505040.

[11] Gomis J and Weinberg S, *Nucl. Phys. B*, 1996, V.469, 473.

[12] Paris J and Troost W, hep-th/9607215.

[13] Paris J, *Nucl. Phys. B*, 1995, V.450, 357.

[14] Barnich G, *Phys. Lett. B*, 1998, V.419, 211.

[15] Tyutin I V and Shahverdijev S S, hep-th/9804088.

[16] Calian V, *Phys. Lett. B*, 1997, V.405, 95.

[17] Calian V and Stoenescu G, *Phys. Lett. B*, 1999, V.450, 136.

[18] Jackiw R, hep-th/9611185.

[19] Pashaev O K, *Nucl. Phys. B (Proc. Suppl.)*, 1997, V.57, 338.

[20] Lee J-H and Pashaev O K, *J. Math. Phys.*, 1998, V.39, 102.