LULEÅ TEKNISKA UNIVERSITET
Avdelningen för fysik

Tentamen i: Statistisk mekanik och termodynamik 2000-12-14 (MTF072)
Lösningsförslag / Suggested solutions

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1. $Z = 1 + e^{\frac{mB}{\tau}} + e^{-\frac{mB}{\tau}} \approx 1 + 1 + \frac{mB}{\tau} + \frac{1}{2}\left(\frac{mB}{\tau}\right)^2 + 1 - \frac{mB}{\tau} + \frac{1}{2}\left(\frac{mB}{\tau}\right)^2 = 3(1 + \frac{1}{3}\left(\frac{mB}{\tau}\right)^2)$

   $F = -\tau \ln Z = -\tau\left[\ln 3 + \ln(1 + \frac{1}{3}\left(\frac{mB}{\tau}\right)^2)\right] \approx -\tau\left[\ln 3 + \frac{1}{3}\left(\frac{mB}{\tau}\right)^2)\right]$ $\sigma = -\frac{\partial F}{\partial \tau}_V =$

   $\ln 3 - \frac{1}{3}\left(\frac{mB}{\tau}\right)^2)$. The decrease in entropy is $\frac{1}{3}\left(\frac{mB}{\tau}\right)^2)$ and $A = \frac{1}{3}\left(mB\right)^2$

2. Count the states in a box of size L. After some steps one reaches at (eq 7 page 185 KK) $\tau_F = (3\pi^2 n)^{2/3}\frac{\hbar^2}{2m}$ this gives $T_F = k_B \tau_F = 5.0$K which much less than $T = 0.5K$ so the approximation of the Fermi-Dirac distribution function as a step function will be fairly good. Calculate U (along lines according to eq 27 page 189 KK) after some steps $C_v = \frac{1}{2}\pi^2 N k_B T/T_F$ is reached. Evaluating the fermi contribution gives $C_v = 0.49 N k_B = 4.10$ J/mole/K at $T = 0.5$K.

3. a) $\frac{1}{\tau} = \frac{\partial \sigma}{\partial E} = \frac{8\pi G M}{\hbar c^3}$

    b) $U = \frac{3}{2}\tau = 1$ eV will give ratio $\frac{18 e^{13.6/3^2\tau}}{8 e^{13.6/2^2\tau}} = \frac{18}{8}e^{20.4(1/9-1/4)/\tau} = \frac{18}{8}e^{-2.833/\tau} = 0.13$

4. The partition function is $Z_{\text{rot}} = \sum_{j=0}^{\infty}(2j+1)e^{-j(j+1)\frac{\hbar^2}{2I\tau}} \approx 1 + 3e^{-\frac{\hbar^2}{I\tau}} = 1 + 3e^{-x}$ where $x = \frac{\hbar^2}{I\tau} >> 1$. For N identical molecules $Z_{\text{rot}}^{(N)} = \frac{1}{N!}Z_{\text{rot}}^N$ And hence $F_{\text{rot}} = -N\tau \ln Z_{\text{rot}} + \tau \ln N! = -N\tau \ln(1 + 3e^{-x}) + \tau \ln N! \approx -3N\tau e^{-x} + \tau \ln N!$ $\sigma_{\text{rot}} = -\frac{\partial F}{\partial \tau}_V \approx 3N(1+x)e^{-x} + \ln N!$ and $(C_v)_{\text{rot}} = \tau\frac{\partial \sigma}{\partial \tau}_V \approx 3Nx^2 e^{-x} = 3N\left(\frac{\hbar^2}{I\tau}\right)^2 e^{-\frac{\hbar^2}{I\tau}}$

5. The total number of atoms is conserved use this to introduce a parameter α controlling the degree of dissociation. $N_A = N_B = \alpha N_0$ and $N_{AB} = (1 - \alpha)N_0$ where N_0 is the number of molecules AB if nothing is dissociated. The total number of particles is hence $(1 + \alpha)N_0$. Initially there are equal number of the A, B and AB and therefore $\alpha = 0.5$ The partial pressures are 1/3 atm. Ideal gas law $pV = n\tau$ gives constant in law of mass action $K_p = (\frac{1}{3})^1 \cdot (\frac{1}{3})^1 \cdot (\frac{1}{3})^{-1} = \frac{1}{3}$ atm We also know $p_0 = 1.5 N_0 \tau/V = 1$ atm. Now double V. The new pressure will be $p_1 = (1 + \alpha)N_0 \tau/2V = (1 + \alpha)p_0/3$ We also have $K_p = \frac{1}{3} = \frac{p_A p_B}{p_{AB}} = \frac{(p_1 \alpha/(1+\alpha))^2}{p_1(1-\alpha)/(1+\alpha)} = \frac{\alpha^2}{3(1-\alpha)}$ This is an equation for α: $\alpha^2 + \alpha - 1 = 0$ with solution $\alpha = \frac{\sqrt{5}-1}{2} \approx 0.618$ (only solutions in interval (0,1)). This gives $p_1 = (\sqrt{5} + 1)/6 \approx 0.54$atm. and $p_A = p_B = (\sqrt{5} - 1)/6 \approx 0.21$atm. $p_{AB} = (3 - \sqrt{5})/6 \approx 0.13$atm.