LULEÅ TEKNISKA UNIVERSITET
Avdelningen för materialvetenskap

Exam in: Statistical Physics and Thermodynamics 2013-03-28 (F7035T)
Suggested solutions

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1. The general relation for the specific heat is $C_v = \tau\left(\frac{\partial\sigma}{\partial\tau}\right)_v$

   a: in case of the conduction electrons we have $C_v = \gamma\tau$ these two relations combine to give $\gamma\tau = \tau\left(\frac{\partial\sigma}{\partial\tau}\right)_v$ leading to $\frac{\partial\sigma}{\partial\tau} = \gamma = constant.$ and hence integrating to $\sigma \propto \tau +$'new constant' where the 'new constant' is zero as the the entropy is zero at temperature absolute zero. If the temperature increases from $\tau = 200$K to 800 K the entropy σ will increase by a factor **4**.

   b: In the case of the electro magnetic field we have that the energy density is $u \propto \tau^4$ (Stefan–Boltzmann T^4 law) and hence we have for the specific heat $C_v \propto \tau^3$ (Note the similarity to phonons at low temperature the Debye T^3 law). As in a) we arrive at $\frac{\partial\sigma}{\partial\tau} \propto \tau^2$ and hence $\sigma \propto \tau^3$. If the temperature is raised from 500K to 1500K the entropy σ will increase by a factor of $(\frac{2000}{500})^3 = 64$ that is a factor of **64**.

2. a) $\frac{1}{\tau} = \frac{\partial\sigma}{\partial E} = \frac{8\pi GM}{\hbar c^3}$

   b) $U = \frac{3}{2}\tau = 1$ eV will give ratio $\frac{18e^{13.6/3^2\tau}}{8e^{13.6/2^2\tau}} = \frac{18}{8}e^{20.4(1/9-1/4)/\tau} = \frac{18}{8}e^{-2.833/\tau} = 0.13$

3. **a**) $<\nu> = \nu_0(1+ <v_x>/c)$ and $<v_x>$ is equal to zero due to symmetry and hence $<\nu> = \nu_0$.

   b) $\sqrt{<(\nu - <\nu>)^2>} = \frac{\nu_0}{c}\sqrt{<v_x^2>}$ and $<v_x^2> = \frac{\int_{-\infty}^{\infty} v_x^2 e^{-\frac{mv_x^2}{2\tau}}dv_x}{\int_{-\infty}^{\infty} e^{-\frac{mv_x^2}{2\tau}}dv_x} = ... = \frac{\tau}{m}$ and hence $\sqrt{<(\nu - <\nu>)^2>} = \frac{\nu_0}{c}\sqrt{\frac{\tau}{m}}$

4. $Z = 1 + e^{\frac{mB}{\tau}} + e^{-\frac{mB}{\tau}} \approx 1 + 1 + \frac{mB}{\tau} + \frac{1}{2}\left(\frac{mB}{\tau}\right)^2 + 1 - \frac{mB}{\tau} + \frac{1}{2}\left(\frac{mB}{\tau}\right)^2 = 3(1 + \frac{1}{3}\left(\frac{mB}{\tau}\right)^2)$ $F = -\tau\ln Z = -\tau\left[\ln 3 + \ln(1 + \frac{1}{3}\left(\frac{mB}{\tau}\right)^2)\right] \approx -\tau\left[\ln 3 + \frac{1}{3}\left(\frac{mB}{\tau}\right)^2)\right]$ $\sigma = -\frac{\partial F}{\partial\tau}_V = \ln 3 - \frac{1}{3}\left(\frac{mB}{\tau}\right)^2)$. The decrease in entropy is $\frac{1}{3}\left(\frac{mB}{\tau}\right)^2)$ and $A = \frac{1}{3}(mB)^2$

5. Entropin före i delsystem 1 och 2 ges av $\sigma_{1,2} = N\left[\ln\left(\frac{n_{Q_{1,2}}}{n}\right) + \frac{5}{2}\right]$ och entropin efter ges av (vid blandningens temperatur τ_e) $\sigma_e = 2N\left[\ln\left(\frac{n_{Q_e}}{n}\right) + \frac{5}{2}\right]$. Bestäm blandningens τ_e, inre energi ändras ej (inget utbyte med omgivning), $\frac{3}{2}\cdot 2N\cdot\tau_e = \frac{3}{2}\cdot N\cdot(\tau_1 + \tau_2)$ ger att $\tau_e = \frac{\tau_1+\tau_2}{2}$. Då blir entropi ändringen (ökning) $\Delta\sigma = \sigma_e - \sigma_1 - \sigma_2 = N\ln\left(\frac{n_{Q_e}^2 n^2}{n^2 n_{Q_1} n_{Q_2}}\right) = N\frac{3}{2}\ln\left(\frac{\tau_e^2}{\tau_1\tau_2}\right) = N\frac{3}{2}\ln\left(\frac{(\tau_1+\tau_2)^2}{4\tau_1\tau_2}\right)$.