LULEÅ TEKNISKA UNIVERSITET
Avdelningen för materialvetenskap

Exam in: Statistical Physics and Thermodynamics 2013-08-31 (F7035T)
Suggested solutions

---

1. **a**) radius=5cm, area $A = 4\pi r^2$, $M_e = \sigma T^4$ total power $P = AM_e = 4\pi r^2 \sigma T^4 = 4\pi 0.05^2 \ 5.6705 \ 10^{-8} \ 10^{24}\text{W} = 1.781 \ 10^{15}\text{W} \approx 2 \ 10^{15}\text{W}$. **b**) flux at distance a=2km, area of sphere $A_2 = 4\pi a^2$ $F = P/A_2 = 1.781 \ 10^{15}/4\pi 2^2 \ 10^6 = 3.544 \ 10^7 \approx 3.5 \ 10^7 \ \text{W/m}^2$. **c**) Wiens displacement law gives $\lambda_m = 2.8978 \ 10^{-3}/10^6\text{m} \approx 3\text{nm}$

2. The equation of state for an ideal gas is $pV = Nk_BT$ and we have to derive the corresponding equation if the particles interact weakly by a van der Waals interaction.

   From the partition function Z the pressure may be derived according to $P = k_BT \left(\frac{\partial \ln Z}{\partial V}\right)_T = \frac{Nk_BT}{V-bN} - \frac{aN^2}{V^2}$ which can be rearranged to $\left(P + \frac{aN^2}{V^2}\right)(V - bN) = Nk_BT$.

   In a similar way the energy can be expressed as a derivative of the logarithm of the partition function with respect to the temperature. The inner energy U is given by $U = k_BT^2 \left(\frac{\partial \ln Z}{\partial T}\right)_V = N\left(\frac{3k_BT}{2} - \frac{aN}{V}\right)$

3. 2 particles A and B, 3 states with energy 0, ϵ and 3ϵ **a**) Classical

state	0	ϵ	3ϵ	energy
1	AB	-	-	0
2	-	AB	-	2ϵ
3	-	-	AB	6ϵ
4	A	B	-	ϵ
5	B	A	-	ϵ
6	A	-	B	3ϵ
7	B	-	A	3ϵ
8	-	A	B	4ϵ
9	-	B	A	4ϵ

and $Z = 1 + 2e^{-\epsilon/\tau} + e^{-2\epsilon/\tau} + 2e^{-3\epsilon/\tau} + 2e^{-4\epsilon/\tau} + e^{-6\epsilon/\tau}$

b) Bosons

state	0	ϵ	3ϵ	energy
1	AA	-	-	0
2	-	AA	-	2ϵ
3	-	-	AA	6ϵ
4	A	A	-	ϵ
6	A	-	A	3ϵ
8	-	A	A	4ϵ

and $Z = 1 + e^{-\epsilon/\tau} + e^{-2\epsilon/\tau} + e^{-3\epsilon/\tau} + e^{-4\epsilon/\tau} + e^{-6\epsilon/\tau}$

c) Fermions

state	0	ϵ	3ϵ	energy
4	A	A	-	ϵ
6	A	-	A	3ϵ
8	-	A	A	4ϵ

and $Z = e^{-\epsilon/\tau} + e^{-3\epsilon/\tau} + e^{-4\epsilon/\tau}$

4. There are n empty lattice sites these can be choosen in $\binom{N}{n} = \frac{N!}{n!(N-n)!}$ ways. There are n interstitial sites occupied, these can be choosen in $\binom{N}{n} = \frac{N!}{n!(N-n)!}$ ways. Hence there are in total $W(n) = \binom{N}{n}^2$ ways to form as configuration with n atoms at interstitial sites, all with energy $E = n\epsilon$. The entropy $\sigma = \ln(W(n)) = \ln(\frac{N!}{n!(N-n)!})^2 = 2\ln(\frac{N!}{n!(N-n)!}) \approx 2[N\ln N - n\ln n - (N-n)\ln(N-n)]$ Use def of temperature: $\frac{1}{\tau} = \frac{\partial\sigma}{\partial E} = \frac{\partial\sigma}{\partial n}\frac{dn}{dE} = \frac{1}{\epsilon}2[-\ln n + \ln(N-n) = \frac{2}{\epsilon}\ln\frac{N-n}{n}$. This gives $\frac{n}{N} = \frac{1}{e^{\epsilon/2\tau}+1} \approx e^{-\epsilon/2\tau}$ (if $\epsilon >> \tau$).

5. In a 3D Ising model on a cubic lattice each spin has 6 nearest neighbours, and in a Mean Field assumption these 6 nearest neighbours will have the magnitude $< m >$ and the spin we perform our calculation on will take the following values $s_i = \pm 1$.. This gives $Z = \sum_i e^{-\epsilon_i/\tau} = e^{+6m/\tau} + e^{-6m/\tau}$ and the magnetisation will be $< m >= \frac{+1e^{+6m/\tau} - 1e^{-6m/\tau}}{e^{+6m/\tau} + e^{-6m/\tau}} = \tanh(\frac{6m}{\tau})$. As m s small we may approximate $\tanh x \approx x - x^3/3$. This gives $m = \frac{6m}{\tau} - (\frac{6m}{\tau})^3\frac{1}{3}$ the equation $m^2 = \frac{3\tau^2}{216}(6-\tau)$ a solution close to $m = 0$ as is $\tau \approx \tau_c$: $m^2 = \frac{3\tau_c^2}{216}(6-\tau)$ and $m = \frac{\sqrt{3}\tau_c}{\sqrt{216}}\sqrt{(6-\tau)}$ and we arrive at the exponent $\beta = \frac{1}{2}$.