LULEÅ TEKNISKA UNIVERSITET
Avdelningen för materialvetenskap

Exam in: STATISTICAL PHYSICS AND THERMODYNAMICS 2015-08-29 (F7035T)
Suggested solutions

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1. 1. Correct: A. At low densities, the pressure would be less than that predicted by the ideal gas law.
   2. Correct: B. The gas in 100 liters
   3. To calculate this, consider a reversible isothermal expansion. Since the internal energy of an ideal gas only depends on its temperaturer T, it doesn't change in the expansion. Thus $dQ = dW = pdV$ and where the equation of state gives $p = nRT/V$, hence

$$\Delta S = \int \frac{dQ}{T} = \int \frac{nRT}{VT} dV = nR \ln(\frac{V_f}{V_i} = R \ln(10) = 8,3145 \cdot 2.3026 = 19.145 \approx 19.1 \text{ J/K} \quad (1)$$

2. See solution to problem 3 chapter 7.

3. Entropin före i delsystem 1 och 2 ges av $\sigma_{1,2} = N \left[\ln \left(\frac{n_{Q_{1,2}}}{n} \right) + \frac{5}{2} \right]$ och entropin efter ges av (vid blandningens temperatur τ_e) $\sigma_e = 2N \left[\ln \left(\frac{n_{Q_e}}{n} \right) + \frac{5}{2} \right]$. Bestäm blandningens τ_e, inre energi ändras ej (inget utbyte med omgivning), $\frac{3}{2} \cdot 2N \cdot \tau_e = \frac{3}{2} \cdot N \cdot (\tau_1 + \tau_2)$ ger att $\tau_e = \frac{\tau_1 + \tau_2}{2}$. Då blir entropi ändringen (ökning) $\Delta\sigma = \sigma_e - \sigma_1 - \sigma_2 = N \ln \left(\frac{n_{Q_e}^2 n^2}{n^2 n_{Q_1} n_{Q_2}} \right) = N \frac{3}{2} \ln \left(\frac{\tau_e^2}{\tau_1 \tau_2} \right) = N \frac{3}{2} \ln \left(\frac{(\tau_1 + \tau_2)^2}{4 \tau_1 \tau_2} \right)$.

4. From the energy $\epsilon(j) = j(j+1)\epsilon_0$ and degeneration $g(j) = 2j + 1$ we arrive at the partition function $Z_R(\tau) = \sum e^{-\epsilon_i/\tau} = \sum_{j=0}^{\infty} (2j+1) e^{-j(j+1)\epsilon_0/\tau}$. The high temperature limit $\tau >> \epsilon_0$ this becomes $Z_R(\tau) \approx \int_0^\infty (2j+1) e^{-j(j+1)\epsilon_0/\tau}$ dj, a change of variables ($j(j+1) = x^2$ and d$j(2j+1) = 2x$ dx), $= \int_0^\infty 2x e^{-x^2 \epsilon_0/\tau}$ d$x = [-\frac{\tau}{\epsilon_0} e^{-x^2\epsilon_0/\tau}]_0^\infty = \frac{\tau}{\epsilon_0}$

   The specific heat C_v, (high temperature) The Free energy $F = -\tau \ln Z_R \approx = -\tau \ln \frac{\tau}{\epsilon_0}$, entropy $\sigma = -\frac{\partial F}{\partial \tau} \approx \ln \frac{\tau}{\epsilon_0} + 1$, and $C_v = \tau \frac{\partial \sigma}{\partial \tau} \approx 1$. ie. $C_v = 1 k_B$ per molecule in the high temperature limit.

   For low temperatures, $\tau << \epsilon_0$ we get $Z_R(\tau) \approx 1 + 3e^{-2\epsilon_0/\tau}$ and $F = -\tau \ln(1 + 3e^{-2\epsilon_0/\tau})$ and $\sigma = -\frac{\partial F}{\partial \tau} = \ln(1 + 3e^{-2\epsilon_0/\tau}) + \tau \frac{1}{1+3e^{-2\epsilon_0/\tau}} \cdot \frac{6\epsilon_0}{\tau^2} e^{-2\epsilon_0/\tau}$. This leads to $C_v = \tau \frac{\partial \sigma}{\partial \tau} = ... = \frac{18\epsilon_0^2 e^{-2\epsilon_0/\tau}}{\tau^2(1+3e^{-2\epsilon_0/\tau})} \left[1 - \frac{e^{-2\epsilon_0/\tau}}{1+3e^{-2\epsilon_0/\tau}} \right]$ approximate $\frac{1}{1+x} \approx 1 - x$ as x is small this gives $C_v \approx \frac{18\epsilon_0^2 e^{-2\epsilon_0/\tau}}{\tau^2}(1-$

$3e^{-2\epsilon_0/\tau})(1 - e^{-2\epsilon_0/\tau}(1 - 3e^{-2\epsilon_0/\tau})) \approx \frac{18\epsilon_0^2 e^{-2\epsilon_0/\tau}}{\tau^2}$. Answer $\tau >> \epsilon_0 : C_v = 1$, and $Z_R(\tau) = \frac{\tau}{\epsilon_0}$ and for $\tau << \epsilon_0 : C_v = \frac{18\epsilon_0^2 e^{-2\epsilon_0/\tau}}{\tau^2}$, and $Z_R(\tau) = 1 + 3e^{-2\epsilon_0/\tau}$

5. The general relation for the specific heat is $C_v = \tau \left(\frac{\partial\sigma}{\partial\tau}\right)_v$

   a: in case of the conduction electrons we have $C_v = \gamma\tau$ these two relations combine to give $\gamma\tau = \tau\left(\frac{\partial\sigma}{\partial\tau}\right)_v$ leading to $\frac{\partial\sigma}{\partial\tau} = \gamma = constant.$ and hence integrating to $\sigma \propto \tau +$'new constant' where the 'new constant' is zero as the the entropy is zero at temperature absolute zero. If the temperature increases from $\tau = 300$K to 800 K the entropy σ will increase by a factor of $\mathbf{\frac{8}{3}} \approx \mathbf{2.67}$.

   b: In the case of the electro magnetic field we have that the energy density is $u \propto \tau^4$ (Stefan–Boltzmann T^4 law) and hence we have for the specific heat $C_v \propto \tau^3$ (Note the similarity to phonons at low temperature the Debye T^3 law). As in a) we arrive at $\frac{\partial\sigma}{\partial\tau} \propto \tau^2$ and hence $\sigma \propto \tau^3$. If the temperature is raised from 500K to 1200K the entropy σ will increase by a factor of $(\frac{1200}{500})^3 = 13.824$ that is a factor of **13.8**.