

1. 1. Correct: A. At low densities, the pressure would be less than that predicted by the ideal gas law.
2. Correct: B. The gas in 100 liters
3. To calculate this, consider a reversible isothermal expansion. Since the internal energy of an ideal gas only depends on its temperature T , it doesn't change in the expansion. Thus $dQ = dW = pdV$ and where the equation of state gives $p = nRT/V$, hence

$$\Delta S = \int \frac{dQ}{T} = \int \frac{nRT}{VT} dV = nR \ln\left(\frac{V_f}{V_i}\right) = R \ln(10) = 8,3145 \cdot 2.3026 = 19.145 \approx 19.1 \text{ J/K} \quad (1)$$

2. From the energy $\epsilon(j) = j(j+1)\epsilon_0$ and degeneration $g(j) = 2j+1$ we arrive at the partition function $Z_R(\tau) = \sum e^{-\epsilon_i/\tau} = \sum_{j=0}^{\infty} (2j+1)e^{-j(j+1)\epsilon_0/\tau}$. The high temperature limit $\tau \gg \epsilon_0$ this becomes $Z_R(\tau) \approx \int_0^{\infty} (2j+1)e^{-j(j+1)\epsilon_0/\tau} dj$, a change of variables $(j(j+1) = x^2$ and $dj(2j+1) = 2x dx$), $= \int_0^{\infty} 2xe^{-x^2\epsilon_0/\tau} dx = [-\frac{\tau}{\epsilon_0} e^{-x^2\epsilon_0/\tau}]_0^{\infty} = \frac{\tau}{\epsilon_0}$

The specific heat C_v , (high temperature) The Free energy $F = -\tau \ln Z_R \approx -\tau \ln \frac{\tau}{\epsilon_0}$, entropy $\sigma = -\frac{\partial F}{\partial \tau} \approx \ln \frac{\tau}{\epsilon_0} + 1$, and $C_v = \tau \frac{\partial \sigma}{\partial \tau} \approx 1$. ie. $C_v = 1k_B$ per molecule in the high temperature limit.

For low temperatures, $\tau \ll \epsilon_0$ we get $Z_R(\tau) \approx 1 + 3e^{-2\epsilon_0/\tau}$ and $F = -\tau \ln(1 + 3e^{-2\epsilon_0/\tau})$ and $\sigma = -\frac{\partial F}{\partial \tau} = \ln(1 + 3e^{-2\epsilon_0/\tau}) + \tau \frac{1}{1+3e^{-2\epsilon_0/\tau}} \cdot \frac{6\epsilon_0}{\tau^2} e^{-2\epsilon_0/\tau}$. This leads to $C_v = \tau \frac{\partial \sigma}{\partial \tau} = \dots = \frac{18\epsilon_0^2 e^{-2\epsilon_0/\tau}}{\tau^2(1+3e^{-2\epsilon_0/\tau})} \left[1 - \frac{e^{-2\epsilon_0/\tau}}{1+3e^{-2\epsilon_0/\tau}}\right]$ approximate $\frac{1}{1+x} \approx 1-x$ as x is small this gives $C_v \approx \frac{18\epsilon_0^2 e^{-2\epsilon_0/\tau}}{\tau^2} (1 - 3e^{-2\epsilon_0/\tau})(1 - e^{-2\epsilon_0/\tau}(1 - 3e^{-2\epsilon_0/\tau})) \approx \frac{18\epsilon_0^2 e^{-2\epsilon_0/\tau}}{\tau^2}$. Answer $\tau \gg \epsilon_0 : C_v = 1$, and $Z_R(\tau) = \frac{\tau}{\epsilon_0}$ and for $\tau \ll \epsilon_0 : C_v = \frac{18\epsilon_0^2 e^{-2\epsilon_0/\tau}}{\tau^2}$, and $Z_R(\tau) = 1 + 3e^{-2\epsilon_0/\tau}$

3. There are five different states of hemoglobin depending on the number of bound oxygen molecules 0, 1, 2, 3, or 4. For each of these the number of different ways to attach the oxygen molecules are 1, 4, 6, 4 and 1. The chemical activity of O_2 is $\lambda = e^{\mu/\tau}$, where ϵ is the energy of bound O_2 . The Grand partition sum is $Z = 1 + 4\lambda e^{-\epsilon/\tau} + 6\lambda^2 e^{-2\epsilon/\tau} + 4\lambda^3 e^{-3\epsilon/\tau} + \lambda^4 e^{-4\epsilon/\tau}$. The probability for one O_2 molecule is $P(1) = \frac{4\lambda e^{-\epsilon/\tau}}{Z}$ and to have four O_2 molecules is $P(4) = \frac{\lambda^4 e^{-4\epsilon/\tau}}{Z}$.

A graph of $P(1)$ vs λ will start at zero and pass a maximum value and approach zero again as λ increases.

A graph of $P(4)$ vs λ will start at zero and start to increase after the maxima of $P(1)$, $P(2)$ and $P(3)$ have passed to the saturation value 1 as λ increases.

4. Funktions formen för trycket är $\ln p = a - \frac{b}{T}$ används längre ner.

Vid trippelpunkten gäller $p_s = p_l$ detta ger temperaturen för trippelpunkten till $T_t = 195.198\text{K} \approx 195.2\text{K}$. **b:** Clausius–Clapeyron $\frac{dp}{dT} = \frac{L}{T\Delta v}$ där $\Delta v = v_g - v_s \approx v_g$ (antagandet att det är gasens volymitet som dominerar) $= \frac{V_g}{N_g}$. Tunn gas antag $pV_g = N_g k_B T$ och därmed $v_g = \frac{k_B T}{p}$. $\frac{dp}{dT} = \frac{L}{T\Delta v} \approx \frac{L}{Tv_g} = \frac{Lp}{k_B T^2}$. vilket ger $\frac{1}{p} \frac{dp}{dT} = \frac{d}{dT} \ln p = \frac{L}{k_B T^2}$ vidare så är $\frac{d}{dT} \ln p = \frac{b}{T^2}$ dvs $L = bk_B$, där b är $\ln p = a - \frac{b}{T}$.

Latent värme för sublimation $b = 3754\text{K}$. $L_s = 3754 \cdot 1.3807 \cdot 10^{-23} \text{J} = 5.183 \cdot 10^{-20} \text{J} = 0.324\text{eV}$.

Latent värme för förångning $b = 3063\text{K}$. $L_s = 3063 \cdot 1.3807 \cdot 10^{-23} \text{J} = 4.229 \cdot 10^{-20} \text{J} = 0.264\text{eV}$.

Skillnaden mellan förångning och sublimation ger smältvärme $(5.183 - 4.229) \cdot 10^{-20} \text{J} = 9.54 \cdot 10^{-21} \text{J} = 0.060\text{eV}$

(Koll mot fysikalia smältvärme = $332 \cdot 10^3 \text{J/kg}$ för NH_3 vilket blir $9.389 \cdot 10^{-21} \text{J}$ per molekyl.
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5. a) Energi tätheten ges av (efter en del räknande enligt KK93-94) $\frac{U}{V} = \frac{\pi^2 \tau^4}{15\hbar^3 c^3}$

b+c) $d\sigma = dU/\tau$ ger efter integrering (KK95) $\sigma = \frac{4\pi^2 \tau^3 V}{45\hbar^3 c^3}$ sedan $\frac{p}{\tau} = \left(\frac{\partial \sigma}{\partial V}\right)_U$. Först måste dock τ elimineras ur uttrycket för σ med hjälp av uttrycket för energitätheten enligt uppgift a. $\tau^3 = \left(\frac{U}{V}\right)^{3/4} \left(\frac{15\hbar^3 c^3}{\pi^2}\right)^{3/4}$ vilket ger för entropin $\sigma = \frac{4\pi^2}{45\hbar^3 c^3} (U)^{3/4} \left(\frac{15\hbar^3 c^3}{\pi^2}\right)^{3/4} (V)^{1/4}$. Detta uttryck innehåller inte variablerna U eller V via variabeln τ .

Nu ges trycket av $\frac{p}{\tau} = \left(\frac{\partial \sigma}{\partial V}\right)_U$ vilket ger $\frac{p}{\tau} = \frac{4\pi^2}{45\hbar^3 c^3} (U)^{3/4} \left(\frac{15\hbar^3 c^3}{\pi^2}\right)^{3/4} \frac{1}{4} (V)^{-3/4} =$ byt tillbaka till τ igen $= \frac{4\pi^2}{45\hbar^3 c^3} \left(\frac{15\hbar^3 c^3}{\pi^2}\right)^{3/4} \frac{1}{4} \left(\frac{15\hbar^3 c^3}{\pi^2}\right)^{-3/4} \tau^3 = \frac{\pi^2}{45\hbar^3 c^3} \tau^3$. Vilket ger $p = \frac{\pi^2}{45\hbar^3 c^3} \tau^4$ och med uttrycket för energitätheten ger detta: $pV = \frac{1}{3}U$