

Course code	F7035T
Examination date	2012-03-23
Time	09.00 - 14.00

Examination in: STATISTICAL PHYSICS AND THERMODYNAMICS

Total number of problems: 5

Teacher on duty: Hans Weber

Examiner: Hans Weber

Tel: (49)2088, Room E304

Tel: (49)2088, Room E304

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Allowed aids: Fysikalia, Physics Handbook, Beta, calculator, COLLECTION OF FORMULAE

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Define notations and motivate assumptions and approximations. Present the solutions so that they are easy to follow. Maximum number of point is 15 p. 7.0 points is required to pass the examination. Grades 3: 7.0, 4: 9.5, 5: 12.0

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### 1. Atomic nuclei at low temperatures

A crystal consists of  $N$  atoms consisting of nuclei with integer spin  $S$ . The energy of the nuclei depends on their spin state with respect to an applied field. The possible energy states are given by  $0, \epsilon, 2\epsilon, \dots, S\epsilon$ , where the ground state is not degenerated and the other states all have degeneracy 2.

The crystal is held at a low temperature. Calculate the contribution to the heat capacity from the spin degrees of freedom of the nucleus in the limit  $S\epsilon/\tau \gg 1$ , ( $\tau = k_B T$ ).

(3p)

### 2. Harmonic oscillator

A three dimensional harmonic oscillator has energy levels

$$\epsilon_{n_1, n_2, n_3} = (n_1 + n_2 + n_3 + \frac{3}{2}) \hbar\omega$$

where  $n_1, n_2, n_3$  är are integers from 0 to  $\infty$ .

- At what temperature is the probability for the oscillator to be in a state of energy  $\frac{3}{2}\hbar\omega$  or  $\frac{5}{2}\hbar\omega$  the same?
- How large is this probability ?

(3p)

### 3. Protons in a magnetic field

A proton rich sample is subjected to a homogeneous magnetic field  $B$ . Each proton has spin  $\frac{1}{2}$  and a magnetic moment  $m_\mu$  and can accordingly have two possible energies,  $\epsilon = \pm Bm_\mu$ , corresponding to the two possible spin states.

An applied radio frequency (of frequency  $\nu$ ) induces transitions between the two levels, if the frequency of the field satisfies the condition  $h\nu = 2Bm_\mu$ .

The absorbed power from the radio frequency field is proportional to the difference between the number of protons in the two levels.

Assume the protons are in thermal equilibrium at temperature  $\tau$ .

How does the absorbed power depend on the temperature? Consider particularly high temperatures where  $m_\mu B \ll \tau$  applies, ( $\tau = k_B T$ ).

(3p)

### 4. Gas-solid equilibrium

In a container of volume  $V$  a substance solid phase is in equilibrium with its gas phase. The atoms have a binding energy  $-\epsilon_0$  to the solid phase.

Use the following approximations. The substance is mono atomic and for the gas phase the ideal gas applies. Further the gas phase has volume  $V$  independent of the amount in the solid phase. Also the entropy of the solid phase is negligible.

Let the total number of atoms be  $N = N_s + N_g$  where  $N_s$  and  $N_g$  are the number of atoms in the solid phase and gas phase.

- Express the free energy of the system  $F$ . (Hint  $F = F_s + F_g$ .)
- Minimize  $F$  with respect to  $N_g$  and derive an expression for  $N_g$ .
- Estimate  $\epsilon_0$  for  $H_2O$  using the data in the table. Answer in electronvolts ! (Assume the ideal gas applies to the gas phase of  $H_2O$ . Hint: Note that the range of temperatures is small if you need to make approximations for to the expression of  $p_g$ .)

T ( $^{\circ}C$ )	saturation pressure (kPa)
-2	0.5176
-6	0.3689
-10	0.2602
-14	0.1815
-20	0.1035
-30	0.0381
-40	0.0129

(3p)

## 5. Maxwell velocity distribution

An experiment is designed to measure the Maxwell velocity distribution for a gas of Sodium (Na) atoms at  $T = 300^\circ\text{C}$ . The experimental set up is according to the figure.

In the oven there is a gas of Sodium atoms. At the slit at A the atoms are allowed to leave the oven. The slit at A is usually closed by the rotating drum. But as the slit D in the drum and the slit A line up Sodium atoms are allowed to exit the oven into the rotating drum.

The drum has a diameter  $d = 10.0\text{cm}$  and rotates with an angular velocity  $\omega$  around the axis C.

Determine the angular velocity  $\omega$  required so that Sodium atoms in the beam travelling at the most probable velocity  $v_{mp}$  in the beam will hit the slit D again as they have travelled across the drum. (ie the drum has rotated half a turn)

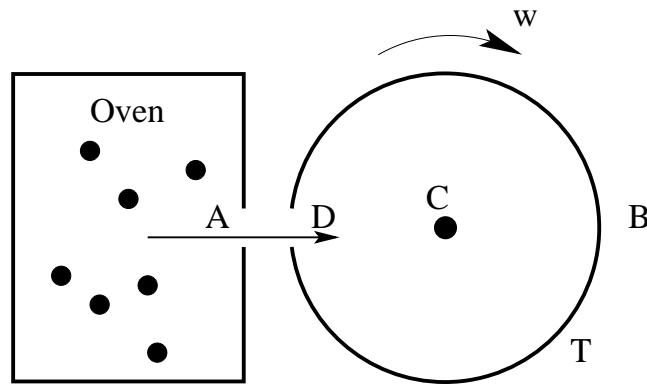


Figure 1: A principal experimental setup to determine the Maxwell velocity distribution.

(3p)

GOOD LUCK !