LULEÅ UNIVERSITY OF TECHNOLOGY
Division of Physics

Course code	F7035T
Examination date Time	2012-05-18 09.00 - 14.00

Examination in: STATISTICAL PHYSICS AND THERMODYNAMICS
Total number of problems: 5
Teacher on duty: Hans Weber — Tel: (49)2088, Room E304
Examiner: Hans Weber — Tel: (49)2088, Room E304

Allowed aids: Fysikalia, Physics Handbook, Beta, calculator, COLLECTION OF FORMULAE

Define notations and motivate assumptions and approximations. Present the solutions so that they are easy to follow. Maximum number of point is 15 p. 7.0 points is required to pass the examination. Grades 3: 7.0, 4: 9.5, 5: 12.0

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1. **van der Waals gas**

   The partition function Z for a gas of N interacting particles is given by

   $$Z = \left(\frac{V - bN}{N}\right)^N \left(\frac{mk_BT}{2\pi\hbar^2}\right)^{\frac{3N}{2}} e^{\frac{aN^2}{Vk_BT}}$$

   where a and b are constants and V is the volume. Derive the equation of state of the gas and also evaluate it's energy U.

   (3p)

2. **Rotation of a di–atomic molecule**

   The kinetic energy of a di–atomic molecule consists of a translational part and a rotational part. The rotational energy $\epsilon(j)$ has quantised levels and for a di–atomic molecule these are given by:

   $$\epsilon(j) = j(j+1)\epsilon_0$$

   where j is an integer with the following values $j = 0, 1, 2, ...$ The degeneracy $g(j)$ of each level is given by:

   $$g(j) = 2j + 1.$$

   a) Calculate the partition function for the rotational degrees of freedom $Z_R(\tau)$.

   b) Approximate $Z_R(\tau)$ in the limit $\tau >> \epsilon_0$ by an integral and calculate the specific heat C_v in this limit.

   c) Approximate $Z_R(\tau)$ in the limit $\tau << \epsilon_0$ by truncating the sum to two terms and calculate the specific heat C_v in this limit.

   d) Draw a figure showing the results from b) and c)

   (3p)

TURN PAGE!

3. **Schottky anomaly**

   A system has two energy levels. The level of higher energy has a two fold degeneracy. The level of lower energy is un degenerate. In a measurement of the heat capacity at constant volume C_v a maximum is found at the temperature $T = 450$K.

   Determine the energy difference (in electron Volts eV) between the two levels. (If you arrive at an equation you cannot solve analytical, solve it grafically or iterate on your pocket calculator)

   (3p)

4. **Freezing of water**

   The latent heat of melting of water is 334 J/g. The density of ice at zero degrees centigrade is 0,9168 g/cm^3. The density of water at zero degrees centigrade is 0,9998 g/cm^3.

   How will the freezing point of water change if the pressure rises from 1,00 to 10,0 atmospheres?

   (3p)

5. **The three dimensional Ising–model in the mean field approximation**

   The three dimensional Ising model on a cubic lattice has the following 'Hamiltonian'

$$H = -J \sum_{<i,j>} s_i s_j,$$

   where the classical spins s have the following states $+1$ and -1. The spins s_i interact with their nearest neighbours. Let $J = 1$ and the system will have a ferro magnetic ground state, ie the magnetisation at temperature $\tau = 0$ is $< m >= \frac{1}{L^3} \sum_i s_i = 1$.

   As the temperature is raised the magnetisation disappears at a specific temperature the Curie temperature τ_c.

   As the temperature approaches τ_c from below the magnetization goes to zero according to $m \propto (\tau_c - \tau)^\beta$. Within the mean field approximation calculate the exponent β for the magentisation.
   ($\tanh(x) \approx x - x^3/3$ for small x).

   (3p)

GOOD LUCK !