## LULEÅ UNIVERSITY OF TECHNOLOGY <br> Division of Physics

| Course code | F7035T |
| :--- | :--- |
| Examination date | $2012-09-01$ |
| Time | $09.00-14.00$ |

Examination in: Statistical Physics and Thermodynamics
Total number of problems: 5
Teacher on duty: Hans Weber
Tel: (49)2088, Room E304
Examiner: Hans Weber
Tel: (49)2088, Room E304
Allowed aids: Fysikalia, Physics Handbook, Beta, calculator, Collection of formulae
Define notations and motivate assumptions and approximations. Present the solutions so that they are easy to follow. Maximum number of point is 15 p .7 .0 points is required to pass the examination. Grades 3: 7.0, 4: 9.5, 5: 12.0

## 1. Harmonic oscillator

A two dimensional harmonic oscillator has energy levels according to

$$
\epsilon_{n_{1}, n_{2}}=\left(n_{1}+n_{2}+1\right) \hbar \omega
$$

where $n_{1}, n_{2}$ are integers $n_{i}=0,1,2,3, \ldots \infty$. The oscillator is coupled to a heatbath of temperature $\tau$ with which the oscillator can exchange energy.
(a) Calculate the partition function of the oscillator for any temperature.
(b) At what temperature equals the probability to find the oscillator in a state of energy $\hbar \omega$ to find it in a state of energy $2 \hbar \omega$ ?
(c) How large is this probability ?
2. Helium ${ }^{3} \mathrm{He}$

Helium ${ }^{3} \mathrm{He}$ has spin $=\frac{1}{2}$ and may at low temperatures to a good approximation be described as an ideal Fermi gas. At these low temperatures ${ }^{3} \mathrm{He}$ is in the liquid phase with a density of $\rho=83 \mathrm{~kg} \mathrm{~m}^{-3}$.
a) Determine the Fermi temperature $T_{F}$ and also the specific heat $C_{v}$ of ${ }^{3} \mathrm{He}$ at $\mathrm{T}=0.2 \mathrm{~K}$.
b) Can you still use the approximations you did in a) if the temperature where say $2-3 \mathrm{~K}$ ? If not why? If yes why?

## 3. Binding of $\mathrm{O}_{2}$ to hemoglobin

A hemoglobin molecule can bind four $\mathrm{O}_{2}$ molecules. Assume $\epsilon$ is the energy of each bound $\mathrm{O}_{2}$, relative to $\mathrm{O}_{2}$ at rest at infinite distance. Let $\lambda$ denote the absolute activity $e^{\mu / \tau}$ of free $\mathrm{O}_{2}$ (in solution).
(a) What is the probability that one and only one $\mathrm{O}_{2}$ is adsorbed on a hemoglobin molecule?
(b) What is the probability that four $\mathrm{O}_{2}$ are adsorbed on a hemoglobin molecule?
(c) Make a sketch of these probabilities as a function of $\lambda$.

## 4. Quantum mechanical rotor

A quantum mechanical rotor (molecule) has energy levels $\epsilon_{j}=j(j+1) \hbar^{2} / 2 I$ where $I$ is the moment of inertia, each level has degeneration $g(j)=2 j+1$ where $j=0,1,2, \ldots$. Calculate the for the rotational degrees of freedom the contribution to the heat capacity for low temperatures $\left(\tau \ll \hbar^{2} / I\right)$. Is the behaviour of exponential or algebraic character at low temperatures?

## 5. Ideal mono atomic gas

An ideal mon atomic gas confined in a box. The box is devided into two sub parts (compartment 1 and 2) according to the figure below. For compartment one 1 we have volume $V_{1}=2 V$, number of particles $N_{1}=N$ and temperature $\tau$. For compartment two 2 we have volume $V_{2}=V$, number of particles $N_{2}=N$ and temperature $\tau$.
Calculate the change of entropy as the wall between compartment 1 and 2 is removed.
The temperature $\tau$ is kept constant.

|  |  |
| :--- | :--- |
| $\mathrm{N}_{1} \mathrm{~V}_{1}$ | $\mathrm{~N}_{2} \mathrm{~V}_{2}$ |
| $\tau$ | $\tau$ |
|  |  |

