LULEÅ UNIVERSITY OF TECHNOLOGY
Division of Physics

| Course code | F7035T |
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| Examination date<br>Time | 2013-08-31<br>09.00 - 14.00 |

Examination in: STATISTICAL PHYSICS AND THERMODYNAMICS
Total number of problems: 5
Teacher on duty: Hans Weber Tel: (49)2088, Room E304
Examiner: Hans Weber Tel: (49)2088, Room E304

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Allowed aids: Fysikalia, Physics Handbook, Beta, calculator, COLLECTION OF FORMULAE

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Define notations and motivate assumptions and approximations. Present the solutions so that they are easy to follow. Maximum number of point is 15 p. 7.0 points is required to pass the examination. Grades 3: 7.0, 4: 9.5, 5: 12.0

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1. **Photons**

   At a blast of an atomic bomb temperatures in the range of $10^6$K can be reached. Assume this is true for a 'fire' ball of diameter $d = 10$cm. Evaluate the following:

   **a)** The total emitted power from this 'fire' ball.

   **b)** What is the radiation flux at a distance of 2.0 km?

   **c)** At what wavelength $\lambda$ peaks the output of power?

   (3p)

2. **van der Waals gas**

   The partition function $Z$ for a gas of $N$ interacting particles is given by

$$Z = \left(\frac{V - bN}{N}\right)^N \left(\frac{mk_BT}{2\pi\hbar^2}\right)^{\frac{3N}{2}} e^{\frac{aN^2}{Vk_BT}}$$

   where $a$ and $b$ are constants and $V$ is the volume. Derive the equation of state of the gas and also evaluate it's energy $U$.

   (3p)

TURN PAGE!

3. **Identical particles**

A system consists of two particles. Each particle can be in one of the following three states with the energies: 0, $\epsilon$ and $3\epsilon$. The system is coupled to a heat reservoir of temperature $\tau$.

**a)** Evaluate an expression for the partition function $Z$ if we consider the particles to be classical (ie we can label the particles as 1 and 2).

**b)** What will $Z$ if the particles are bosons?

**c)** What will $Z$ if the particles are fermions?

(3p)

4. **Interstitial atoms**

The atoms in a crystal of a monoatomic substance can be assumed to sit in either their original lattice positions or in so called interstitial positions. Atoms sitting at a interstitial position have a higher energy compared to if they had been at an ordinary site. The difference in energy is denoted by $\epsilon$. The crystal has $N$ atoms, $N$ lattice sites and $N$ interstitial positions. At a temperature $\tau$, $n$ interstitial sites are occupied by atoms.
Calculate the fraction $n/N$ if $\tau << \epsilon$ and $N$ and $n >> 1$ .
(use the approximation $\ln n! = n \ln n - n$)

(3p)

5. **The three dimensional Ising–model in the mean field approximation**

The three dimensional Ising model on a cubic lattice has the following 'Hamiltonian'

$$H = -J \sum_{<i,j>} s_i s_j,$$

where the classical spins $s$ have the following states $+1$ and $-1$. The spins $s_i$ interact with their nearest neighbours. Let $J = 1$ and the system will have a ferro magnetic ground state, ie the magnetisation at temperature $\tau = 0$ is $< m >= \frac{1}{L^3} \sum_i s_i = 1$.

As the temperature is raised the magnetisation disappears at a specific temperature the Curie temperature $\tau_c$.

As the temperature approaches $\tau_c$ from below the magnetization goes to zero according to $m \propto (\tau_c - \tau)^\beta$. Within the mean field approximation calculate the exponent $\beta$ for the magentisation.
($\tanh(x) \approx x - x^3/3$ for small $x$).

(3p)

GOOD LUCK !