Allowed aids: Fysikalia, Physics Handbook, Beta, calculator, COLLECTION OF FORMULAE

Define notations and motivate assumptions and approximations. Present the solutions so that they are easy to follow. Maximum number of point is 15 p. 7.0 points is required to pass the examination. Grades 3: 7.0, 4: 9.5, 5: 12.0

1. **Diatomic molecule**

   An ideal gas consists of N identical molecules. Each molecule consists of two atoms with the following rotational energy levels: $E(j) = j(j+1)\frac{\hbar^2}{2I}$, $j = 0, 1, 2, ...$ Where I is the moment of inertia. Each level is $(2j+1)$ times degenerate. For low temperatures determine to lowest order in temperature the contribution to C_v from the rotational degrees of freedom.   (3p)

2. **Pressure and energy of a gas of Photons**

   In this problem you are asked to derive an expression for the pressure of a photon gas confined to a cube with the side L.

   a) Start by deriving an expression for the energy density $\frac{U}{V}$ (Stefan–Boltzmanns T^4 law). (Hint use the Planck distribution and that the relation between the frequency of the photon ω_n and the mode $n = \sqrt{n_x^2 + n_y^2 + n_z^2}$ is given by $\omega_n = n\pi c/L$.)

   b) Derive the expression for the pressure p of the photon gas.
   (Hint use the entropy σ.)

   c) The energy of a mono atomic gas is given by $pV = \frac{2}{3}U$. Derive the corresponding expression for the photon gas.

   (3p)

TURN PAGE!

3. **Entropy**

Use simple arguments concerning the specific heat C_v to answer the following questions.

a) For a metal the temperature is changed from $300K$ to $800K$. By how large a factor will the entropy change for the conduction electrons?

b) For the electromagnetic radiation inside a cavity the temperature is changed from $500K$ to $1200K$. By how large a factor will the entropy change for the radiation field inside the cavity?

(3p)

4. **Maxwell velocity distribution**

An experiment is designed to measure the Maxwell velocity distribution for a gas of Sodium (Na) atoms at $T = 280^oC$. The experimental set up is according to figure 1.

In the oven there is a gas of Sodium atoms. At the slit at A the atoms are allowed to leave the oven. The slit at A is usually closed by the rotating drum. But as the slit D in the drum and the slit A line up Sodium atoms are allowed to exit the oven into the rotating drum.

The drum has a diameter $d = 10.0$cm and rotates with an angular velocity ω around the axis C.

Determine the angular velocity ω required so that Sodium atoms in the beam travelling at the most probable velocity v_{mp} in the beam will hit the slit D again as they have travelled across the drum. (ie the drum has rotated half a turn) (3p)
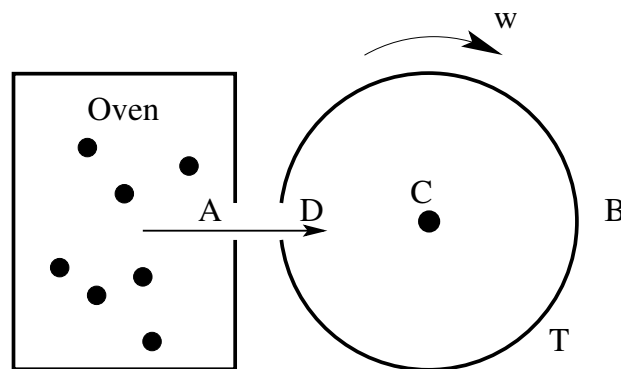

Figure 1: A principal experimental setup to determine the Maxwell velocity distribution.

5. The DNA – molecule

The DNA–molecule consists, as we know of two chains (see figure 2) which are held together by bonds between the base pairs adenine and thymine (AT - bonds) and cytosine and guanine (CG - bonds). This means that the DNA – molecule can function much like a zipper and opened by the bindings sequentially broken from one end or from both ends. For a DNA – molecule containing N bonds, the energy cost to break a bond is ϵ, and a broken bond can have g different orientations.

a) Show that the partition sum for a DNA–zipper that only can be opened from one of its ends is given by:

$$Z^{(N)} = \sum_{N_b=0}^{N} g^{N_b} e^{-N_b \epsilon/\tau},$$

where N_b is the number of broken bonds.

b) Evaluate explicitly and show that the fraction of broken bonds as $N \to \infty$ is 0 if $ge^{-\epsilon/\tau} < 1$ and 1 if $ge^{-\epsilon/\tau} > 1$.

(3p)

This is an example of a *first–order phase transition* of the same kind as when water changes from a liquid to a gas. The phase transition occurs at the temperature $T_c = \frac{\epsilon}{k_B \ln(g)}$. The phase transition from a closed to open an DNA–zipper and vice versa occurs not because of body temperature, but because the bonding energy ϵ and hence T_c is changed due to catalytic influence on ϵ, which gives the same result.
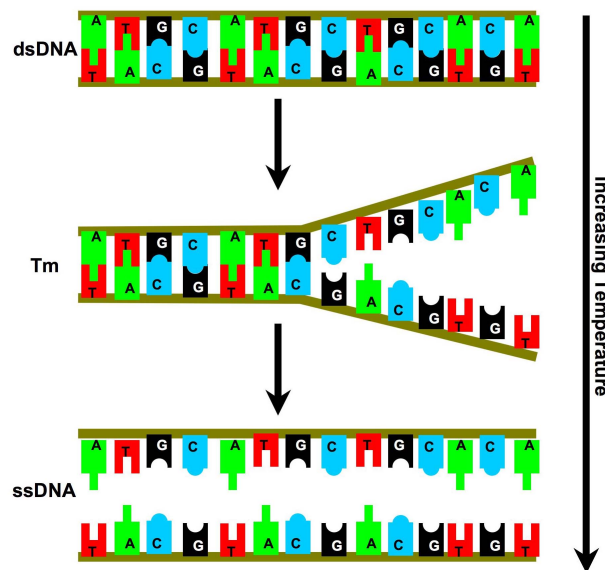

Figure 2: How DNA melts, taken from http://www.patentlens.net/daisy/RiceGenome/3663/3612.html
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GOOD LUCK !