LULEÅ UNIVERSITY OF TECHNOLOGY<br>Division of Physics

| Course code | F7035T |
| :--- | :--- |
| Examination date | $2014-05-17$ |
| Time | $09.00-14.00$ |

Examination in: Statistical Physics and Thermodynamics
Total number of problems: 5
Teacher on duty: Hans Weber
Tel: (49)2088, Room E304
Examiner: Hans Weber
Tel: (49)2088, Room E304
Allowed aids: Fysikalia, Physics Handbook, Beta, calculator,Collection of formulae
Define notations and motivate assumptions and approximations. Present the solutions so that they are easy to follow. Maximum number of point is 15 p .7 .0 points is required to pass the examination. Grades 3: 7.0, 4: 9.5, 5: 12.0

## 1. Freezing point of water

The latent heat of melting of water is $333.9 \mathrm{~J} / \mathrm{g}$. Ice at zero degrees has a density of 0,9168 $\mathrm{g} / \mathrm{cm}^{3}$, whereas liquid water at zero degrees has a density of $0,9998 \mathrm{~g} / \mathrm{cm}^{3}$.
Determine by how much (amount and direction) the freezing point will change in temperature if the pressure increases from 1.00 atmospheres to 10.0 atmospheres.

## 2. Atomic nuclei at low temperatures

A crystal consists of $N$ atoms consisting of nuclei with integer spin $S$. The energy of the nuclei depends on their spin state with respect to an applied field. The possible energy states are given by $0, \epsilon, 2 \epsilon, \ldots, S \epsilon$, where the ground state is not degenerated and the other states all have degeneracy 2 .
The crystal is held at a low temperature. Calculate the contribution to the heat capacity from the spin degrees of freedom of the nucleus in the limit $S \epsilon / \tau \gg 1,\left(\tau=k_{B} T\right)$.

## 3. Schottky anomaly

A system has two energy levels. The level of higher energy has a two fold degeneracy. The level of lower energy is not degenerated. In a measurement of the heat capacity at constant volume $C_{v}$ a maximum is found at the temperature $T=450 \mathrm{~K}$.
Determine the energy difference (in electron Volts eV ) between the two levels. (If you arrive at an equation you cannot solve analytical, solve it graphically or iterate on your pocket calculator)

## 4. Gas-solid equilibrium

In a container of volume $V$ a substance solid phase is in equilibrium with its gas phase. The atoms have a binding energy $-\epsilon_{0}$ to the solid phase.
Use the following approximations. The substance is mono atomic and for the gas phase the ideal gas applies. Further the gas phase has volume $V$ independent of the amount in the solid phase. Also the entropy of the solid phase is negligible.
Let the total number of atoms be $N=N_{s}+N_{g}$ where $N_{s}$ and $N_{g}$ are the number of atoms in the solid phase and gas phase.
a) Express the free energy of the system $F$. (Hint $F=F_{s}+F_{g}$.)
b) Minimize $F$ with respect to $N_{g}$ and derive an expression for $N_{g}$.
c) Estimate $\epsilon_{0}$ for $\mathrm{H}_{2} \mathrm{O}$ using the data in the table. Answer in electron volts ! (Assume the ideal gas applies to the gas phase of $\mathrm{H}_{2} \mathrm{O}$. Hint: Note that the range of temperatures is small if you need to make approximations for to the expression of $p_{g}$.)

| $\mathrm{T}\left({ }^{\circ} \mathrm{C}\right)$ | saturation pressure $(\mathrm{kPa})$ |
| :--- | :--- |
| -2 | 0.5176 |
| -6 | 0.3689 |
| -10 | 0.2602 |
| -14 | 0.1815 |
| -20 | 0.1035 |
| -30 | 0.0381 |
| -40 | 0.0129 |

## 5. Identical particles

A system consists of two particles. Each particle can be in one of the following three states with the energies: $0, \epsilon$ and $3 \epsilon$. The system is coupled to a heat reservoir of temperature $\tau$.
a) Evaluate an expression for the partition function $Z$ if we consider the particles to be classical (ie we can label the particles as 1 and 2 ).
b) What will $Z$ if the particles are bosons?
c) What will $Z$ if the particles are fermions?

